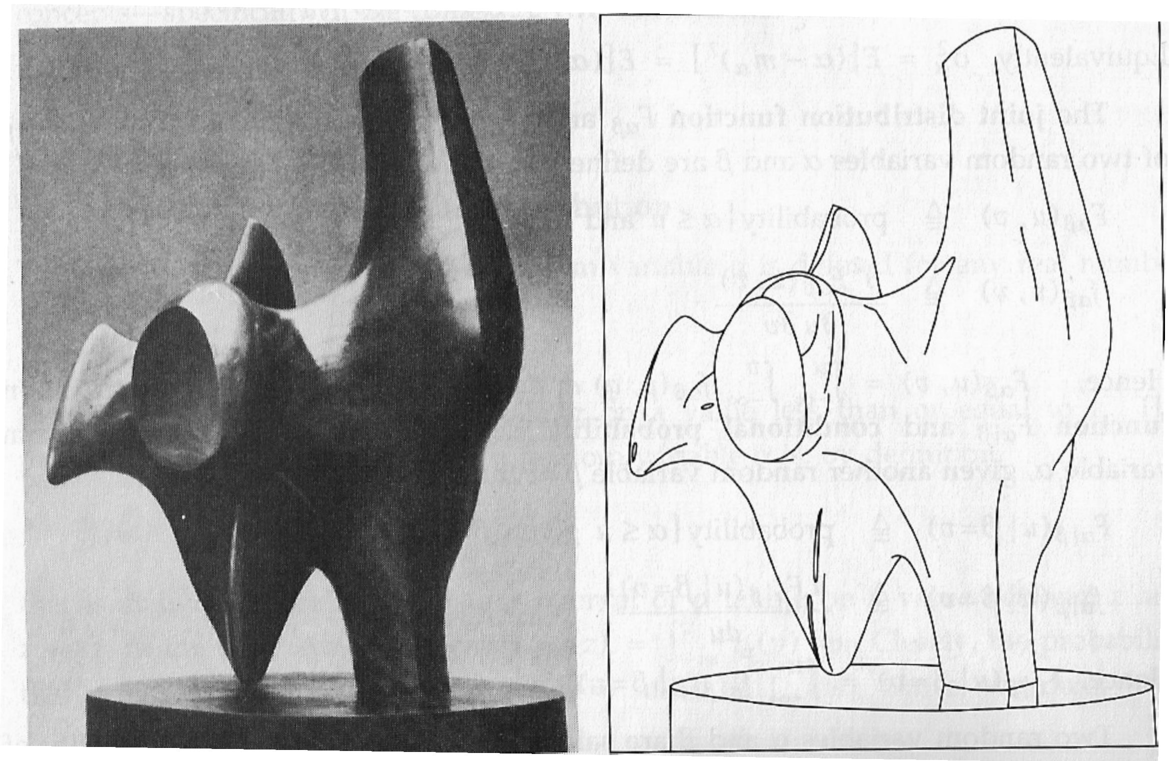
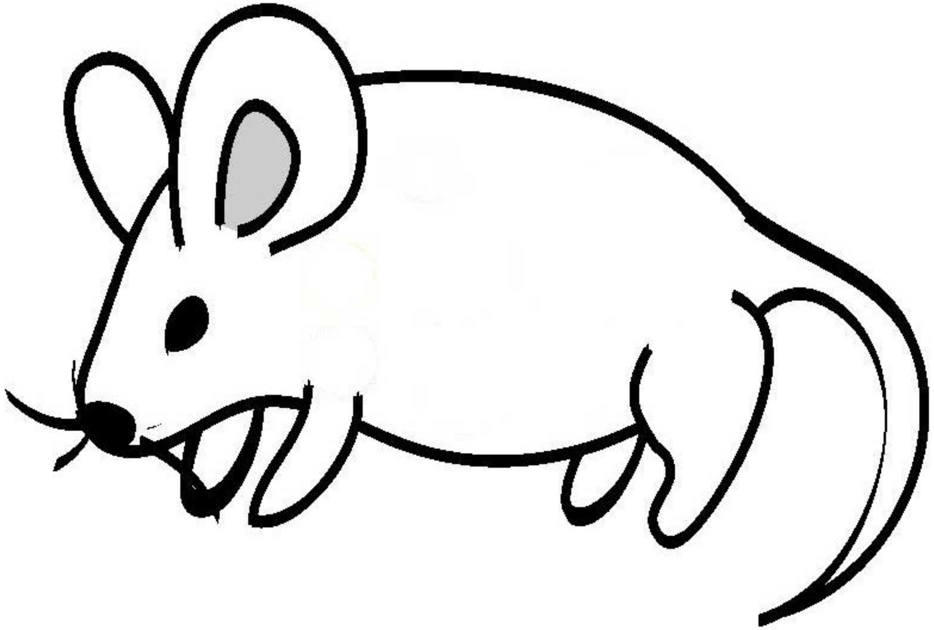
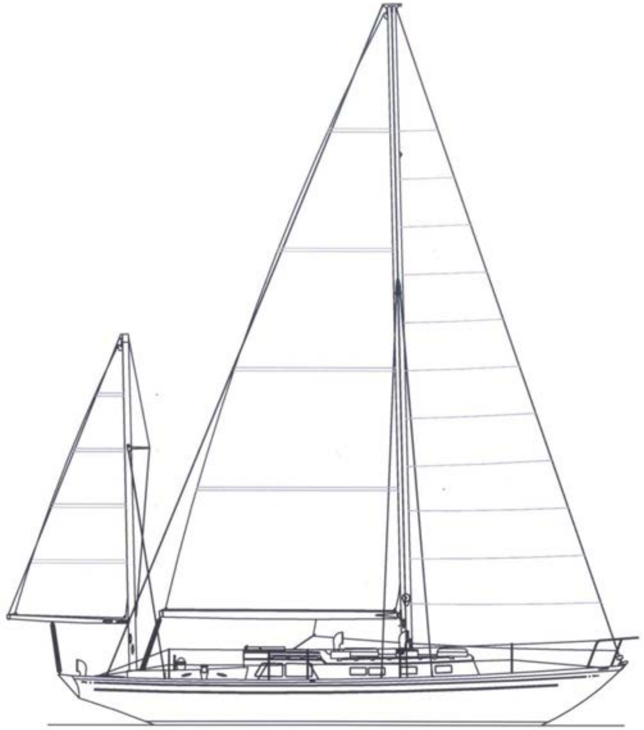
Edge Detection



- What's an edge
- Image gradients
- Edge operators

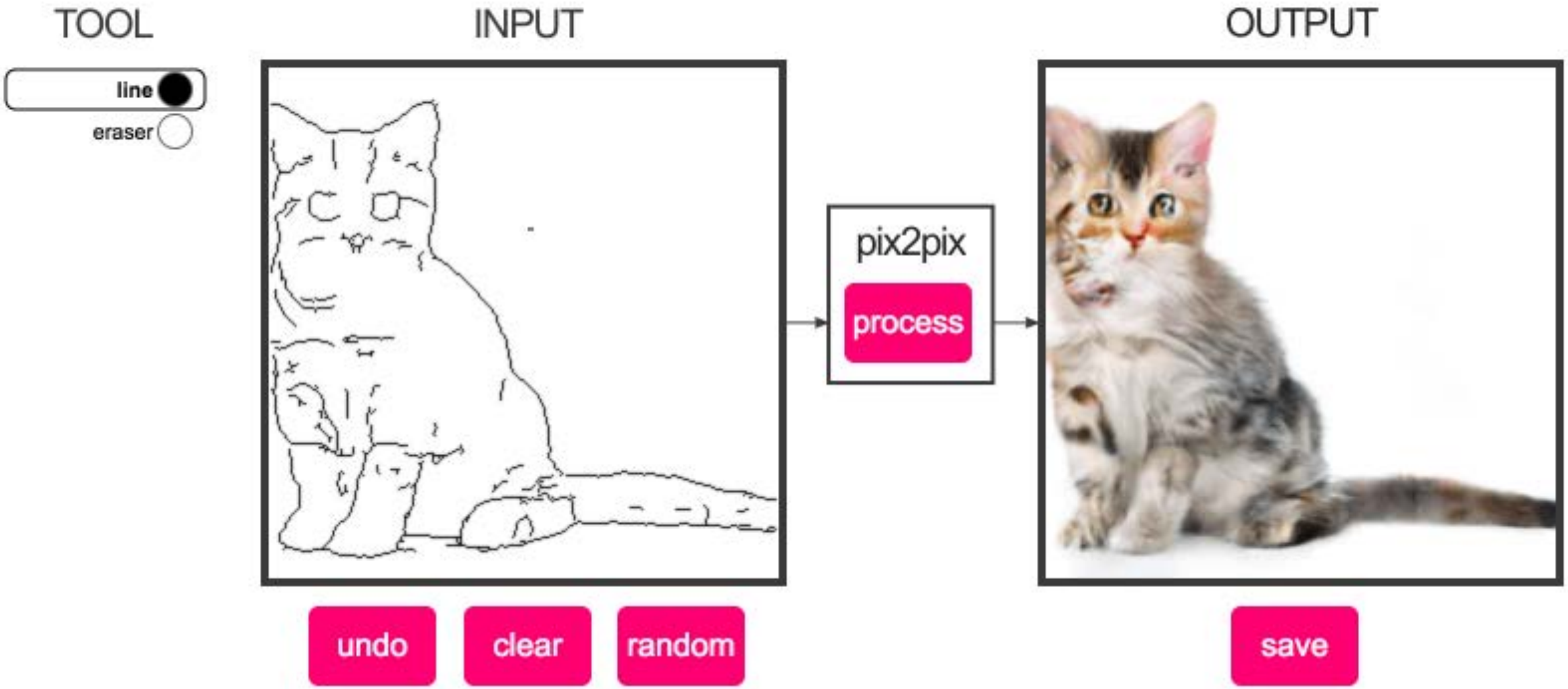
Line Drawings





- Edges seem fundamental to human perception.
- They form a compressed version of the image.

From Edges To Cats



Deep-Learning based generative model.

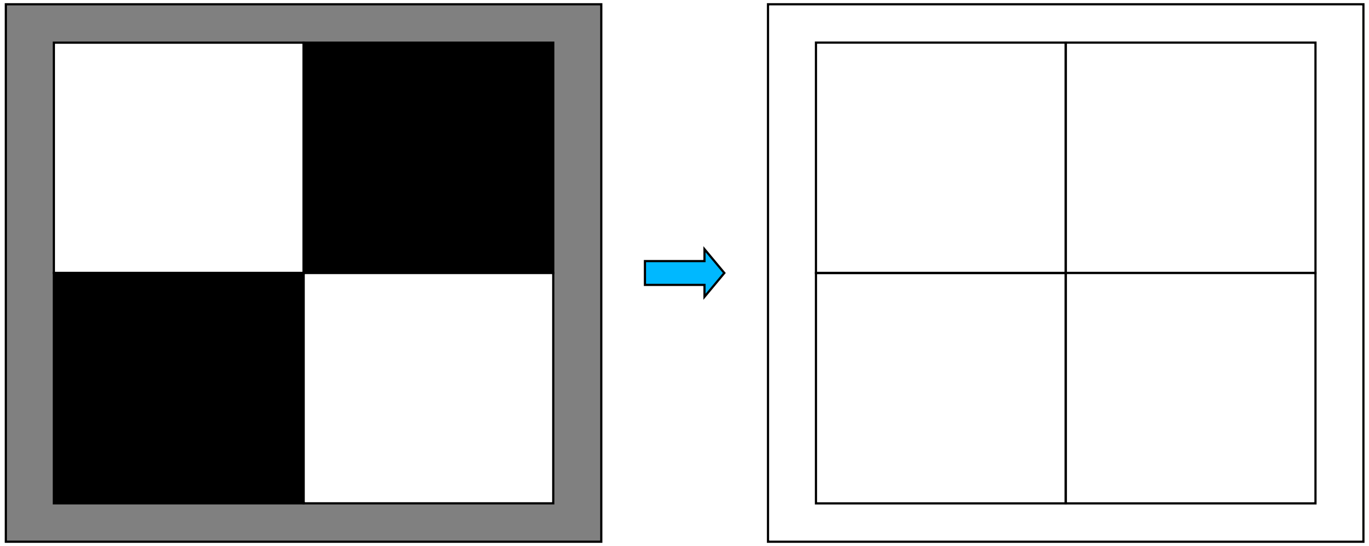
Demo https://affinelayer.com/pixsrv/

Maps and Overlays

Corridor

Corridor

Edges and Regions



Edges:

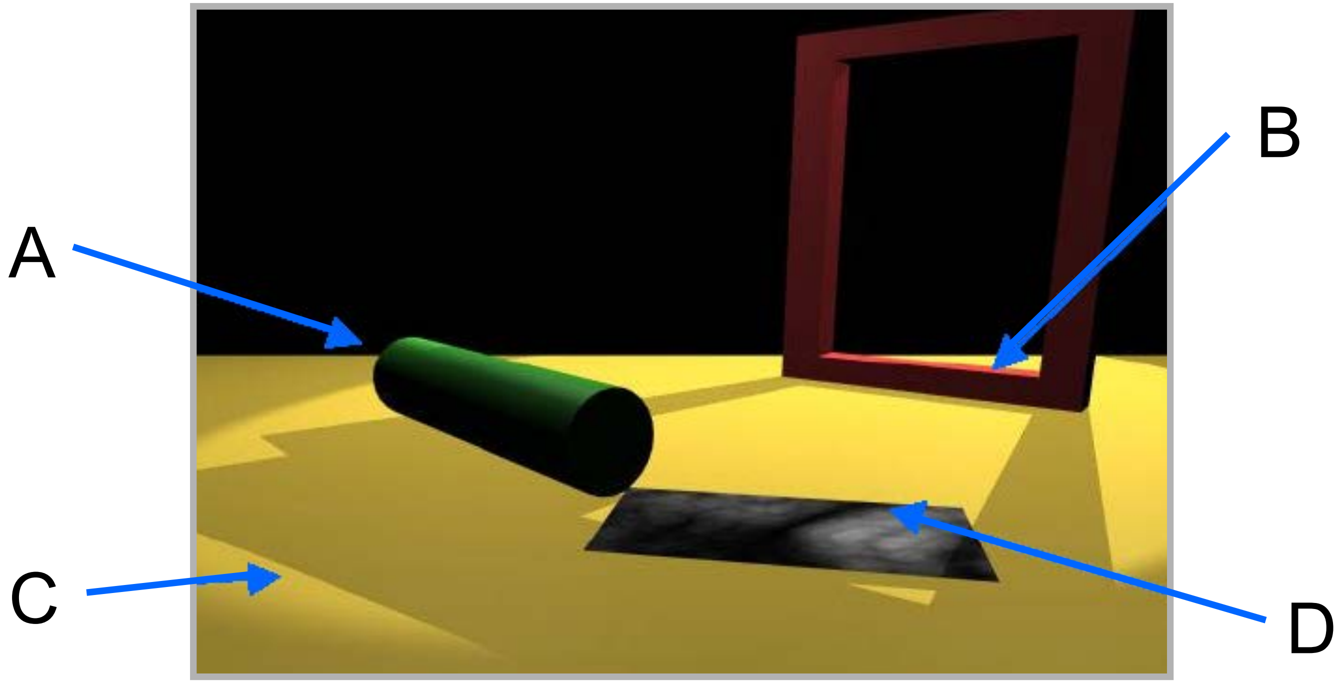
- Boundary between bland image regions.

Regions:

- Homogenous areas between edges.

→ Edge/Region Duality.

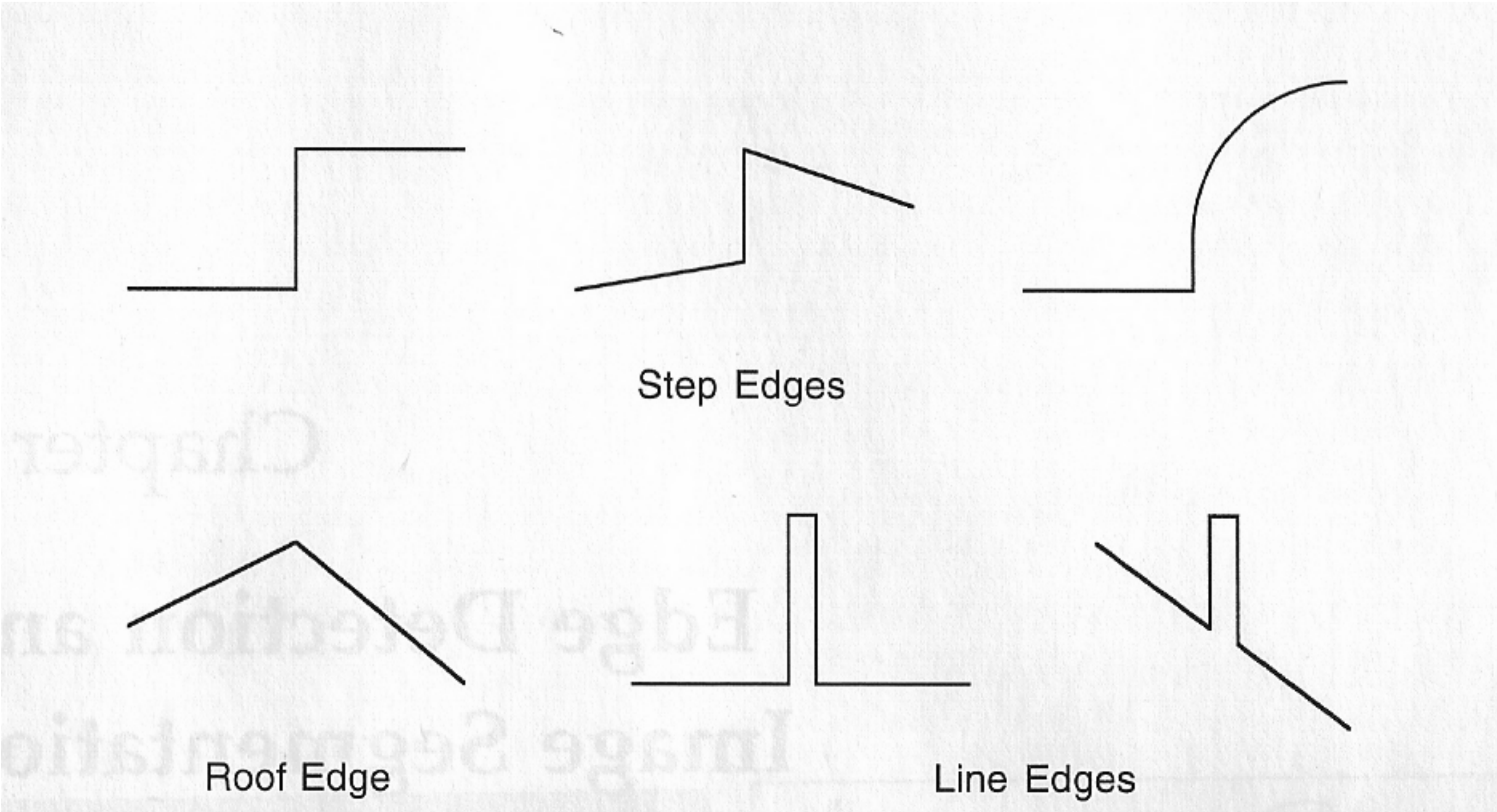
Discontinuities



- A. Depth discontinuity: Abrupt depth change in the world
- B. Surface normal discontinuity: Change in surface orientation
- C. Illumination discontinuity: Shadows, lighting changes
- D. Reflectance discontinuity: Surface properties, markings

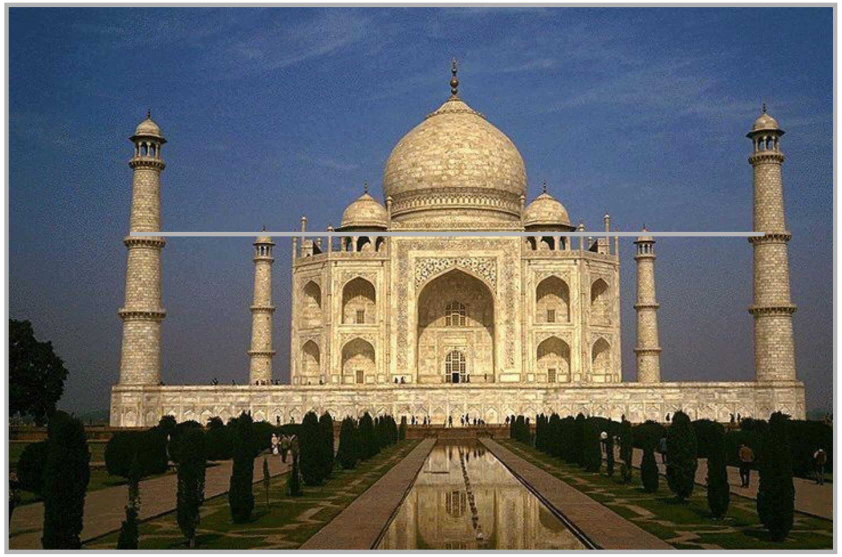
→ Sharply different Gray levels on both sides

