Problem 1: Spectroscopy Basics

a) How is the unit ppm defined, and why is the spectral dimension often given in ppm?

In an ex-vivo experiment we would like to identify two molecules $a$ and $b$, (T1$_a$ = 1520 ms, T2$_a$ = 390 ms, T1$_b$ = 1615 ms, T2$_b$ = 233 ms) dissolved in oil (T1$_{oil}$ = 360 ms, T2$_{oil}$ = 32 ms). The two molecules have a resonance frequency of 1.4 ppm and 1.5 ppm respectively.

b) If the line width (the full width at half maximum, FWHM) is 8 Hz, what is the minimum required magnetic field strength to be able to discern the peaks from each other?

c) Since the T$_2$ of the oil is short, the resonance of oil will cover a broad range of chemical shifts. After the first experiment, the two peaks cannot be distinguished from the frequency of the background oil resonances. Describe an experiment and its parameters to cancel the oil signal.

Problem 2: MRS Sensitivity

In this problem, you are going to investigate different factors that influence signal intensity, especially field strength and the gyromagnetic ratio of the investigated nucleus.

a) Calculate the SNR ratio between a $^1$H-MRS spectrum acquired at 3T and 14T at 37°C (Hint: SNR ~ M$_0$ ~ N$_1$-N$_2$).

b) Two $^1$H resonances are resonating at 3.0 and 3.1 ppm; calculate the difference in Hertz at 3T and 14T.

c) How does a high magnetic field strength affect spectroscopy?

d) Calculate the SNR ratio between a $^1$H-MRS spectrum and a $^{13}$C-MRS at 14T at 37°C.

e) How can one achieve the same sensitivity for both $^1$H- and $^{13}$C- localized spectroscopy at 14T?

Problem 3: TR Optimisation

One is interested in quantifying metabolites at 14.1T. Knowing that the two metabolites with the longer and shorter T$_1$ are Taurine (T$_1$= 2.2s) and total Creatine (T$_1$= 1s) and that a TR of 2 seconds is used, which flip angle would you choose in order to optimize the metabolite signals?

Problem 4: Water and Lipid Suppression

For magnetic resonance spectroscopy it can be crucial to remove the water or lipid signal to be able to quantify metabolite signals. Let’s investigate the two cases below.

a) Using the vector model, show the effects of the pulse sequence given below on the thermal equilibrium magnetization for two cases:

First, for a water proton exactly on resonance (that is, stationary in the rotating reference frame). Second for a fat proton, which has a precession frequency in the rotating frame of $\omega$ rad/s, where the value of $\omega$ is given by $\pi/\tau$. The final answer should include x, y and z components of magnetization and the pulse sequence is: $45^\circ_x - \tau - 45^\circ_{-x}$. The relaxation processes during $\tau$ can be neglected.

b) The water signal resonates at around 4.8 ppm and the lipid signal below 1 ppm. One wants to remove both signals by exciting only the frequency range between 1 and 4.6 ppm. Calculate the length of the needed hard pulse at 14.1T and at 9.4T.