Texture

- What is texture?
- Texture analysis
- Deep Texture
Reminder: Homogeneous or Not?

What is homogeneous in some parts of these images are the statistical properties, not the actual pixel values.
Ideally, we would like to:

• Assign to individual pixels whose texture is similar the same values to form a textural image.

• Evaluate homogeneity both in the original image and in the textural one.
Similarly, we would like to be able to find boundaries between textures.
What is Texture?

Repetition of a basic pattern:
- Structural
- Statistical
→ Non local property, subject to distortions.
Structural Textures

Repetitive Texture Elements (Texels)

A texel represents the smallest graphical element in a two-dimensional texture that creates the impression of a textured surface.
Statistical Textures

Homogeneous Statistical Properties
Textured vs Smooth

A “featureless” surface can be regarded as the most elementary spatial texture:

- Microstructures define reflectance properties.
- They may be uniform or smoothly varying.

→ Texture is a scale dependent phenomenon
At these two different scales, the texture seems very different.
Structural vs Statistical

- Segmenting out texels is difficult or impossible in most real images.

- Numeric quantities or statistics that describe a texture can be computed from the gray levels or colors alone.

→ The statistical approach is less intuitive, but more effective in practice.

What are the fundamental texture primitives in this image?
Creating Textural Images

Because texture is non-local, the texture of individual pixels must be estimated using neighborhoods that surround them:

- For each pixel, compute a feature vector using either an image patch or a set of filters.
- Run a classification algorithm to assign a texture value to each pixel.
• Compute image statistics for each superpixel.
• Train a classifier to assign a probability to be within a mitochondria.

→ We used the super pixels to compute local statistics.
Textural Metrics

Spectral metrics:
• Texture is characterized by the properties of its Fourier transform.

Statistical Metrics:
• Texture is as statistical property of the pixels’ intensity and color in a region.

Deep Net Metrics:
• They have now mostly superseded the others.
• They encompass the earlier concepts.
Reminder: Discrete Fourier Transform

\[ F(\mu, \nu) = \frac{1}{\sqrt{M \times N}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)e^{-2i\pi(\mu x/M + \nu y/N)} \]

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The DFT is the discrete equivalent of the 2D Fourier transform:
- The 2D function \( f \) is written as a sum of sinusoids.
- The DFT of \( f \) convolved with \( g \) is the product of their DFTs.
Spectral Analysis

Lines in the DFT modulus images capture the main orientations in the image.
Texture Analysis

Angular and radial bins in the Fourier domain capture the directionality and fluctuation speed of an image texture, respectively.
For some types of textures, the Fourier spectra are easily distinguishable. A classifier can be trained to tell them apart. However, one must have the same texture in the whole image patch.
Limitations

• DFT on small patches is subject to severe boundary effects.

• Only applicable if texture is uniform over large areas.

• Results can be improved by using wavelets instead, but only up to a point.

—> More local metrics are required.
Statistical Metrics

First order gray-level statistics:
• Statistics of single pixels in terms of histograms.
• Insensitive to neighborhood relationships.

Second order gray-level statistics:
• Statistics of pixel pairs.

Filter-based measures:
• Statistics of whole neighborhoods.
Simple First Order Measure

The orientation histogram gives a clue to the orientation of the underlying plane.
More First Order Measures

Edge Density and Direction:
- Edge detection as a first step in texture analysis.
- The number of edge pixels in a fixed-size region tells us how busy that region is.
- The directions of the edges also help characterize the texture.

Edgeness per unit area:
- \( \frac{\{ p : \text{gradient}_\text{magnitude}(p) \geq \text{threshold}\}}{N} \) where \( N \) is the unit area or region.

Edge magnitude and direction histograms:
- \( \{ H_G, H_\tau \} \)
Second Order Measures

Histogram of the co-occurrence of particular intensity values in the image.

- Specified in terms of geometric relationships between pixel pairs:
  - Distance
  - Orientation

- $P(i,j,d,\theta)$ Frequency with which a pixel with value $j$ occurs at distance $d$ and orientation $\theta$ from a pixel with value $i$. 
Simple Example

If \[ I = \begin{bmatrix} 0 & 1 & 2 & 3 & 1 \\ 2 & 1 & 3 & 1 & 1 \\ 0 & 0 & 2 & 2 & 1 \\ 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}, \]

then \[ H = \begin{bmatrix} 4 & 1 & 1 & 2 \\ 0 & 1 & 2 & 1 \\ 1 & 2 & 1 & 1 \\ 0 & 3 & 0 & 0 \end{bmatrix}, \]

and \[ P(l,m,1,0) = \frac{H(l,m)}{20}, \]
No need to distinguish between
\[ P(m, l, \Delta i, \Delta j) \]
and
\[ P(l, m, \Delta i, \Delta j) \]
→ Co-Occurrence matrix C:
\[ C = H + H^T \]
Second Order Measures