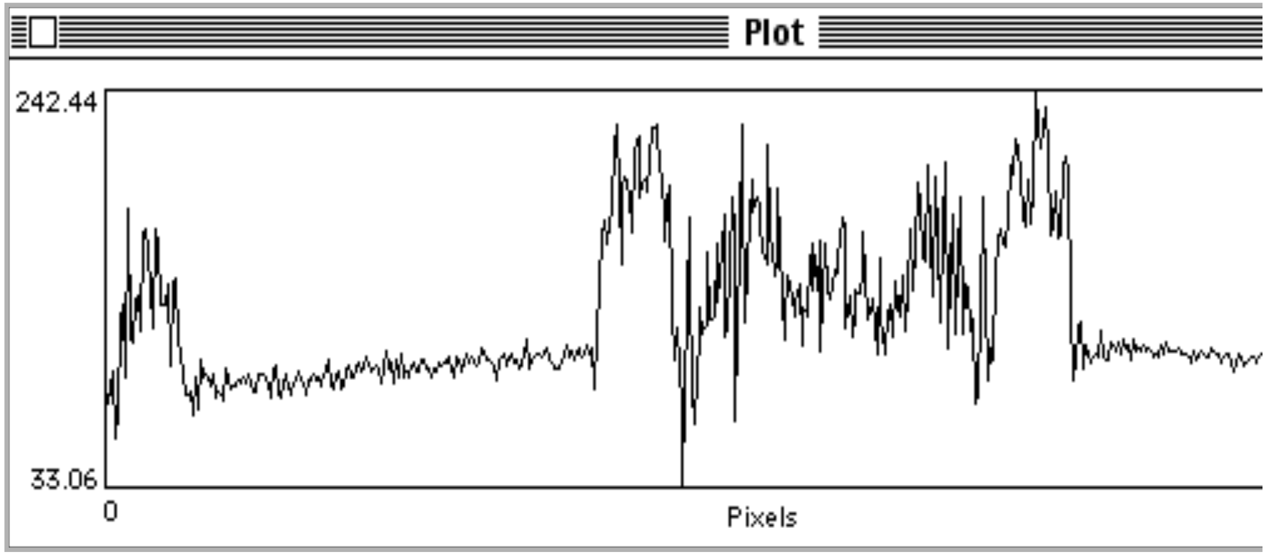
EDGE PROFILES

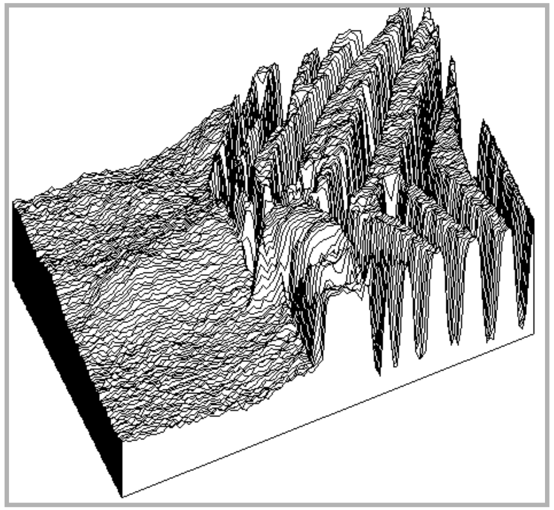


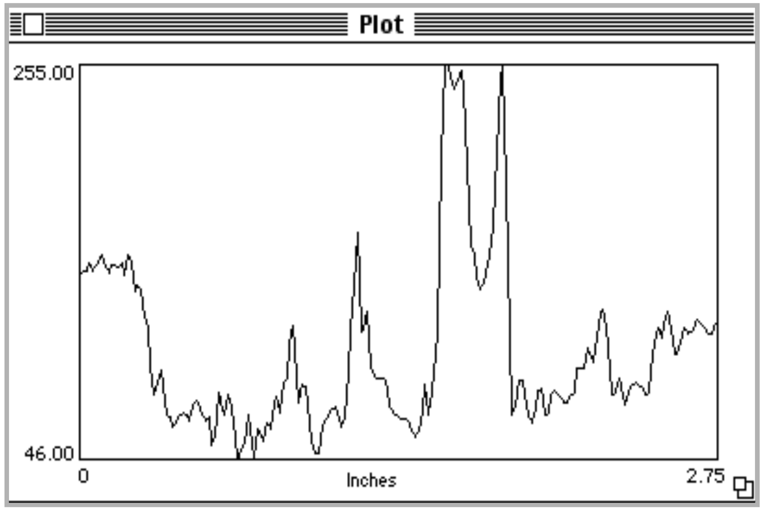
Edges are where a change occurs

REALITY

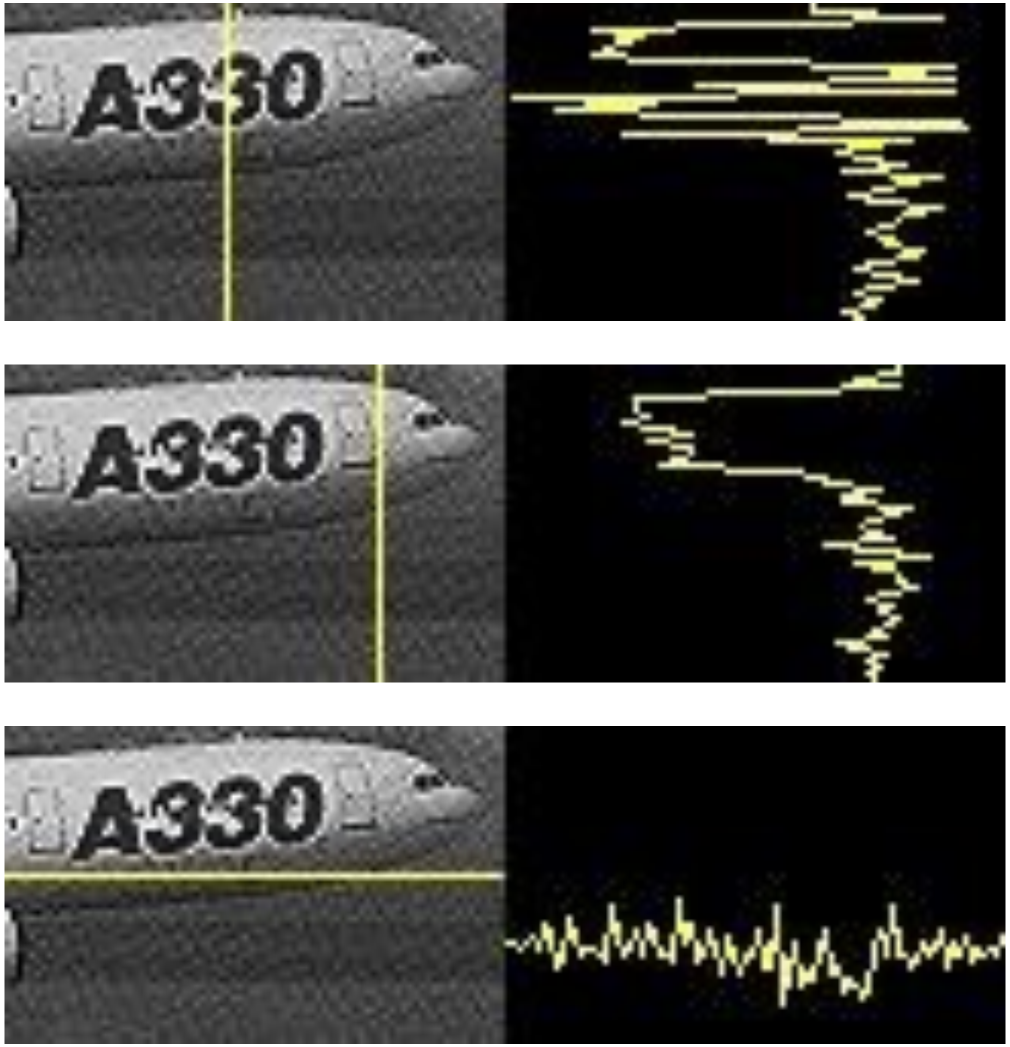








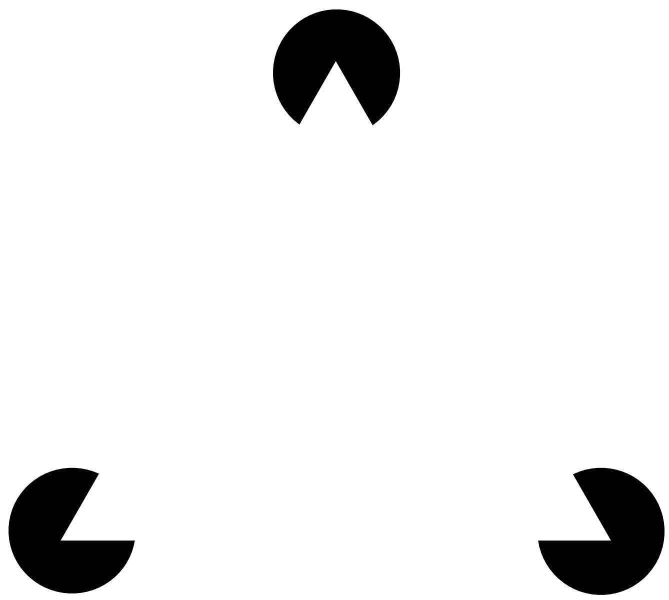
More Reality



Very noisy signals

→ Prior knowledge is required!!

Optional: Illusory Contours

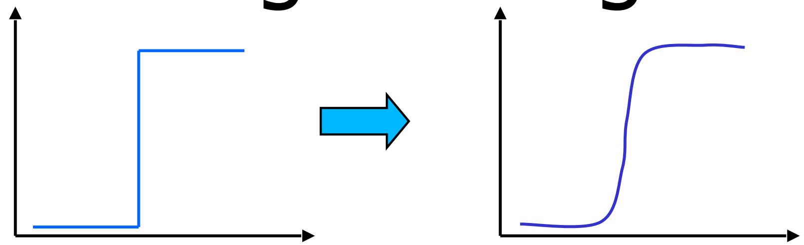


- No closed contour, but we still perceived an edge.
- This will not be further discussed in this class.

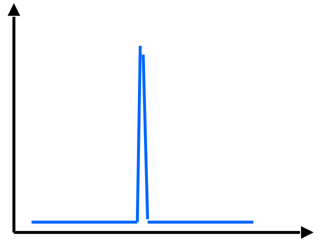
Ideal Step Edge

Rapid change in image => High local gradient

f(x) = step edge

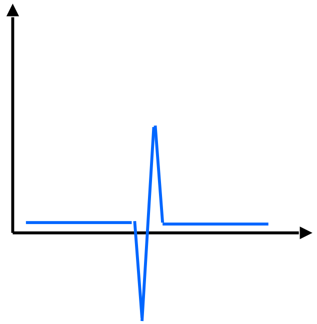


1st Derivative f’(x)



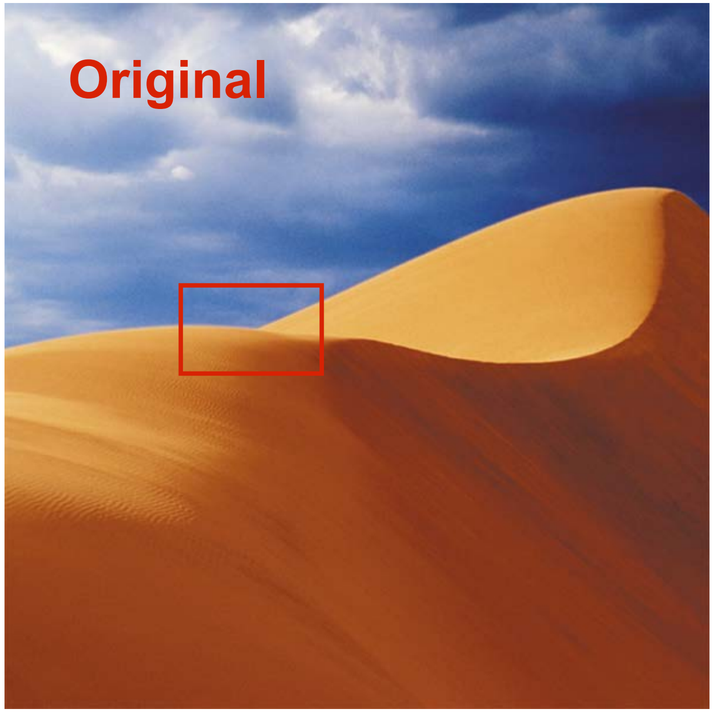
maximum

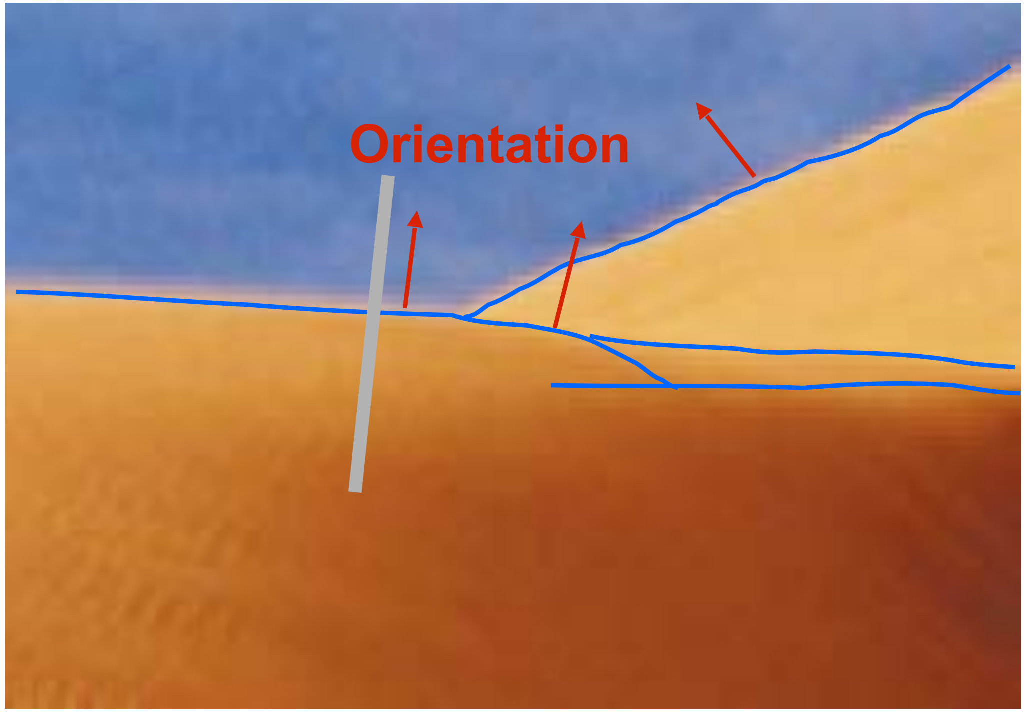
2nd Derivative f’’(x)

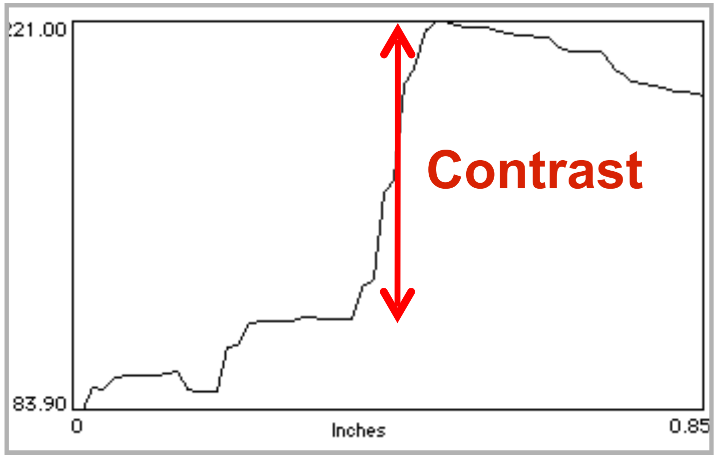


zero crossing

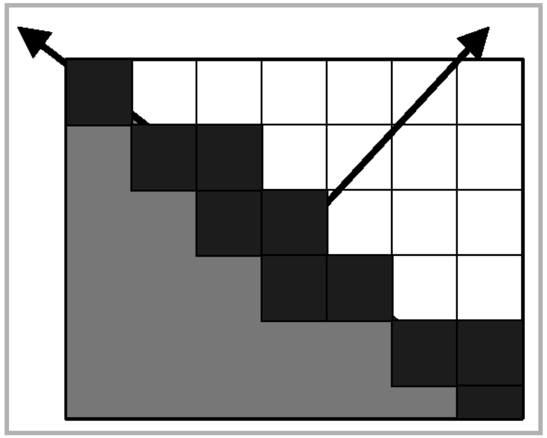
Edge Properties





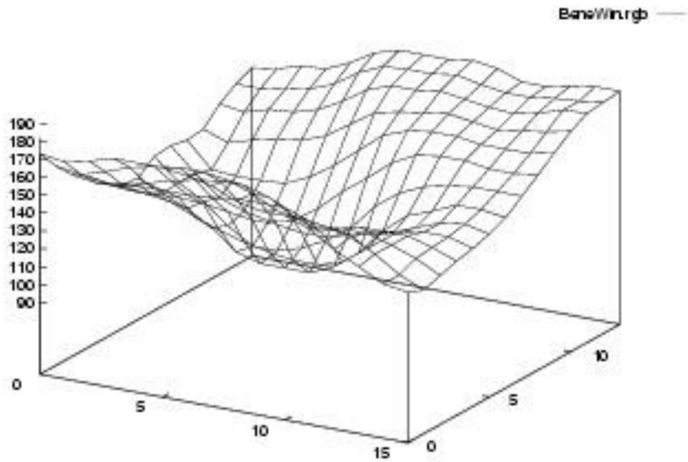


Edge Descriptors

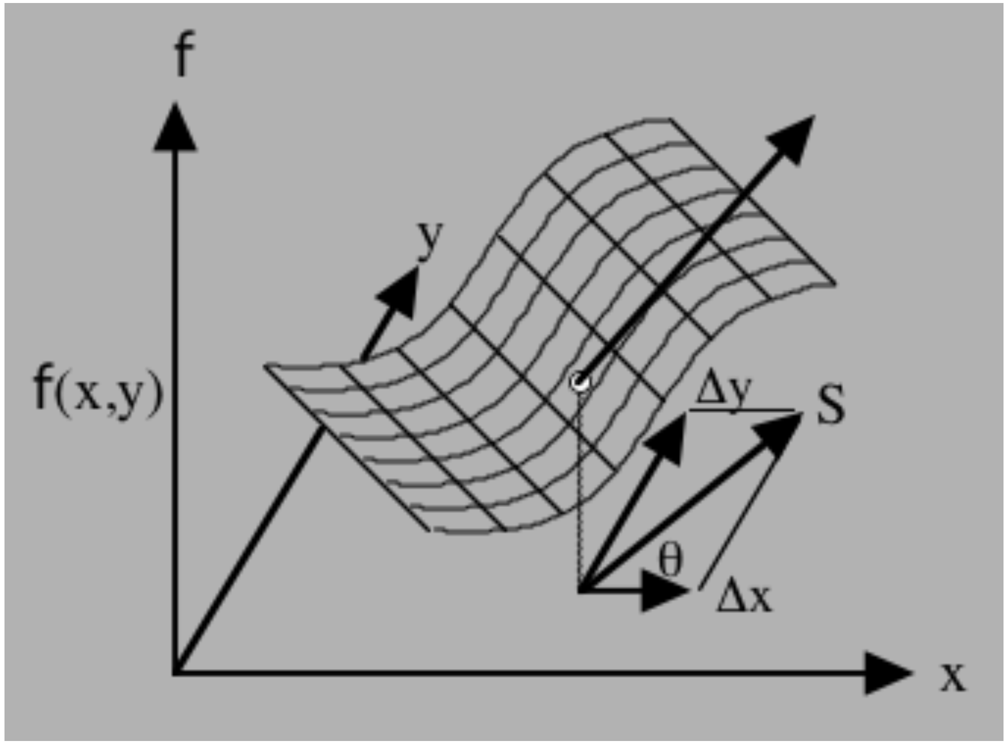


- Edge Normal:
  - Unit vector in the direction of maximum intensity change
- Edge Direction:
  - Unit vector perpendicular to the edge normal
- Edge position or center
  - Image location at which edge is located
- Edge Strength
  - Speed of intensity variation across the edge.

Images as 3-D Surfaces



Geometric Interpretation



Since I(x,y) is not a continuous function:

1. Locally fit a smooth surface.
2. Compute its derivatives.

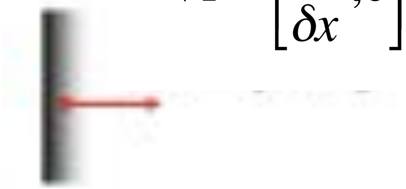
Image Gradient

The gradient of an image

$$\nabla I = \left[\frac{\delta I}{\delta x}, \frac{\delta I}{\delta y}\right]$$

points in the direction of most rapid change in intensity.

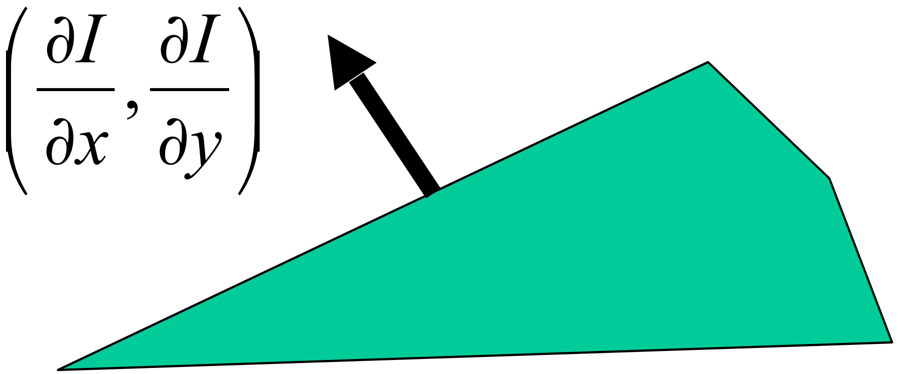
$$\nabla I = \left[\frac{\delta I}{\delta x}, 0\right]$$



$$\nabla I = \left[0, \frac{\delta I}{\delta y}\right]$$

$$\nabla I = \left[\frac{\delta I}{\delta x}, \frac{\delta I}{\delta y}\right]$$

Magnitude And Orientation



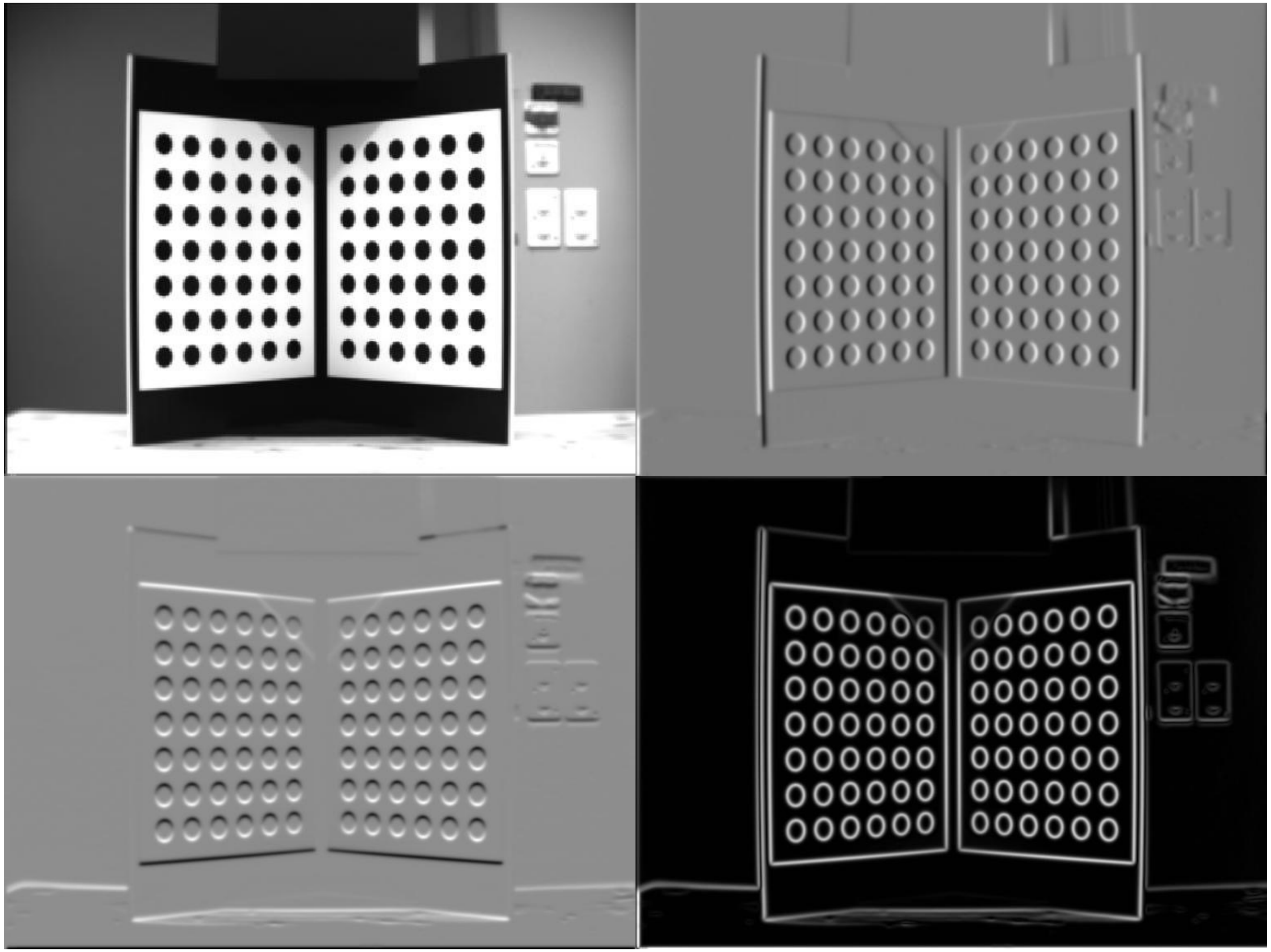
$$\text{Measure of contrast: } G = \sqrt{\frac{\partial I}{\partial x}^2 + \frac{\partial I}{\partial y}^2}$$

$$\text{Edge orientation: } \theta = \arctan\left(\frac{\partial I}{\partial y}, \frac{\partial I}{\partial x}\right)$$

Gradient Images

I

$\frac{\partial I}{\partial x}$



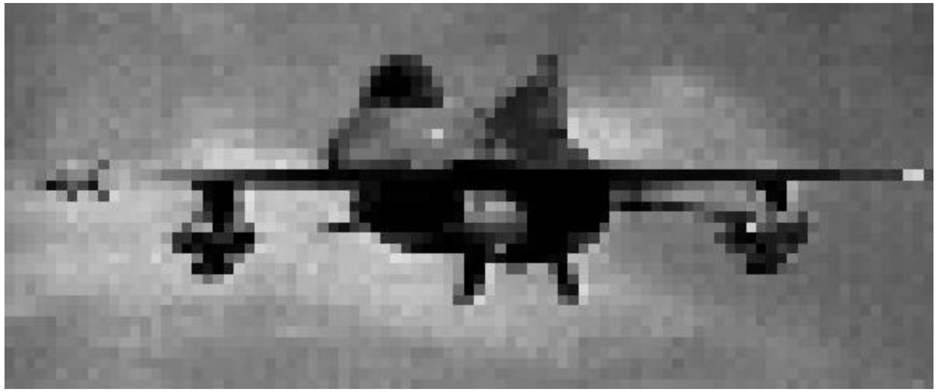
$\frac{\partial I}{\partial y}$

$\sqrt{\frac{\partial I}{\partial x}^2 + \frac{\partial I}{\partial y}^2}$

The gradient magnitude is unaffected by orientation ….

Real Images





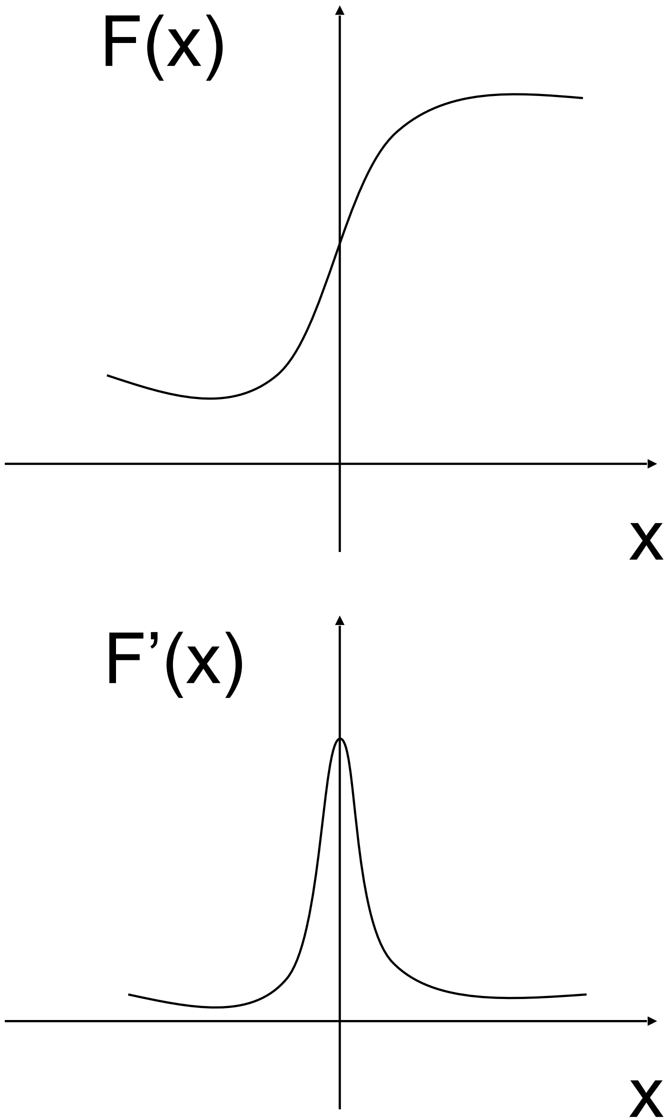

… but not directly usable in most real-world images.

Edge Operators

- Difference Operators
- Convolution Operators
- Trained Detectors
- Deep Nets

EPFL

Gradient Methods

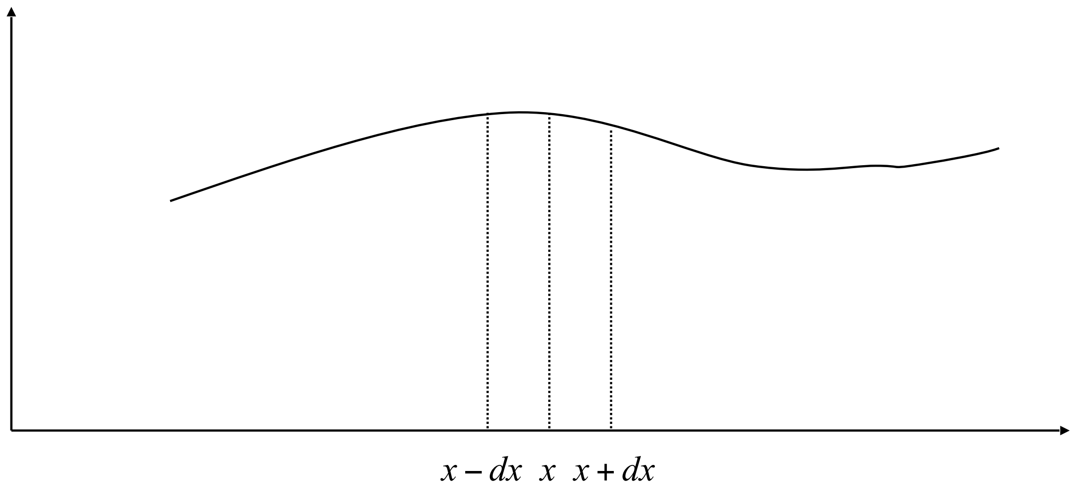


Edge = Sharp variation

Large first derivative

1D Finite Differences

In one dimension:

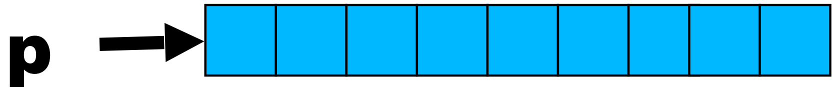


$$\frac{df}{dx} \approx \frac{f(x+dx)-f(x)}{dx} \approx \frac{f(x+dx)-f(x-dx)}{2dx}$$

$$\frac{d^2 f}{dx^2} \approx \frac{f(x+dx)-2f(x)+f(x-dx)}{dx^2}$$

Coding 1D Finite Differences

Line stored as an array:

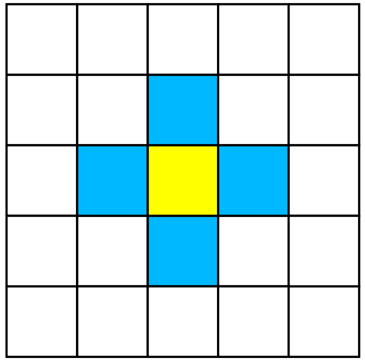


- for i in range(n-1):
  q[i]=(p[i+1]-p[i])

- for i in range(1,n-1):
  q[i]=(p[i+1]-p[i-1])/2

- q=(p[2:]-p[:-2])/2

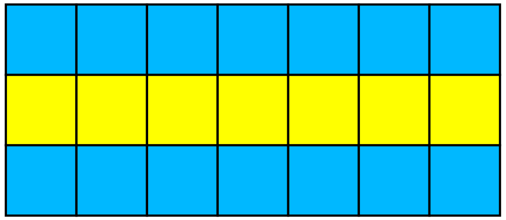
2D Finite Differences



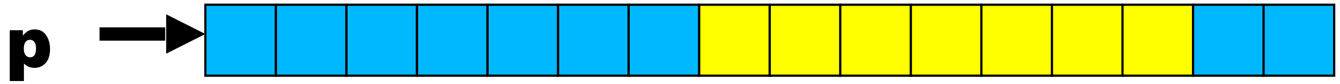
$$\frac{\partial f}{\partial x} \approx \frac{f(x+dx,y)-f(x,y)}{dx} \approx \frac{f(x+dx,y)-f(x-dx,y)}{2dx}$$

$$\frac{\partial f}{\partial y} \approx \frac{f(x,y+dy)-f(x,y)}{dy} \approx \frac{f(x,y+dy)-f(x,y-dy)}{2dy}$$

Coding 2D Finite Differences



Python



C

Image stored as a 2D array:

- dx = p[1:,:]-p[:-1,:]
  dy = p[:,1:]-p[:,:-1]

- dx = (p[2:,:]-p[:-2,:])/2
  dy = (p[:,2:]-p[:,:-2])/2

Image stored in raster format:

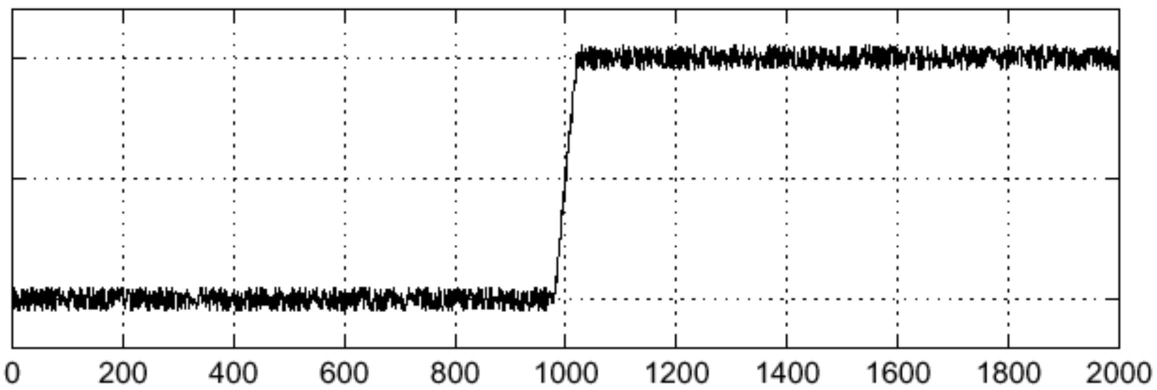
```
{
  int i;
  for(i=0;i<xdim;i++){
    dx[i] = p[i+1]     -p[i];
    dy[i] = p[i+xdim]-p[i];
  }
}
```

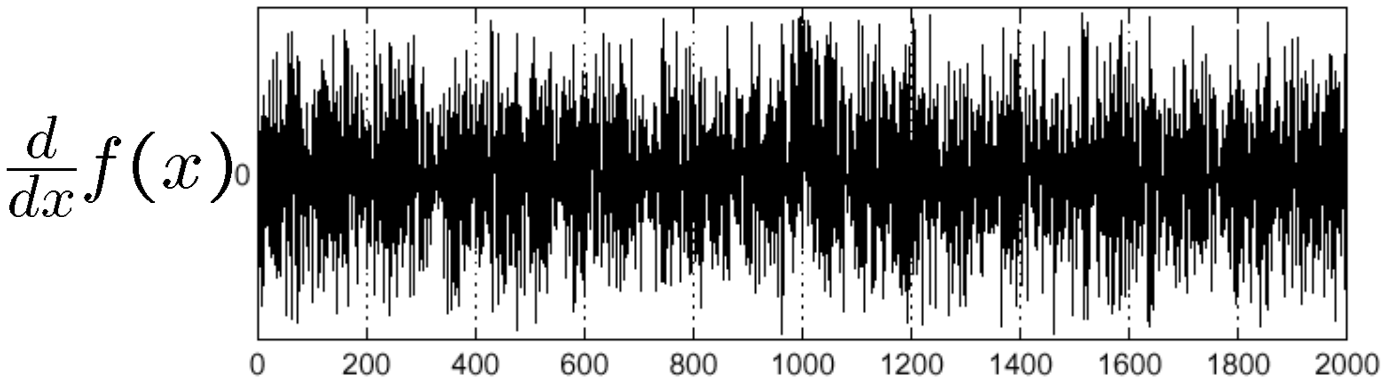
- Only 1D array accesses
- No multiplications

—> Can be exploited to increase speed.