Contrast:
\[ \sum_{i,j=0}^{N-1} P_{i,j} (i - j)^2 \]

Dissimilarity:
\[ \sum_{i,j=0}^{N-1} P_{i,j} |i - j| \]

Homogeneity:
\[ \sum_{i,j=0}^{N-1} \frac{P_{i,j}}{1 + (i - j)^2} \]

Angular Second Moment (Energy, Uniformity):
\[ \sum_{i,j=0}^{N-1} P_{i,j}^2 \]

Entropy:
\[ \sum_{i,j=0}^{N-1} P_{i,j} (-\ln P_{i,j}) \]

and many more ......
The image is excerpted from Path 41, Row 25 of Landsat 7 ETM+, dated 4 September 1999. This is an area in the Rocky Mountain Foothills near Waterton National Park, Alberta. The western edge of the image contains steep slopes and deep valleys. To the east is both grassland and annual crops, mostly grains. The eastern area is bisected by numerous small streams.
Let us consider two areas, one in the hills, the other in the plain.
Using Second Order Measures

The various measures within the two areas are sufficiently different for a classifier to easily distinguish them.
Classification

Used to identify eight terrain classes:
- Old residential
- New residential
- Urban
- Lake
- Swamp
- Scrub
- Wood
Parameter Choices

Using co-occurrence matrices requires choosing:
- window size,
- direction of offset,
- offset distance,
- what channels to use,
- what measures to use.

How do we choose these parameters?
- Critical question for all statistical texture methods.
- Can be addressed using Machine Learning.
Filter Based Measures

Represent image textures using the responses of a collection of filters.

- An appropriate filter bank will extract useful information such as spots and edges.
- Traditionally one or two spot filters and several oriented bar filters.
Gaussian Filter Derivatives

Gaussian Derivative  x and y derivatives at different scales

These filters respond to horizontal and vertical edges.
Horizontal and Vertical Structures

vertical filter

image

horizontal filter

Squared responses  Spatially blurred

Threshold squared, blurred responses, then categorize texture based on those two bits
Oriented Filters

\[
\frac{\partial I}{\partial \theta} = \cos(\theta) \frac{\partial I}{\partial x} + \sin(\theta) \frac{\partial I}{\partial y}
\]
Oriented filters respond to edges in a specific direction.
Higher-derivative filters of the Gaussian filters can be used to compute higher-order image derivatives.
Filter Bank

- Different scales.
- Different orientations.
- Derivatives order 0, 1, 2 ..

→ For every image pixel, compute a vector of responses to each filter.
Filter Responses: Small Scales

Gaussian filters with a small $\sigma$. Capture local details.
Filter Responses: Large Scales

Gaussian filters with a large $\sigma$. Capture larger details.
Gabor Filters

Gabor filters are the products of a Gaussian filter with oriented sinusoids. They come in pairs, each consisting of a symmetric filter and an anti-symmetric filter:

\[ G_{\text{sym}}(x, y) = \cos(k_x x + k_y y) \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) \]

\[ G_{\text{asym}}(x, y) = \sin(k_x x + k_y y) \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) \]

where \( k_x \) and \( k_y \) determine the spatial frequency and the orientation of the filter and \( \sigma \) determines the scale.

→ A filter bank is formed by varying the frequency, the scale, and the filter orientation.
Vertical Derivatives
Gabor Responses

Images:

Filters:

Responses:
GABOR FILTER CHARACTERISTICS

• Respond strongly at points in an image where there are components that locally have a particular spatial frequency and orientation.

• In theory, by applying a very large number of Gabor filters at different scales, orientations and spatial frequencies, one can analyze an image into a detailed local description.

• In practice, it is not known how many filters, at what scale, frequencies, and orientations, to use. This tends to be application dependent.
Use AdaBoost to perform classification on the output of Gabor filters.

Shotton et al., ECCV’06
Using Decision Forests to perform classification on the output of Gabor filters works better in this case.

But what works even better, is ......
Reminder: ConvNets
Reminder: Convolutional Layer

\[
\sigma \left( b + \sum_{x=0}^{n_x} \sum_{y=0}^{n_y} w_{x,y} a_{i+x, j+y} \right)
\]
Reminder: Feature Maps

28 × 28 input neurons

first hidden layer: 3 × 24 × 24 neurons

Filters:
Learned Feature Maps

- Some of these convolutional filters look very Gabor like.
- The network requires a large training set to learn an effective filter bank.
- The older techniques still have their place in the absence of such training sets.
Reminder: U-Net Architecture
Potential Interpretation

A key role of the ConvNet is to generate for every output pixel a feature vector containing the output of all the intermediate layers.
In Short

Texture is a key property of objects which is

- Non local
- Non trivial to measure
- Subject to deformations

➡ Hard to characterize formally and best used in conjunction with effective Machine Learning techniques.
➡ This seems to be exactly what Convolutional Neural Nets do.