Noise in 1D

Consider a single row or column of the image:



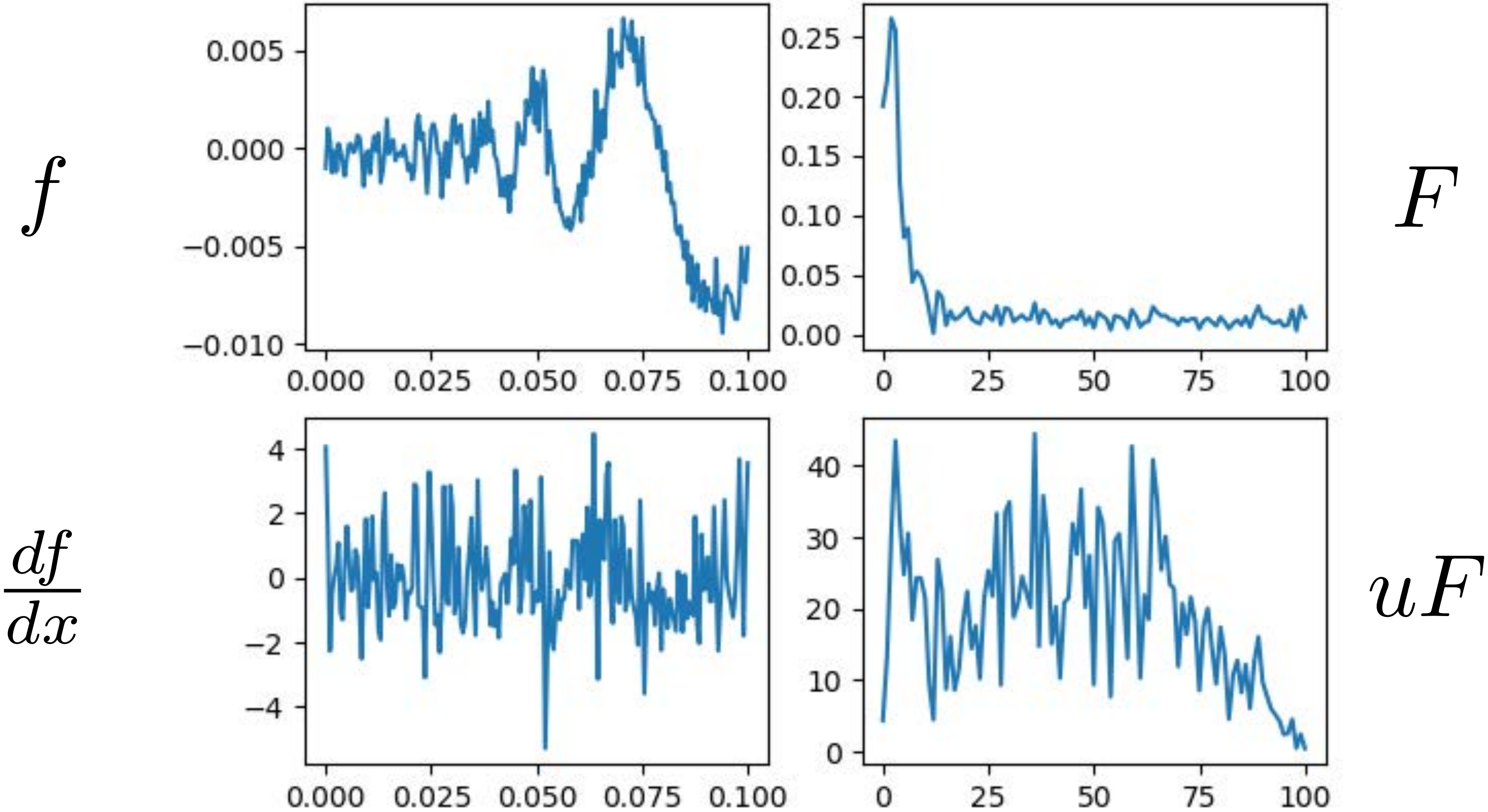


Fourier Interpretation

Function	Fourier Transform
$\frac{df}{dx}(x)$	$uF(u)$
$\frac{\delta f}{\delta x}(x,y)$	$uF(u,v)$
$\frac{\delta f}{\delta y}(x,y)$	$vF(u,v)$

→ Differentiating emphasizes high frequencies and therefore noise!

$f(x) = x^2 \sin(1/x)$



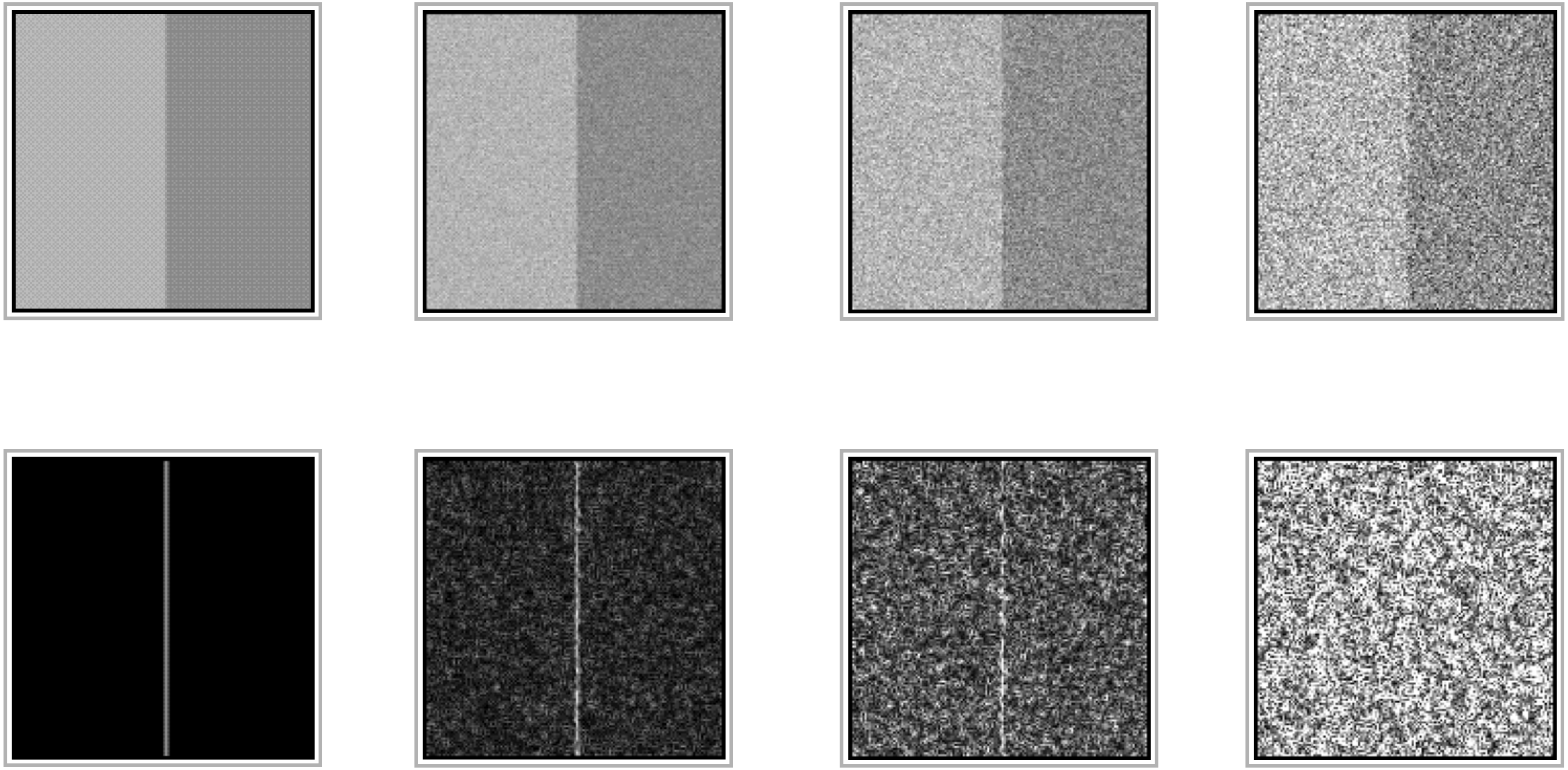
Original function + Noise

Fourier transform

Noise in 2D

Ideal step edge

Step edge + noise



Increasing noise level →

As the amount of noise increases, the derivatives stop being meaningful.

Removing Noise

Problem:

- High frequencies and differentiation do not mix well.

Solution:

- Suppress high frequencies by
  - using the Discrete Fourier Transform.

Discrete Fourier Transform

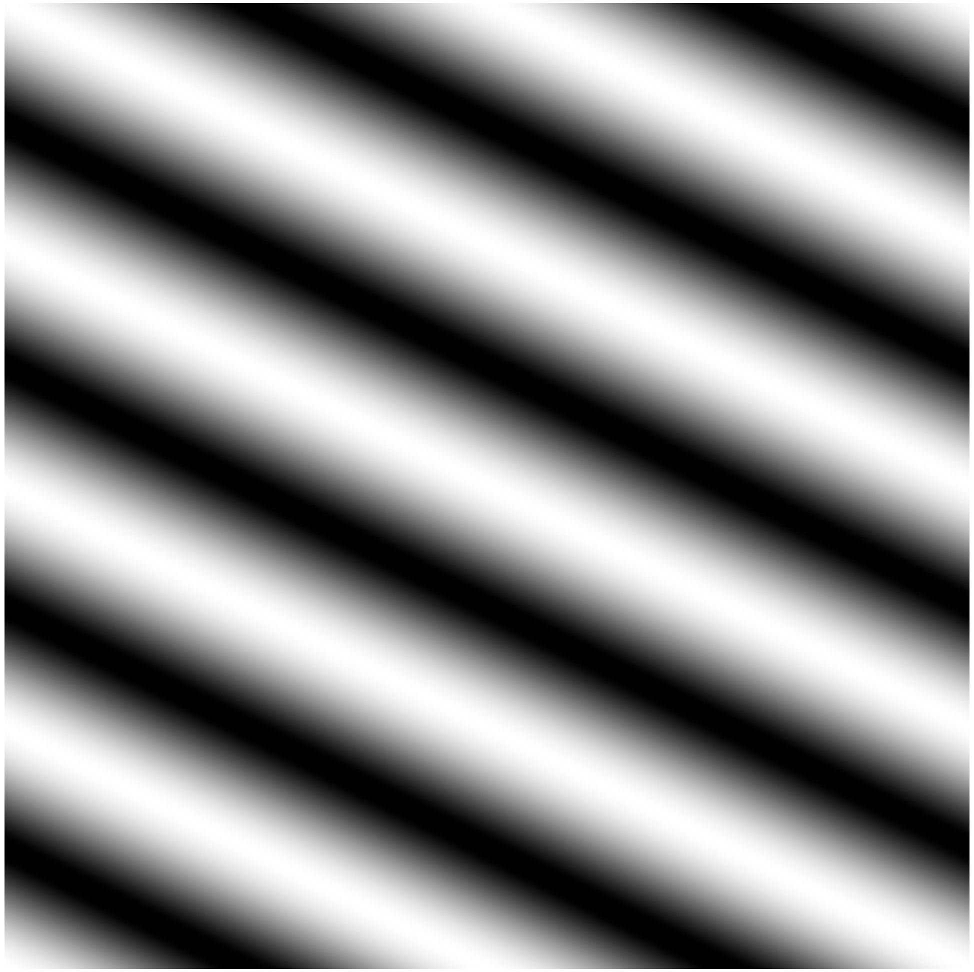
$$F(\mu,\nu) = \frac{1}{\sqrt{M*N}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-2i\pi(\mu x/M + \nu y/N)}$$

$$f(x,y) = \frac{1}{\sqrt{M*N}} \sum_{\mu=0}^{M-1} \sum_{\nu=0}^{N-1} F(\mu,\nu) e^{+2i\pi(\mu x/M + \nu y/N)}$$

The DFT is the discrete equivalent of the 2D Fourier transform:

- The 2D function f is written as a sum of sinusoids.
- The DFT of f convolved with g is the product of their DFTs.

Fourier Basis Element



Real part of

$$e^{+2i\pi(ux+vy)}$$

where

- $\sqrt{u^2 + v^2}$ represents the frequency,
- $\text{atan}(v, u)$ represents the orientation.

Fourier Basis Element

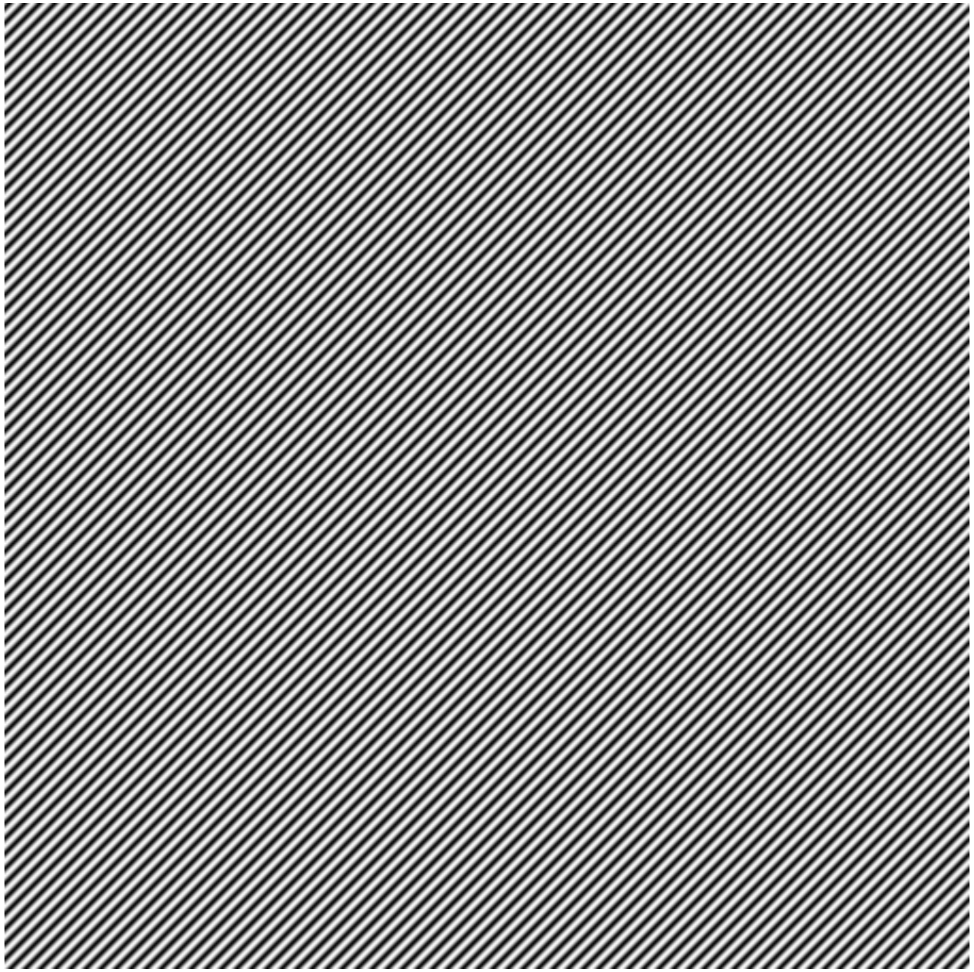
Real part of

$$e^{+2i\pi(ux+vy)}$$

where

- $\sqrt{u^2 + v^2}$ is larger than before.

Fourier Basis Element



Real part of

$$e^{+2i\pi(ux+vy)}$$

where

- $\sqrt{u^2 + v^2}$ is larger still.

Truncated Inverse DFT

$$F(\mu, \nu) = \frac{1}{\sqrt{M * N}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-2i\pi(\mu x/M + \nu y/N)}$$

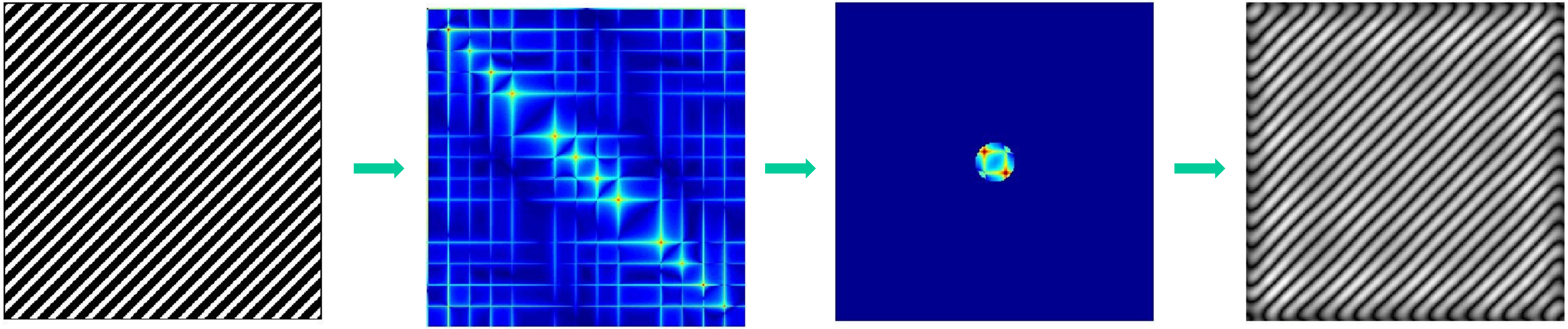
$$f(x, y) = \frac{1}{\sqrt{M * N}} \sum_{\mu=0}^{M-1} \sum_{\nu=0}^{N-1} F(\mu, \nu) e^{+2i\pi(\mu x/M + \nu y/N)}$$

$$f(x, y) = \frac{1}{\sqrt{M * N}} \sum_{\mu^2+\nu^2<T} F(\mu, \nu) e^{+2i\pi(\mu x/M + \nu y/N)}$$

T is a hand-specified threshold.

- The sinusoids corresponding to $\mu^2 + \nu^2 \geq T$ depict high frequencies.
- Removing them amounts to removing high-frequencies.

Smoothing by Truncating the IDFT



Rotated stripes:

- Dominant diagonal structures
- Discretization produces additional harmonics

—> Removing higher frequencies and reconstructing yields a smoothed image.

Removing Noise

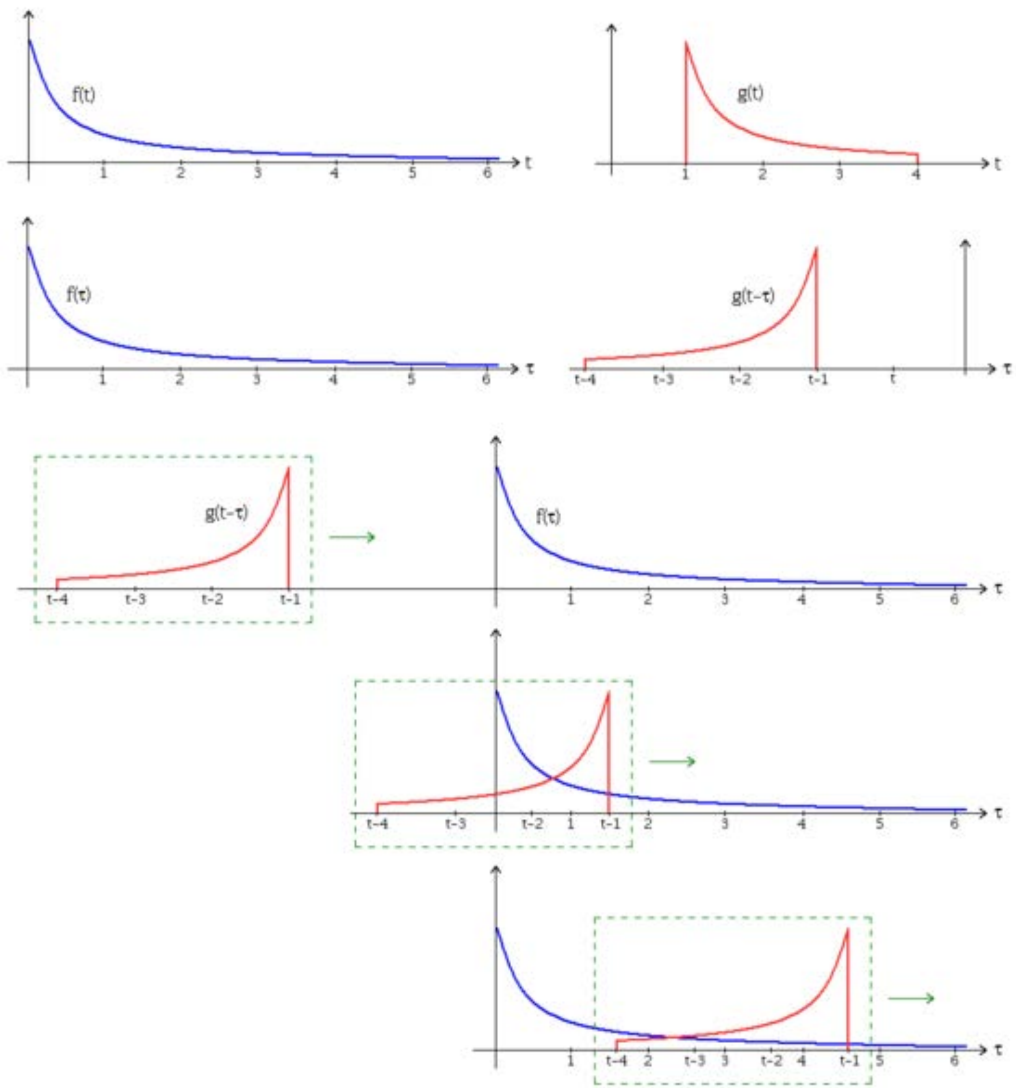
Problem:

- High frequencies and differentiation do not mix well.

Solution:

- Suppress high frequencies by
  - using the Discrete Fourier Transform,
  - convolving with a low-pass filter.

1D Convolution



$$g * f(t) = \int_{\tau} g(t-\tau) f(\tau) d\tau$$

EPFL

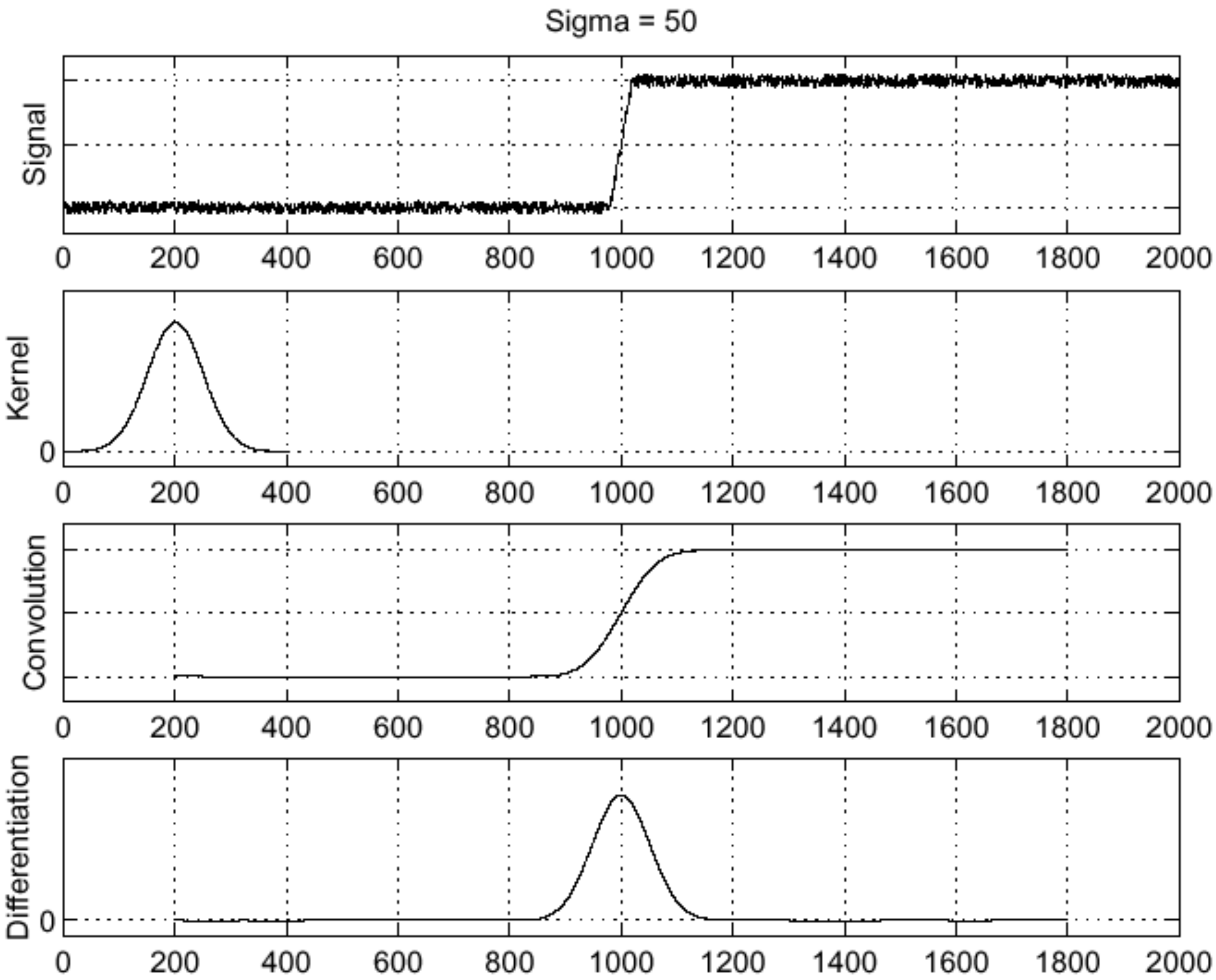
Smooth Before Differentiating

f

g

$g * f$

$\frac{\partial}{\partial x}(g * f)$

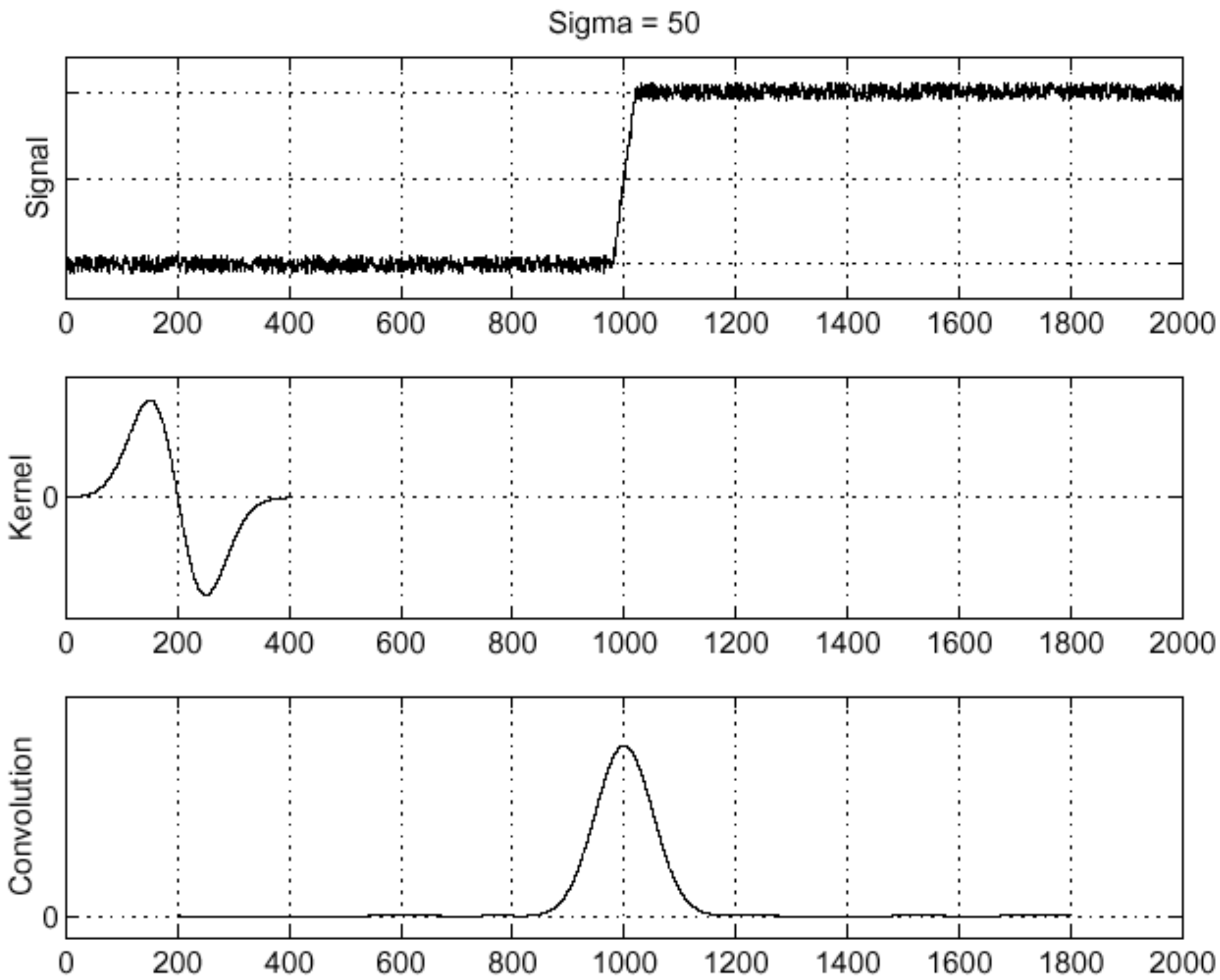


Simultaneously Smooth and Differentiate

f

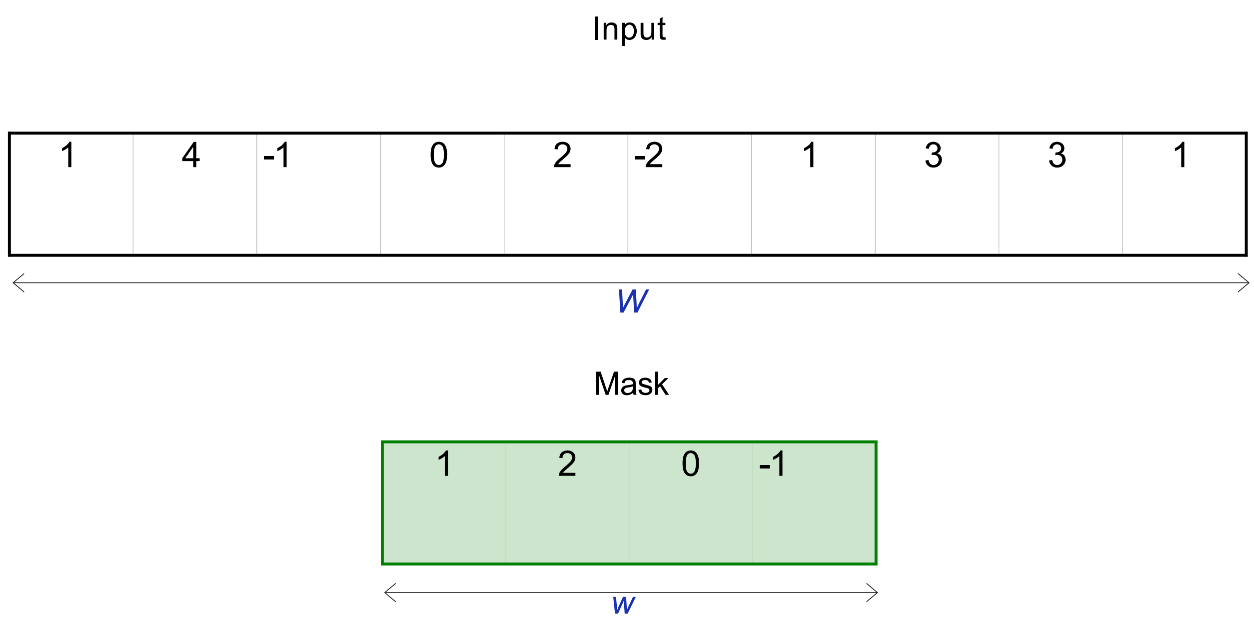
$\frac{\partial g}{\partial x}$

$$\frac{\partial}{\partial x}(g * f) = \frac{\partial g}{\partial x} * f$$

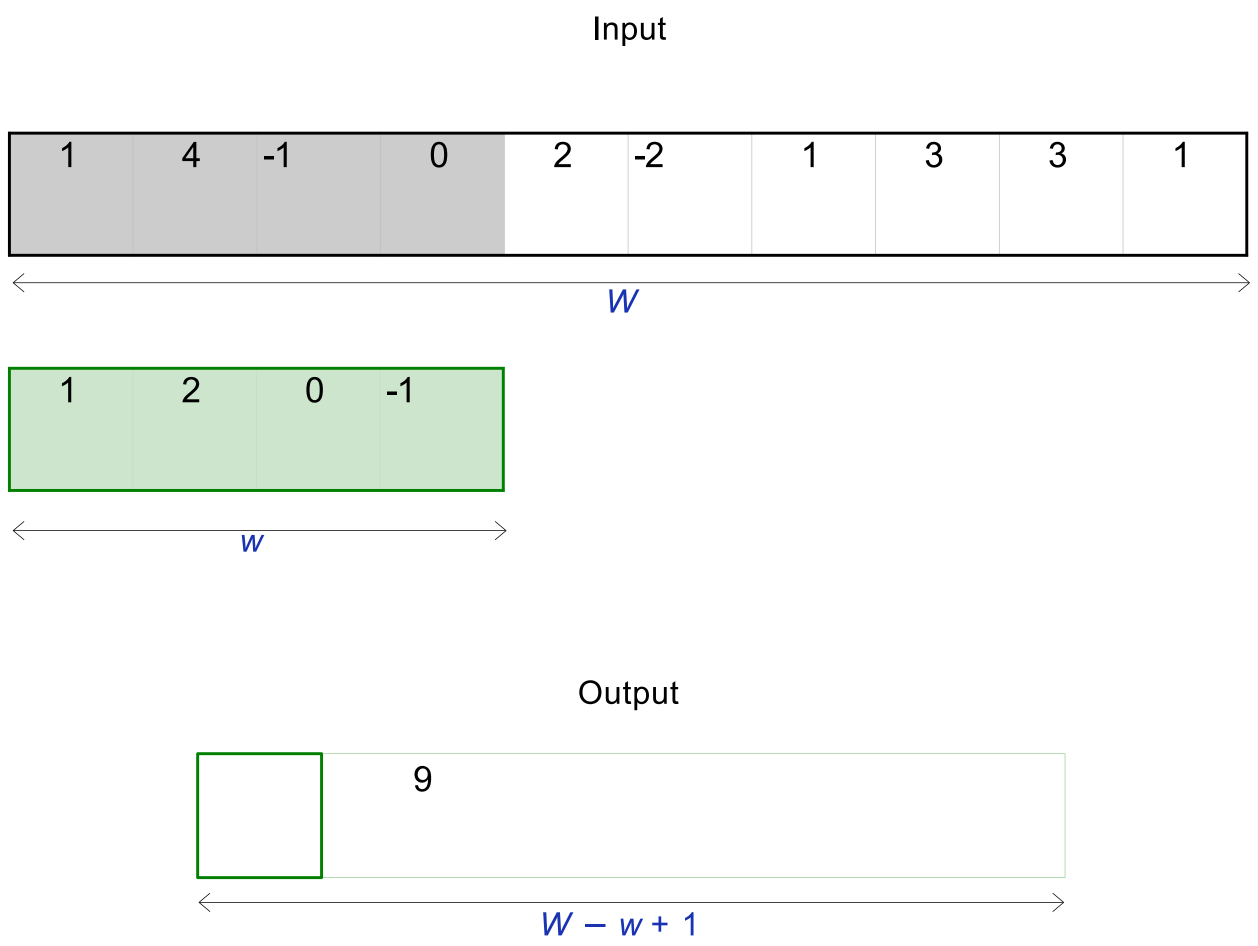


--> Faster because dg/dx can be precomputed.

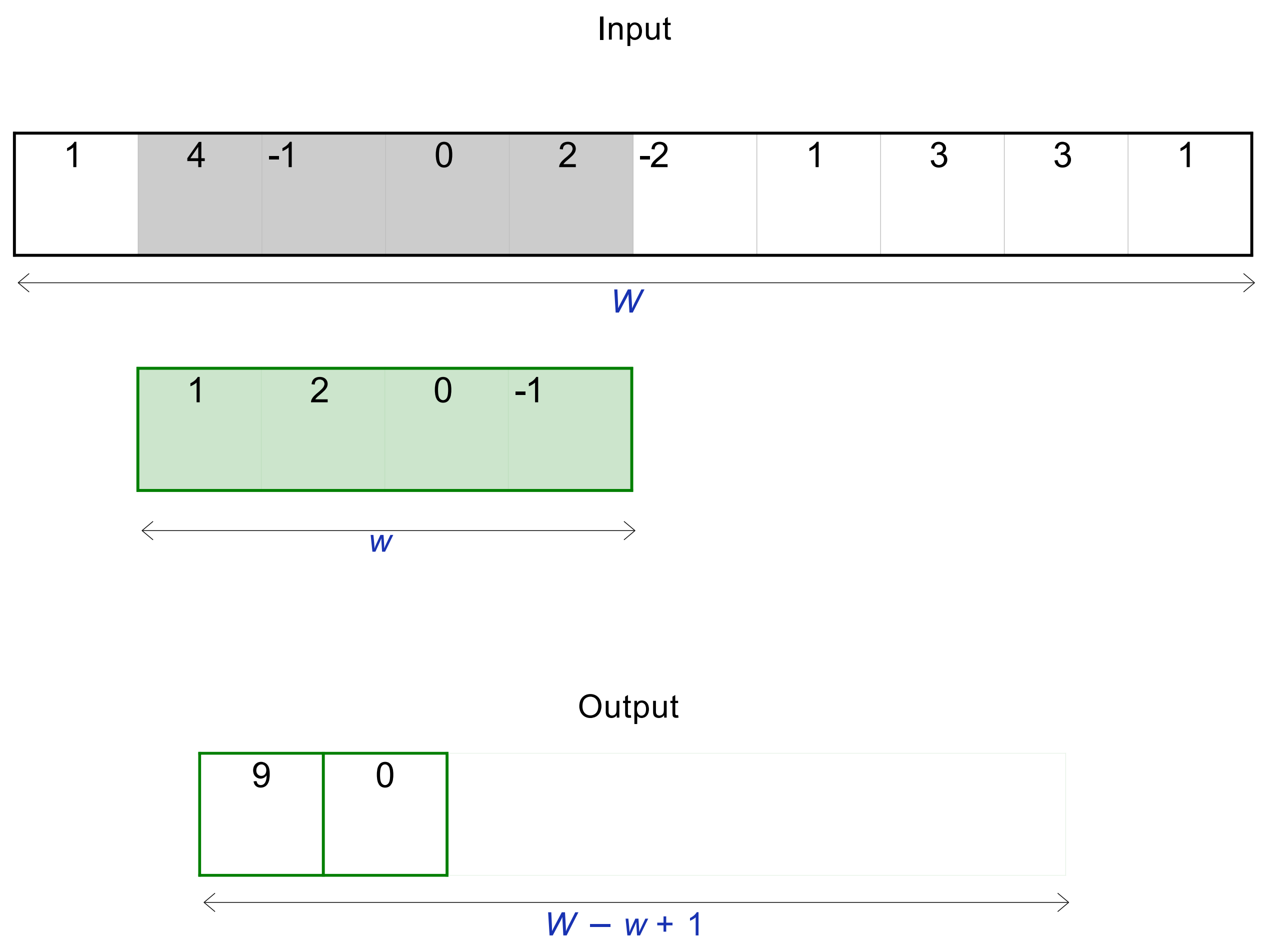
Discrete 1D Convolution



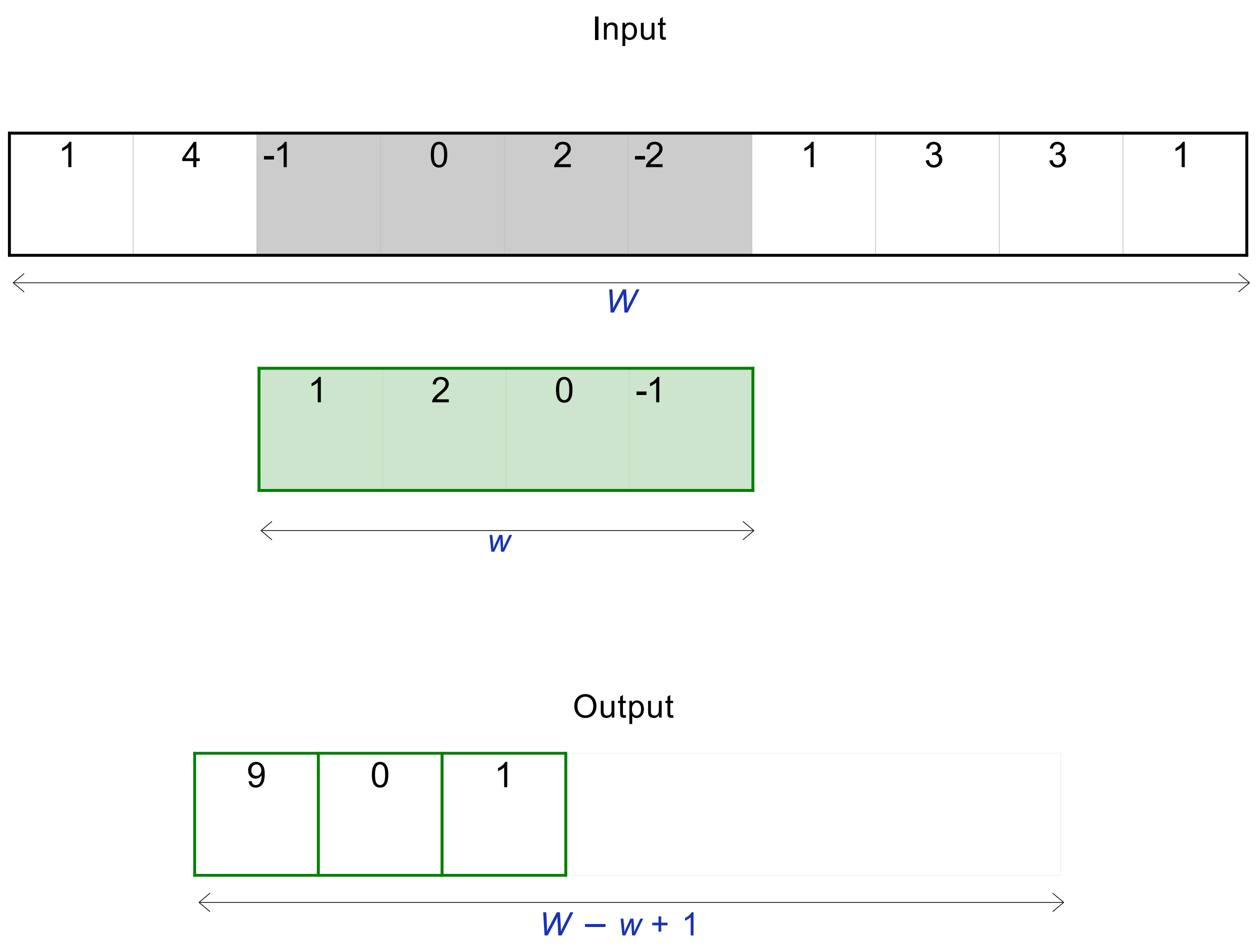
Discrete 1D Convolution



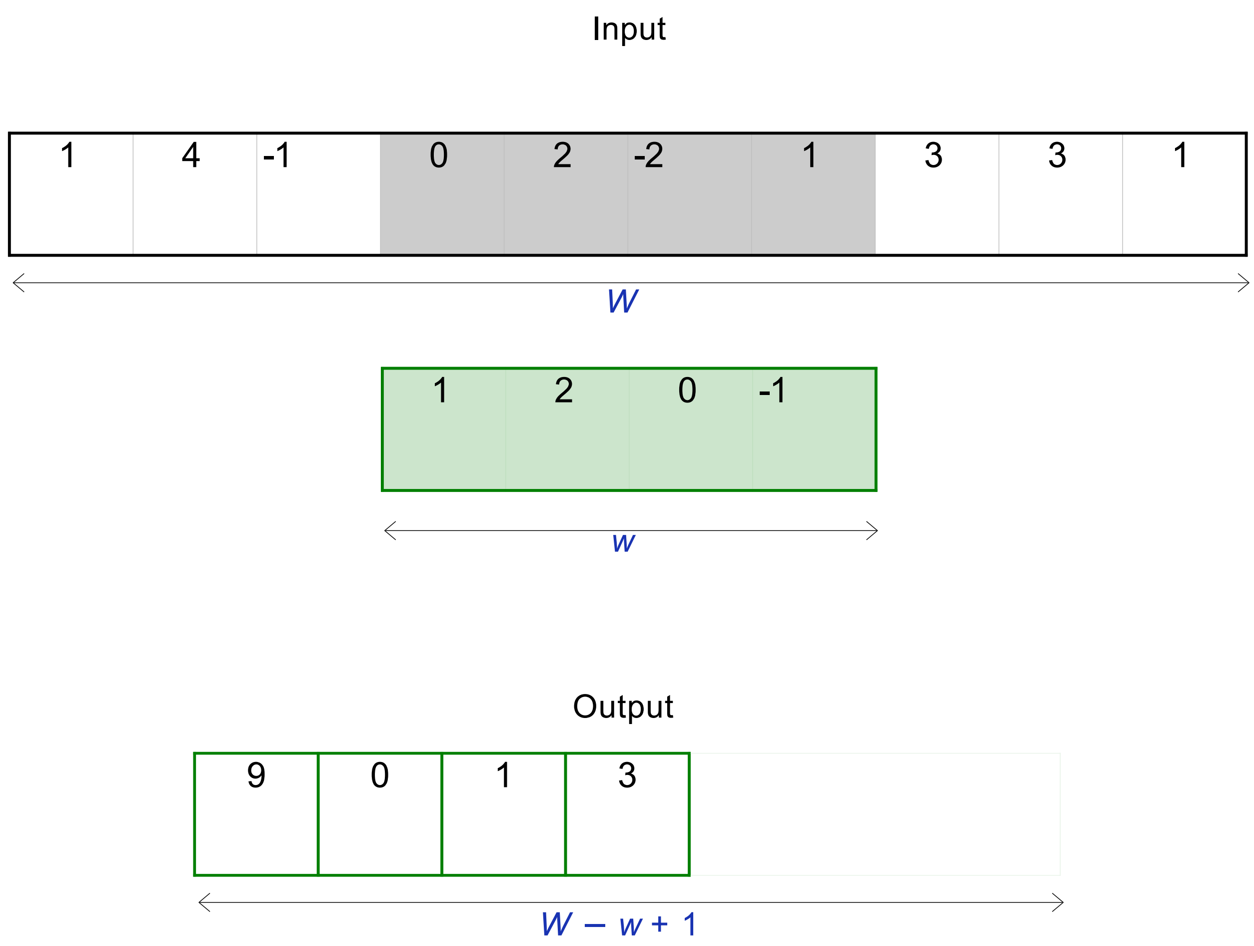
Discrete 1D Convolution



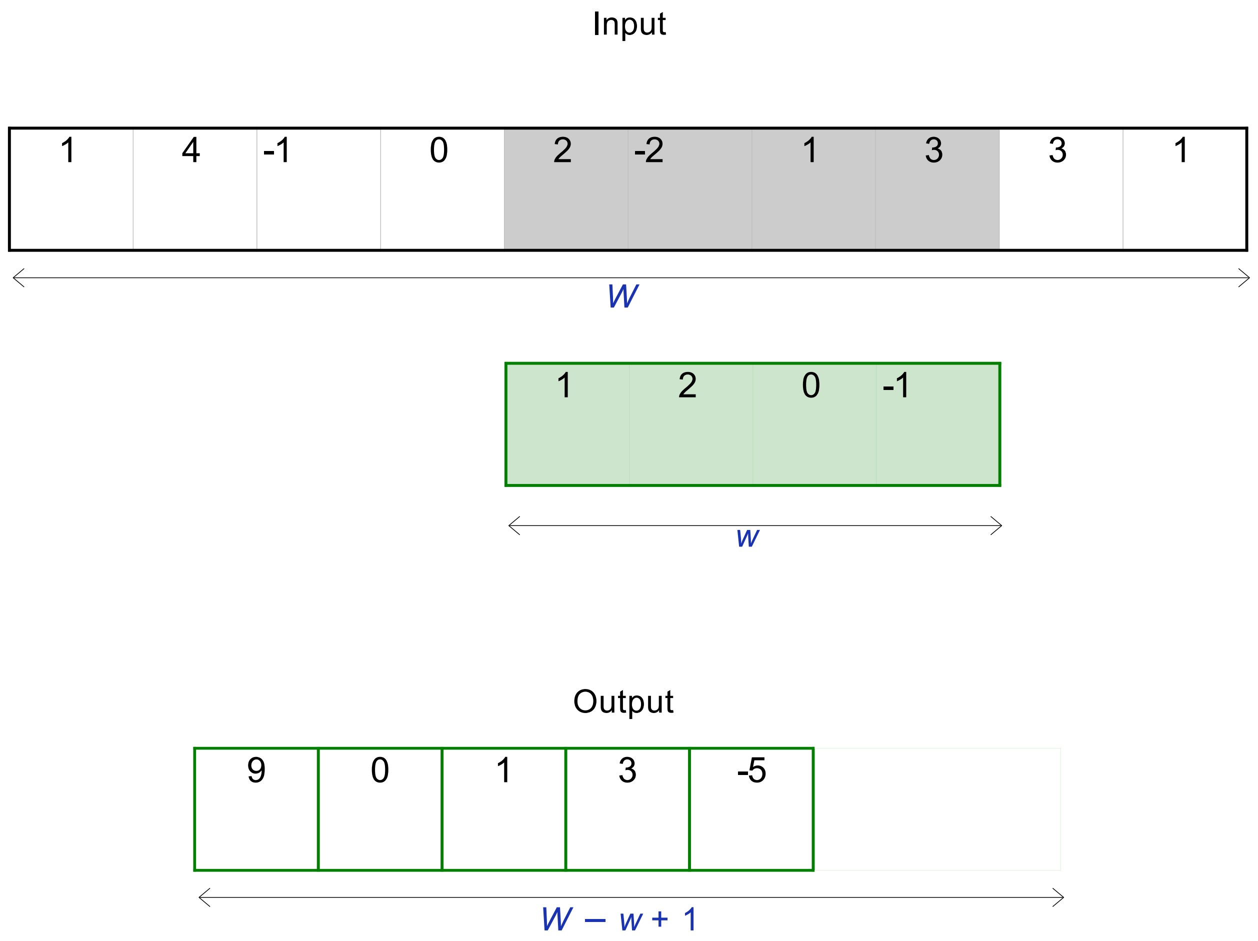
Discrete 1D Convolution



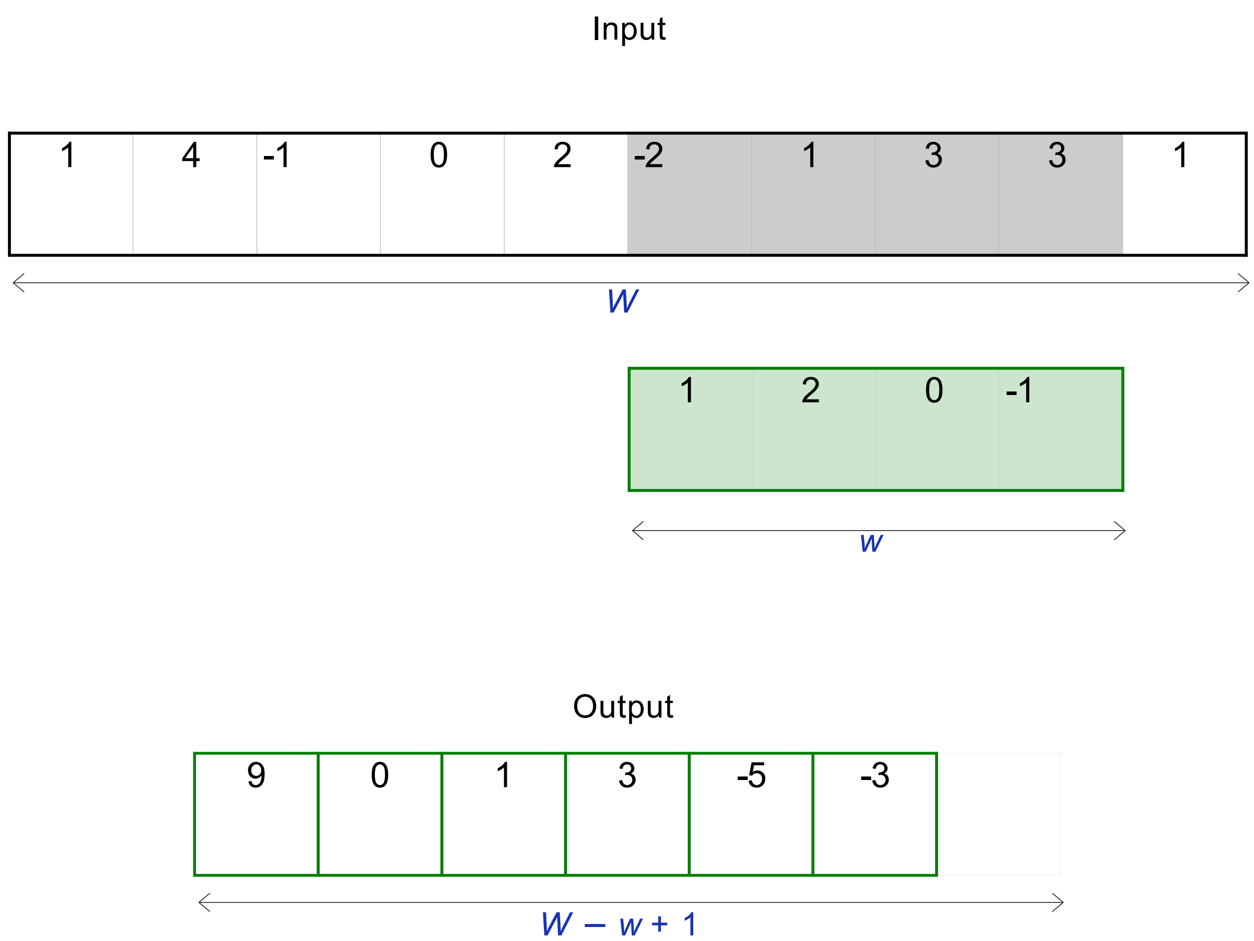
Discrete 1D Convolution



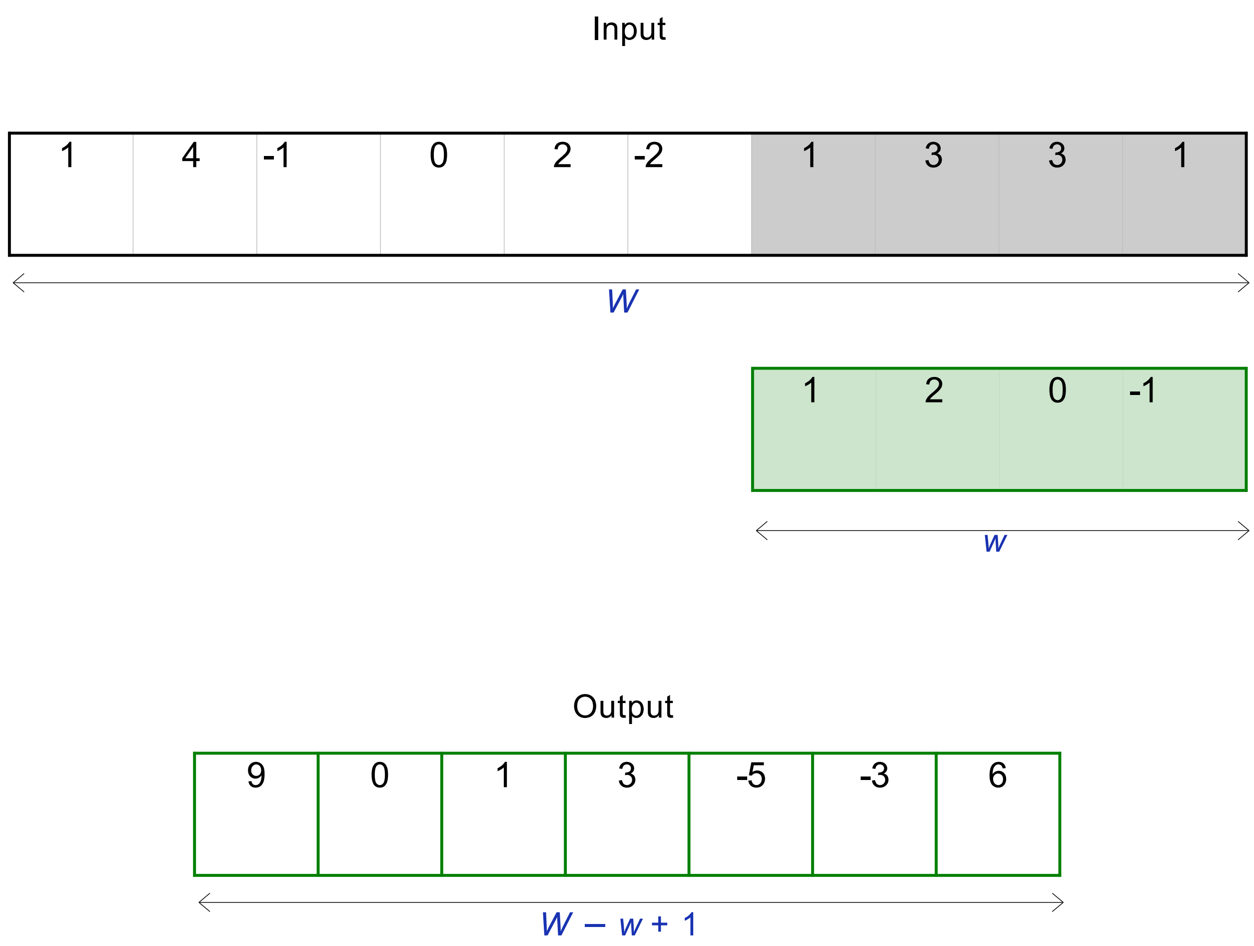
1D Convolution



Discrete 1D Convolution



Discrete 1D Convolution



Discrete 1D Convolution

Input

f

1	4	-1	0	2	-2	1	3	3	1

W

Mask

m

1	2	0	-1

w

Output

m*f

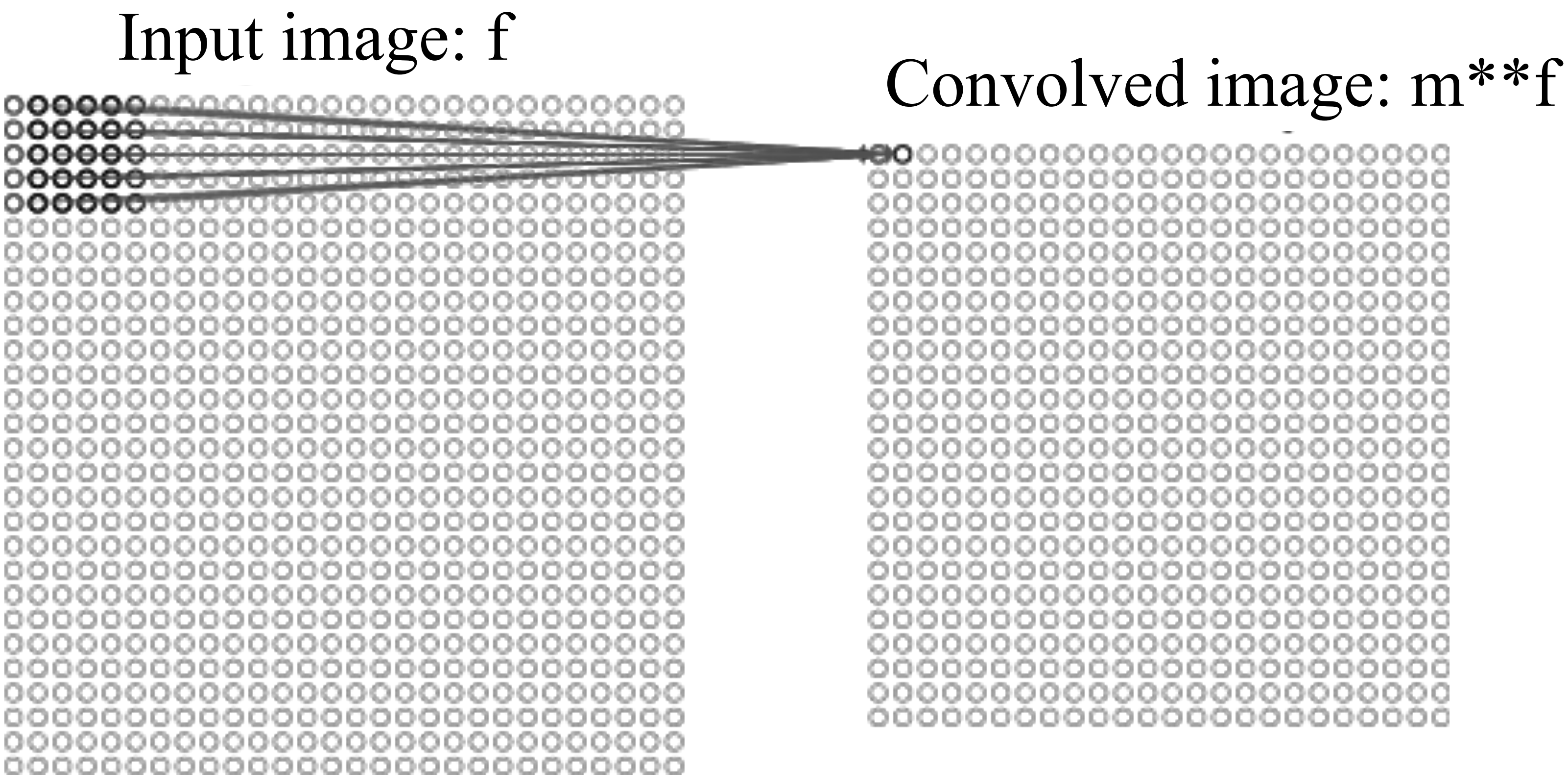
9	0	1	3	-5	-3	6

$W - w + 1$

$$m * f(x) = \sum_{i=0}^{w} m(i) f(x - i)$$

EPFL

Discrete 2D Convolution



Convolution mask m, also known as a *kernel*.

$$\begin{bmatrix} m_{11} & \dots & m_{1w} \\ \dots & \dots & \dots \\ m_{w1} & \dots & m_{ww} \end{bmatrix}$$

$$m**f(x,y) = \sum_{i=0}^{w} \sum_{j=0}^{w} m(i,j) f(x-i, y-j)$$

Convolution In C

Naive C implementation:

```
static double g[][]={{-1.0,-2.0,-1.0},{0.0,0.0,0.0},{1.0,2.0,1.0}};
{
        for(i=i0;i<N;i++)
            for(j=j0;j<N;j++){
              q[i][j]=0;
              for(a=a0;a<W;a++)
                for(b=b0;b<W,b++)
                  q[i][j]+=g[a][b]*p[i-a][j-b];
    }
}
```

Computational complexity:

- Lots of memory access

→ Slow, but can be sped up when the filters are separable.

N^2W^2 multiplications for a $N \times N$ image and a $W \times W$ mask.

Differentiation As Convolution

$$[-1,1] \rightarrow \frac{\partial f}{\partial x} \approx \frac{f(x+dx,y)-f(x,y)}{dx}$$

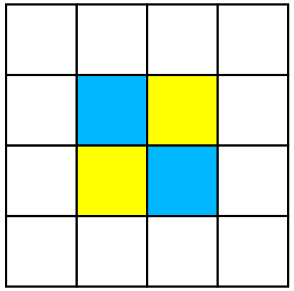
$$[-0.5,0,0.5] \rightarrow \frac{\partial f}{\partial x} \approx \frac{f(x+dx,y)-f(x-dx,y)}{2dx}$$

$$\begin{bmatrix} -1 \\ 1 \end{bmatrix} \rightarrow \frac{\partial f}{\partial y} \approx \frac{f(x,y+dy)-f(x,y)}{dy}$$

$$\begin{bmatrix} -0.5 \\ 0 \\ 0.5 \end{bmatrix} \rightarrow \frac{\partial f}{\partial y} \approx \frac{f(x,y+dy)-f(x,y-dy)}{2dy}$$

→ Use wider masks to add some smoothing

2x2 Masks:
Roberts Operator

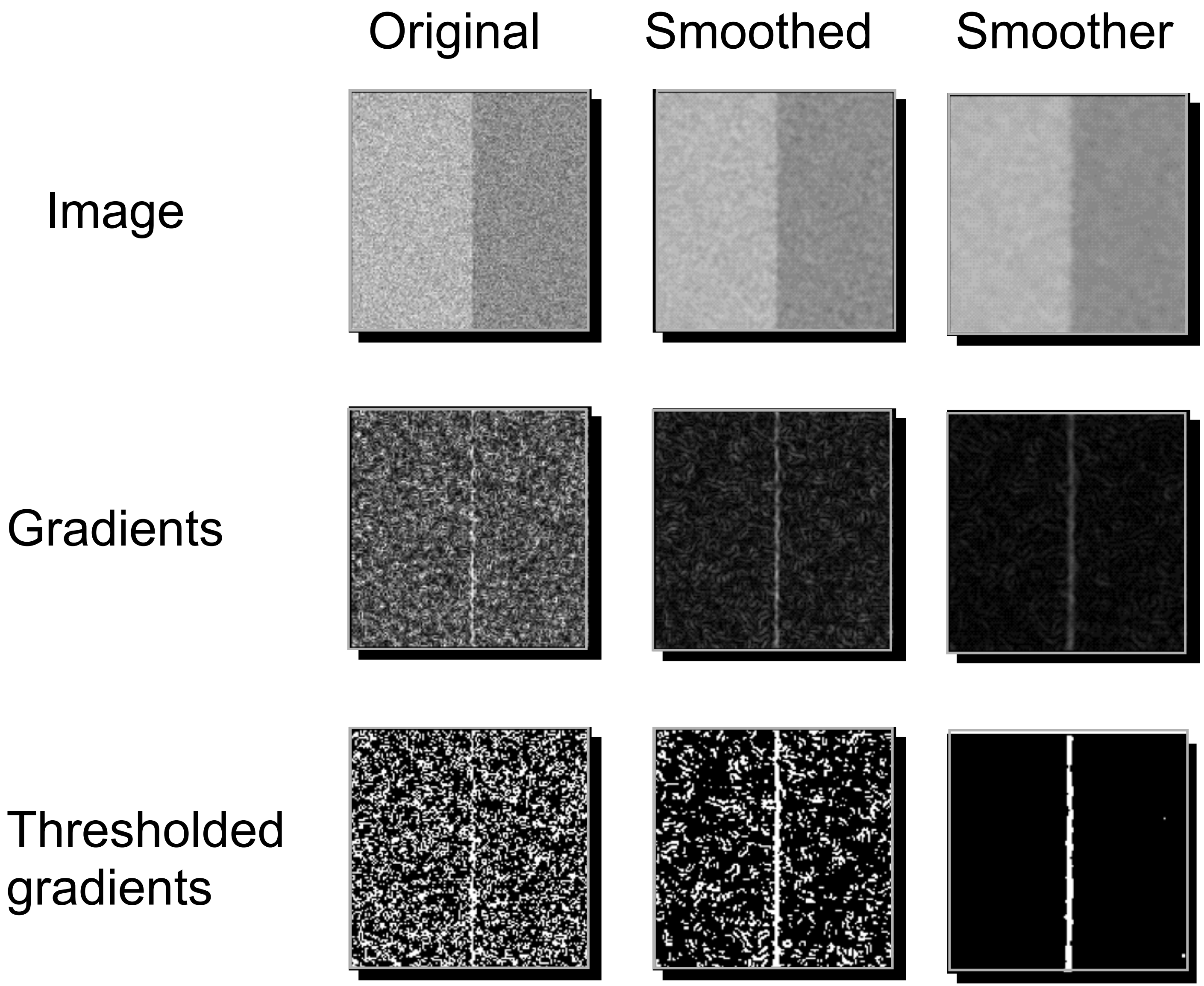


$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \text{and} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

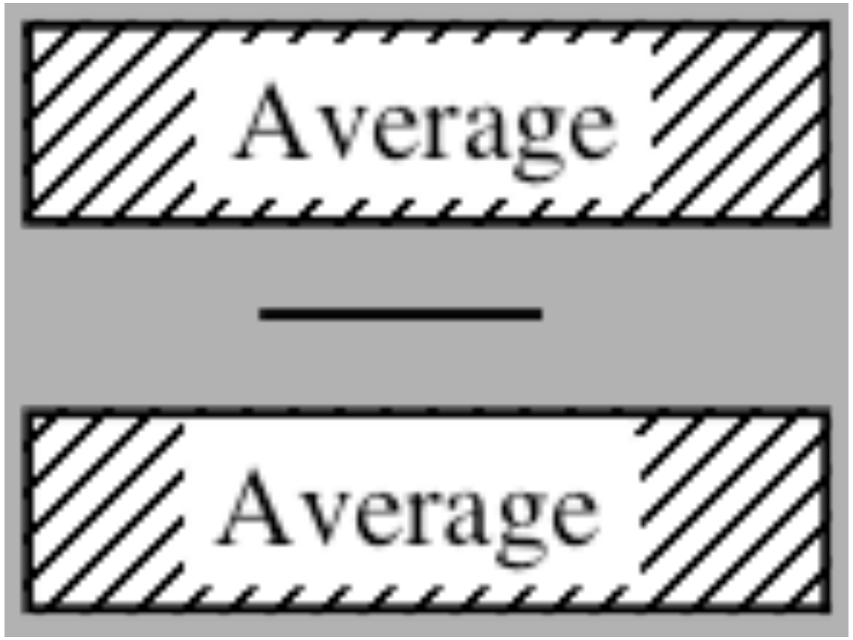
$$G = \sqrt{\left[I(x+1,y+1) - I(x,y) \right]^2 + \left[I(x+1,y-1) - I(x-1,y+1) \right]^2}$$

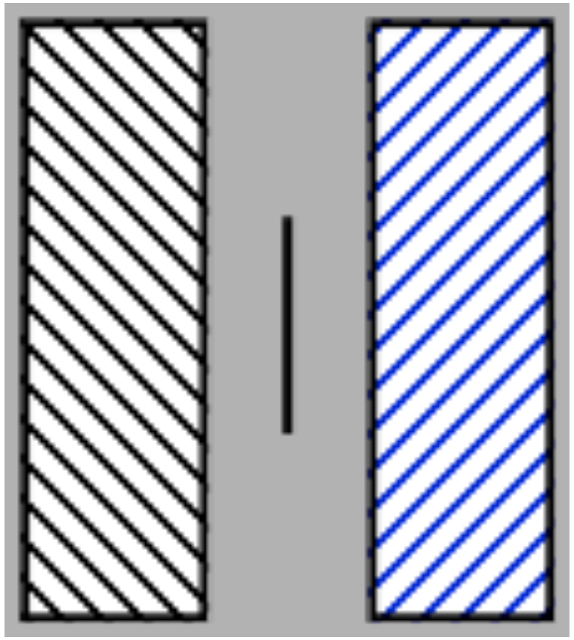
→ Equivalent to fitting plane to patch.

Benefits of Smoothing



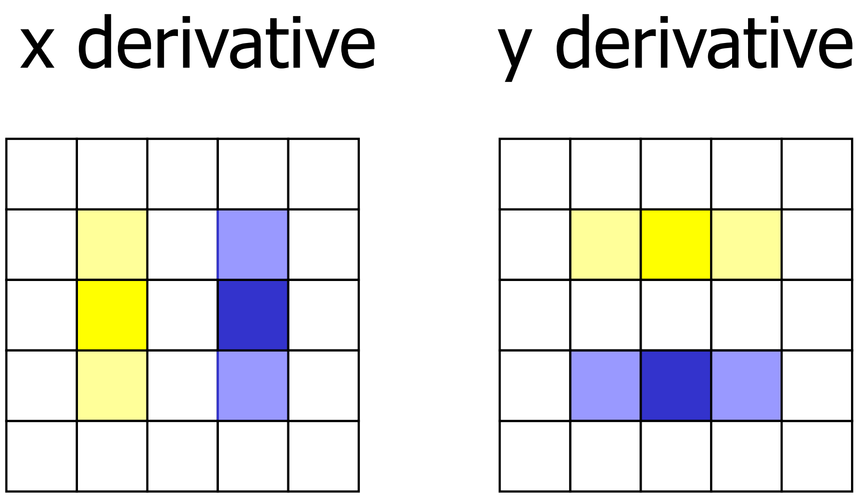
Smoothing and Differentiating





Compute the difference of averages on either side of the central pixel.

3X3 Masks



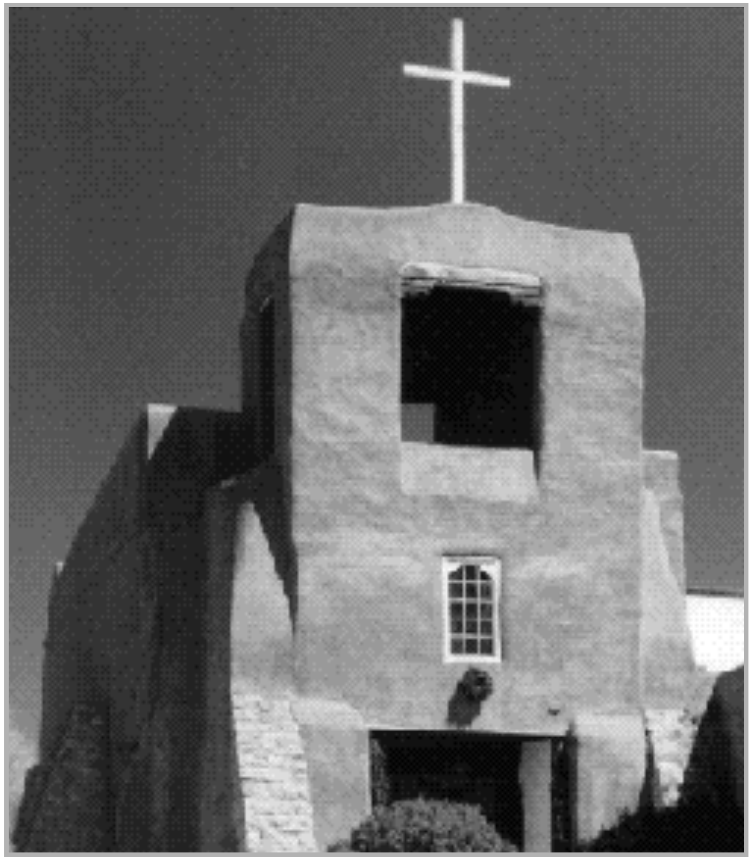
$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \text{and} \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Prewitt operator

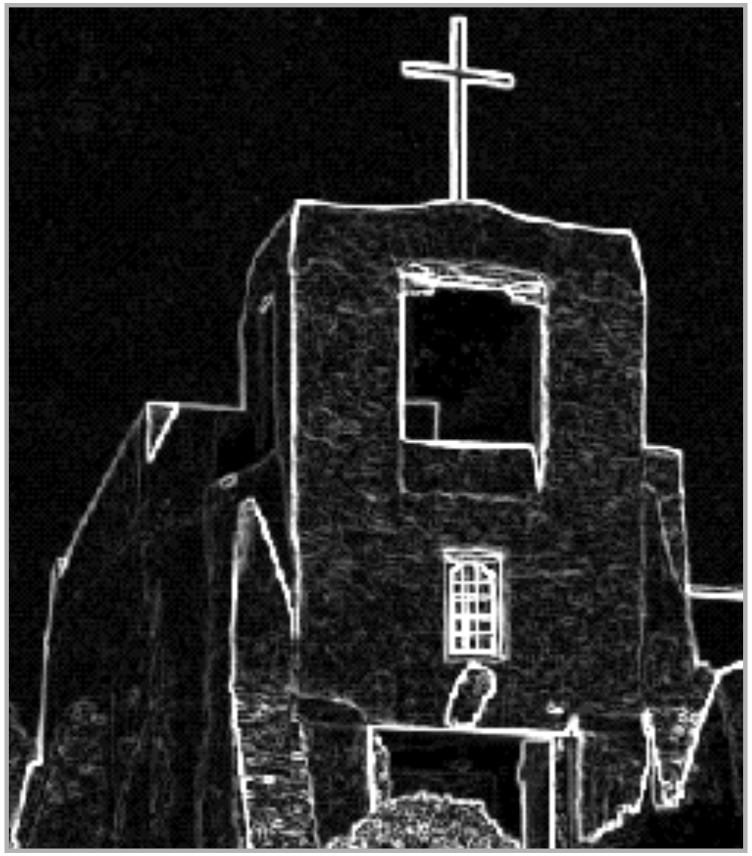
$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \text{and} \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Sobel operator

Prewitt Example

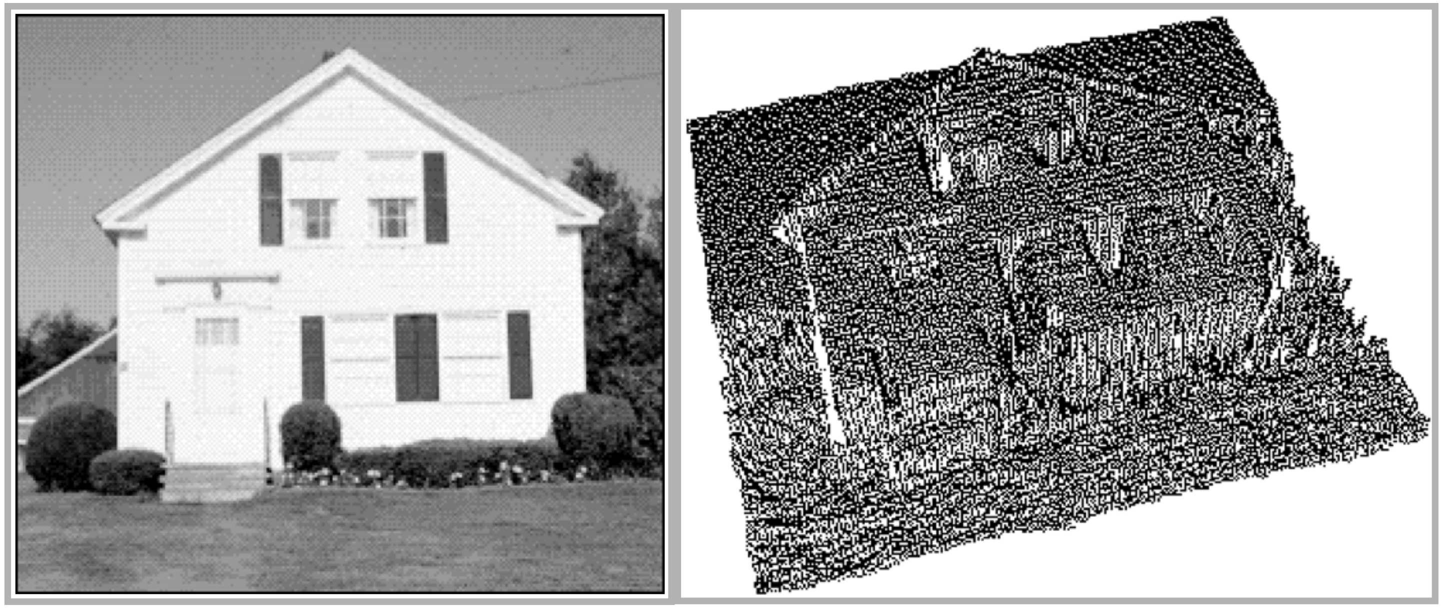


Santa Fe Mission

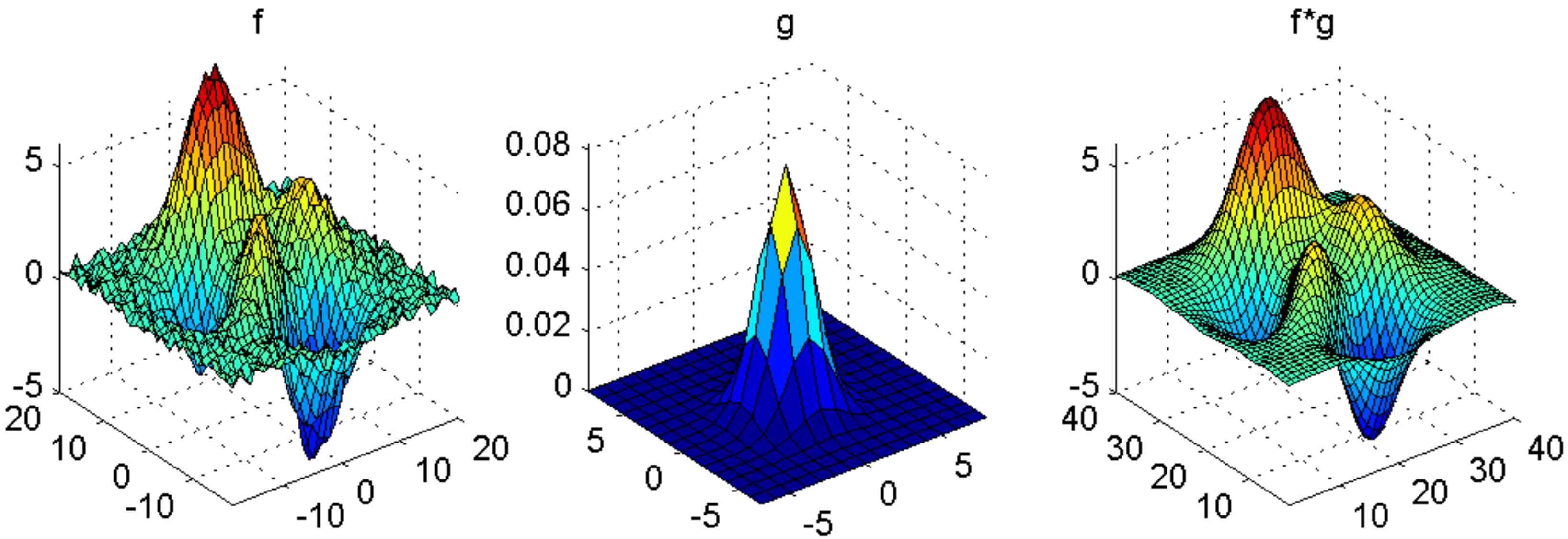


Gradient Image

Sobel Example

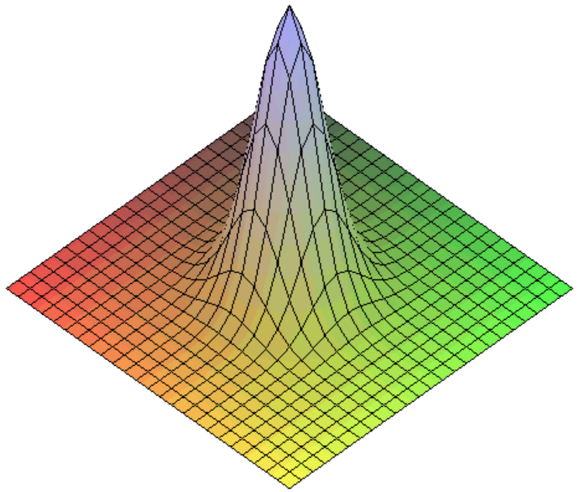


Gaussian Smoothing

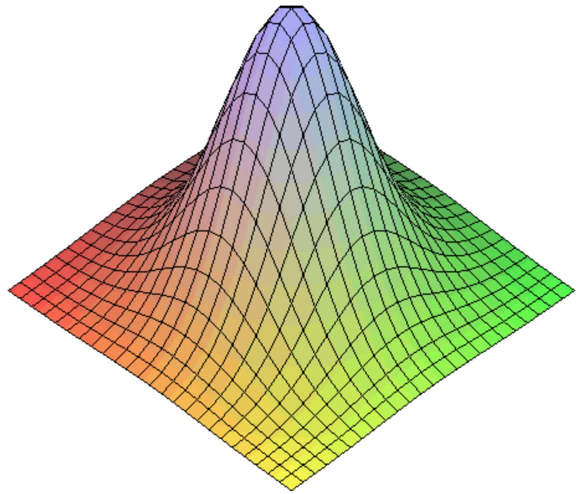


- More principled way to eliminate high frequency noise.
- Is fast because the kernel is
  - small,
  - separable.

Gaussian Functions



$\sigma = 1$

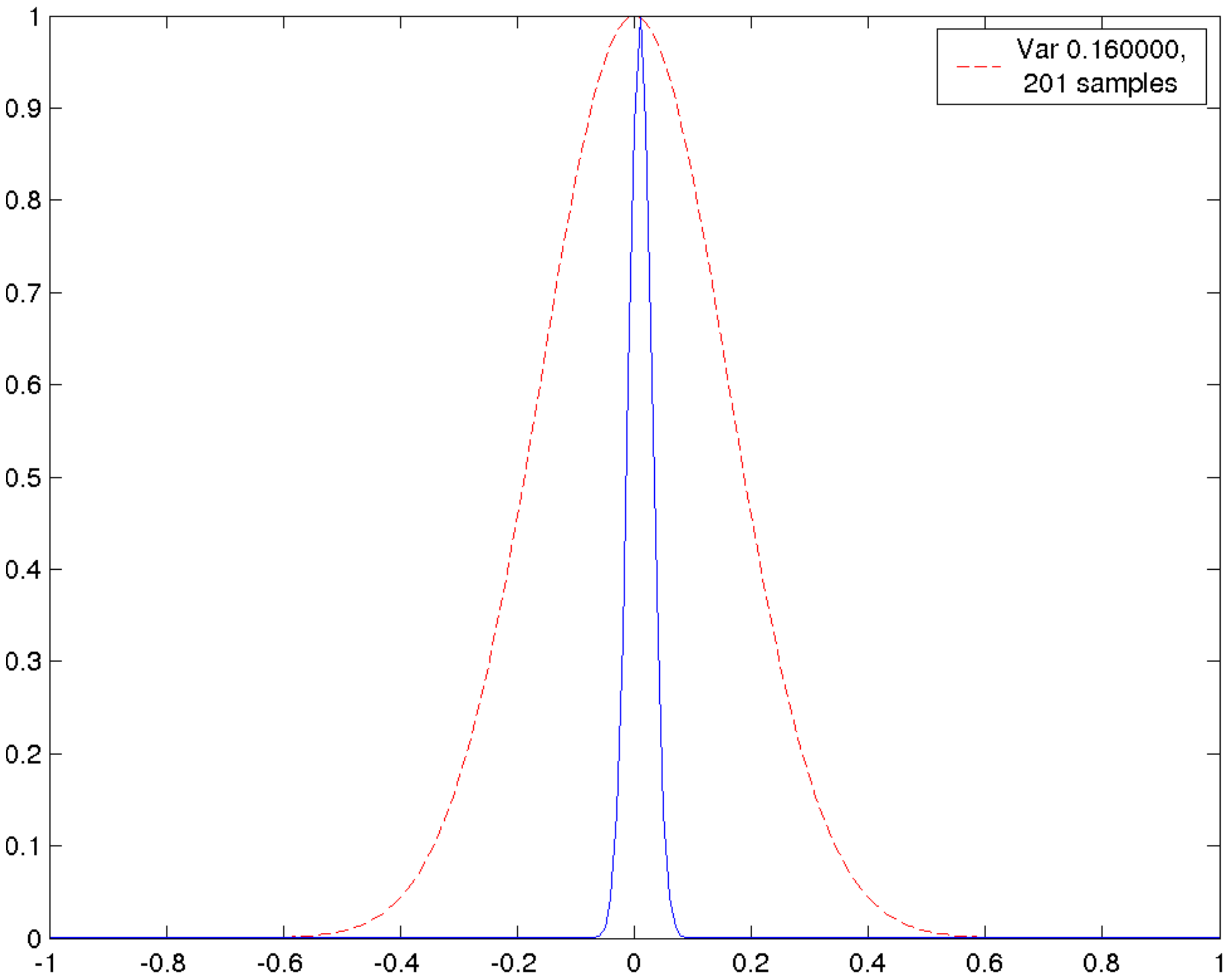


$\sigma = 2$

$$g_2(x, y) = \frac{1}{2\pi\sigma^2} \exp(-(x^2 + y^2)/2\sigma^2)$$

- The integral is always 1.0
- The larger σ, the broader the Gaussian is.
- As σ approaches 0, the Gaussian approximates a Dirac function.

DFT of a Gaussian



- The DFT of a Gaussian is a Gaussian.
- It has finite support.
- Its width is inversely proportional to that of the original Gaussian.

Gaussians as Low-Pass Filters

- The Fourier transform of a convolution is the product of their Fourier transforms: $\mathscr{F}(g * f) = \mathscr{F}(g)\mathscr{F}(f)$.
- If g is a Gaussian, so is $\mathscr{F}(g)$.
- Furthermore if g is broad, the support of $\mathscr{F}(g)$ is small.
- So is the support of $\mathscr{F}(g * f)$.
- There are no more high-frequencies in $g * f$.

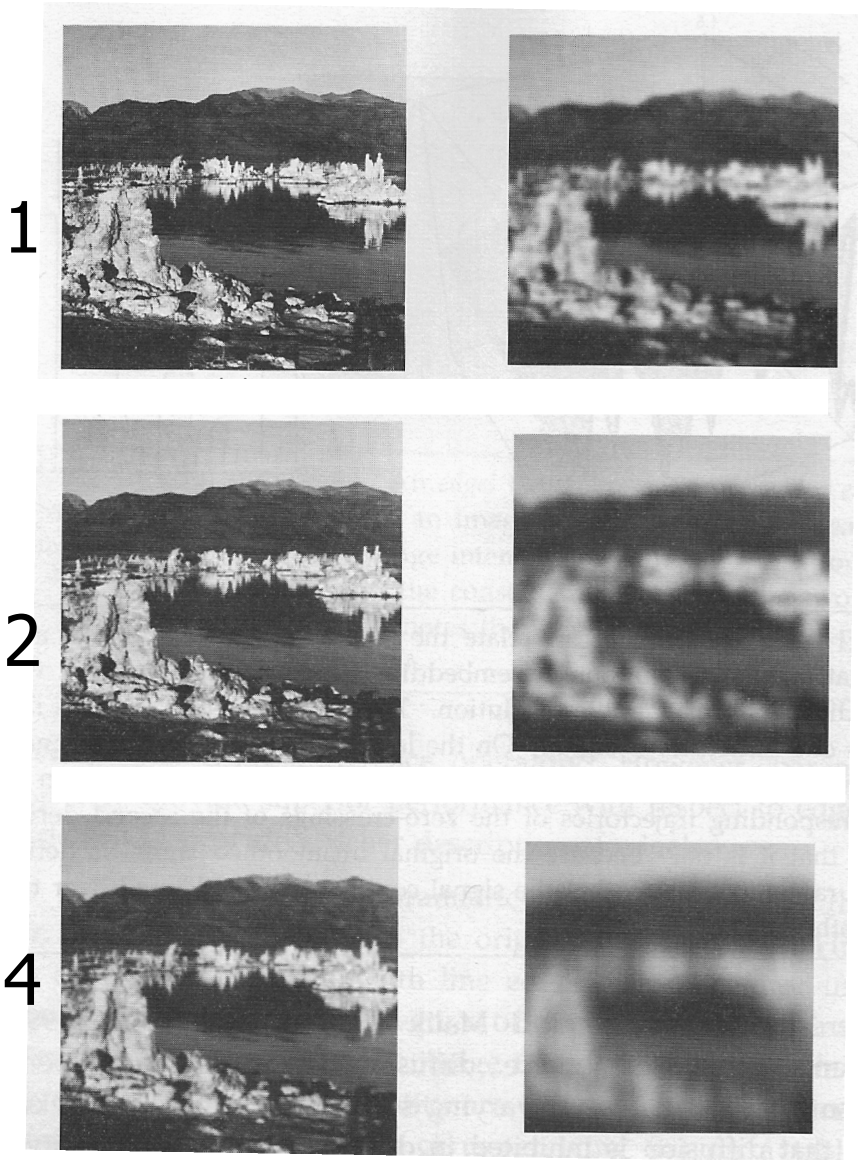
—> Convolving with a Gaussian suppresses the high frequencies.

Gaussian Smoothed Images

σ =1

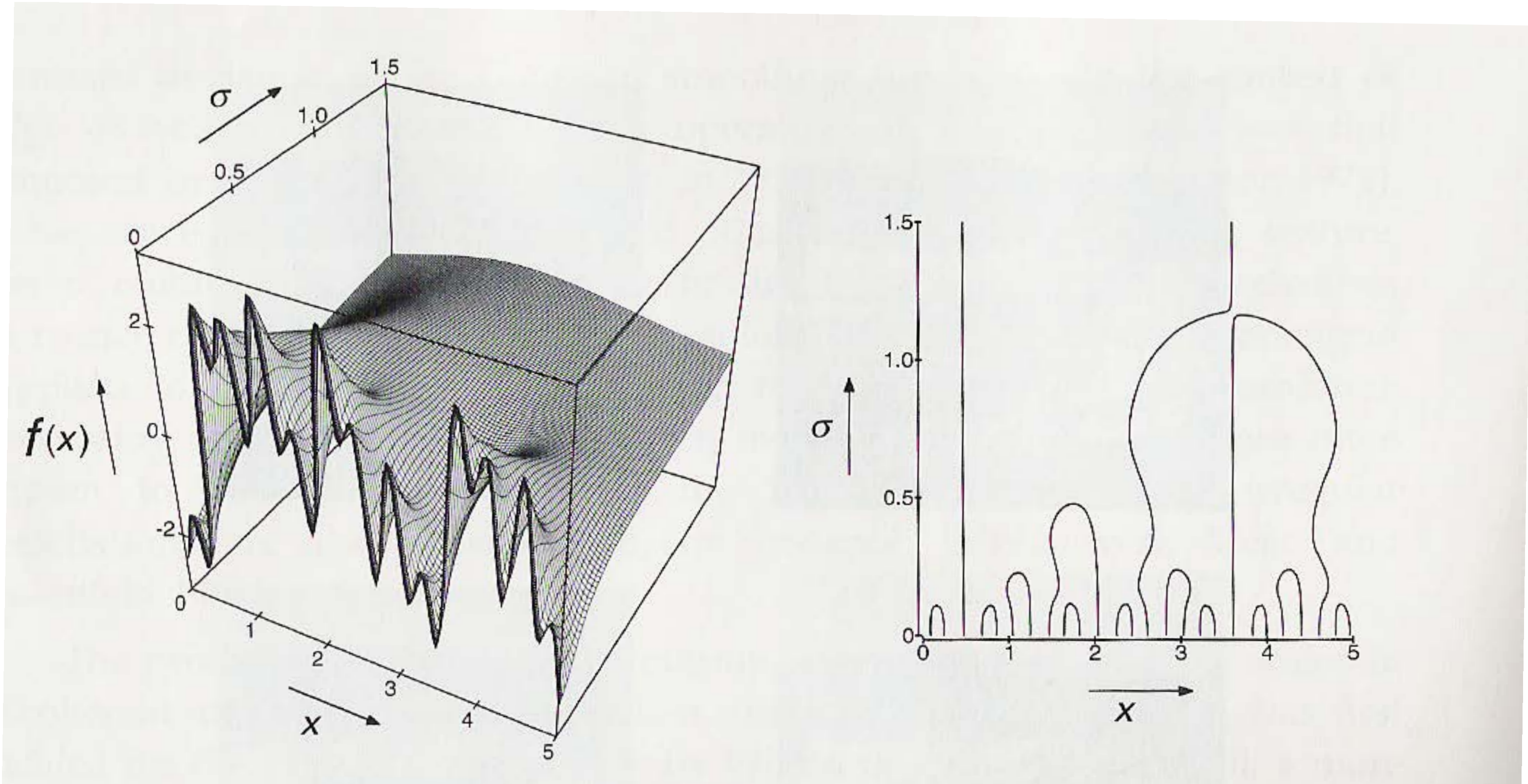
σ =2

σ =4



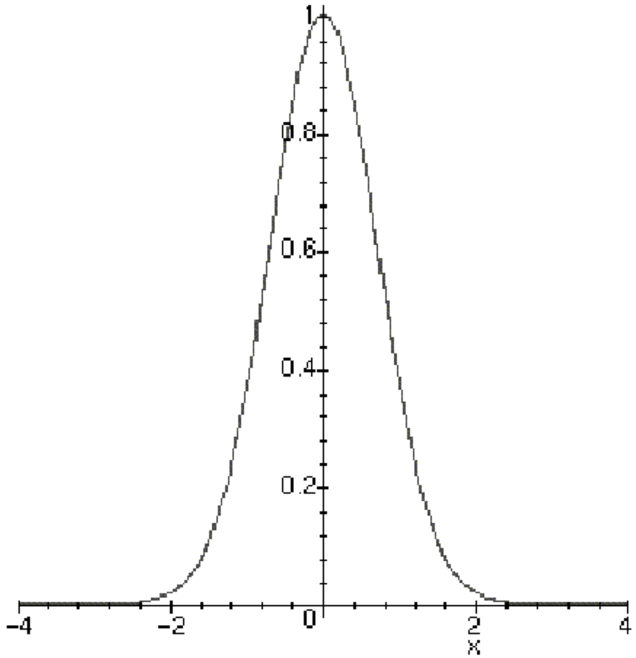
Original image Blurred image

Scale Space



Increasing scale (σ) removes high frequencies (details) but never adds artifacts.

Separability



$$g_1(x) = \frac{1}{\sqrt{\pi}\sigma} \exp(-x^2/\sigma^2)$$

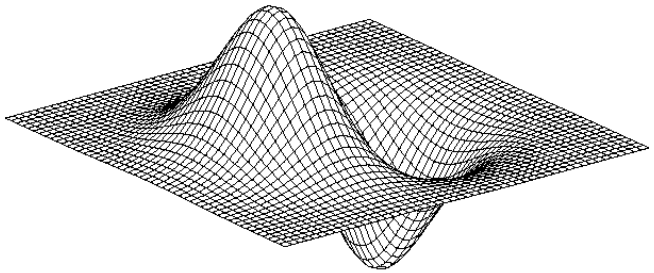
$$g_2(x,y) = g_1(x)g_1(y)$$

$$\int_u \int_v g_2(u,v) f(x-u, y-v)\,du\,dv = \int_u g_1(u) \left(\int_v g_1(v) f(x-u, y-v)\,dv \right) du$$

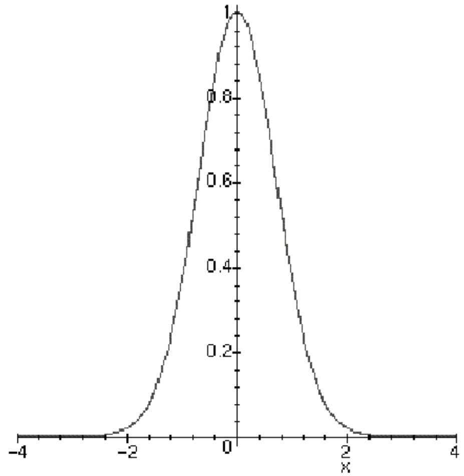
$$= \int_v g_1(v) \left(\int_u g_1(u) f(x-u, y-v)\,du \right) dv$$

—> 2D convolutions are never required. Smoothing can be achieved by successive 1D convolutions, which is faster.

Continuous Gaussian Derivatives



=



x

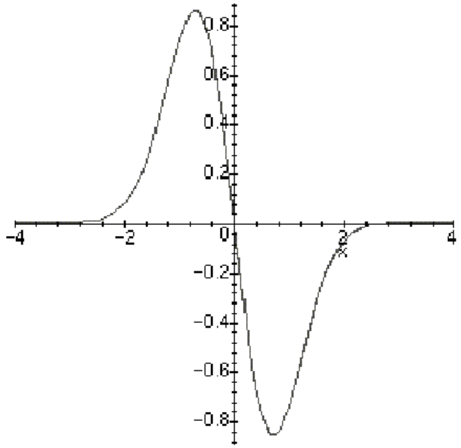
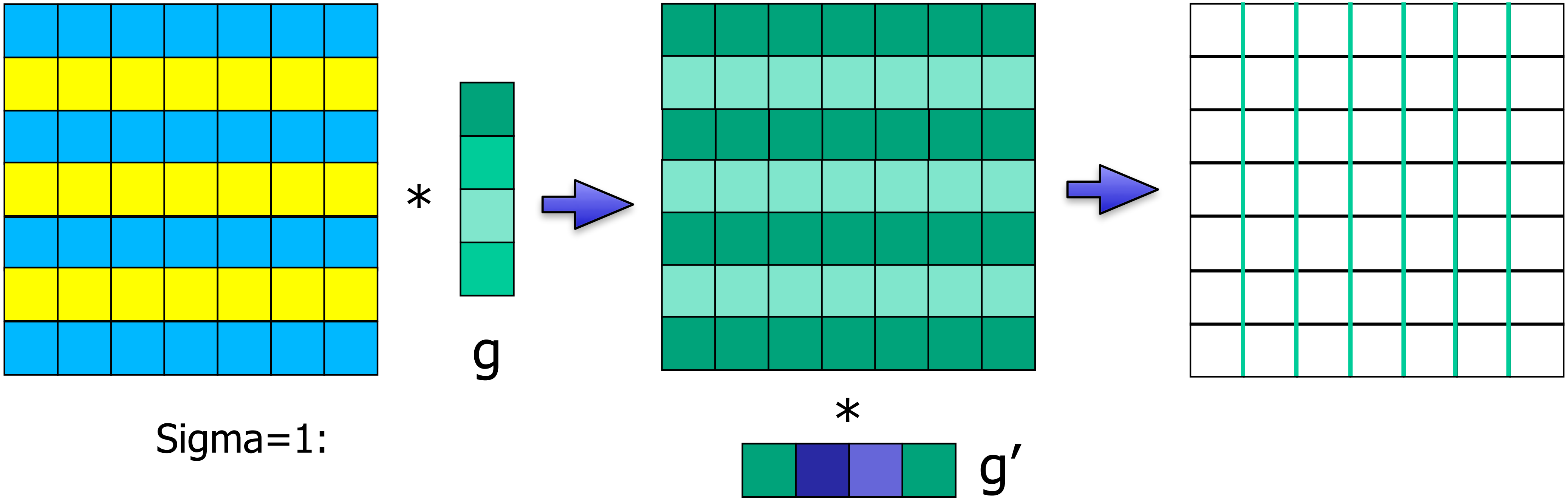


Image derivatives computed by convolving with the derivative of a Gaussian:

$$\frac{\partial}{\partial x}\int_u\int_v g_2(u,v)f(x-u,y-v)dudv = \int_u g_1^{'}(u)\left(\int_v g_1(v)f(x-u,y-v)dv\right)du$$

$$\frac{\partial}{\partial y}\int\int g_2(u,v)f(x-u,y-v)dudv = \int_v g_1^{'}(v)\left(\int_u g_1(u)f(x-u,y-v)du\right)dv$$

Discrete Gaussian Derivatives



Sigma=1:

g : 0.000070 0.010332 0.207532 0.564131 0.207532 0.010332 0.000070

g′: 0.000418 0.041330 0.415065 0.000000 -0.415065 -0.041330 -0.000418

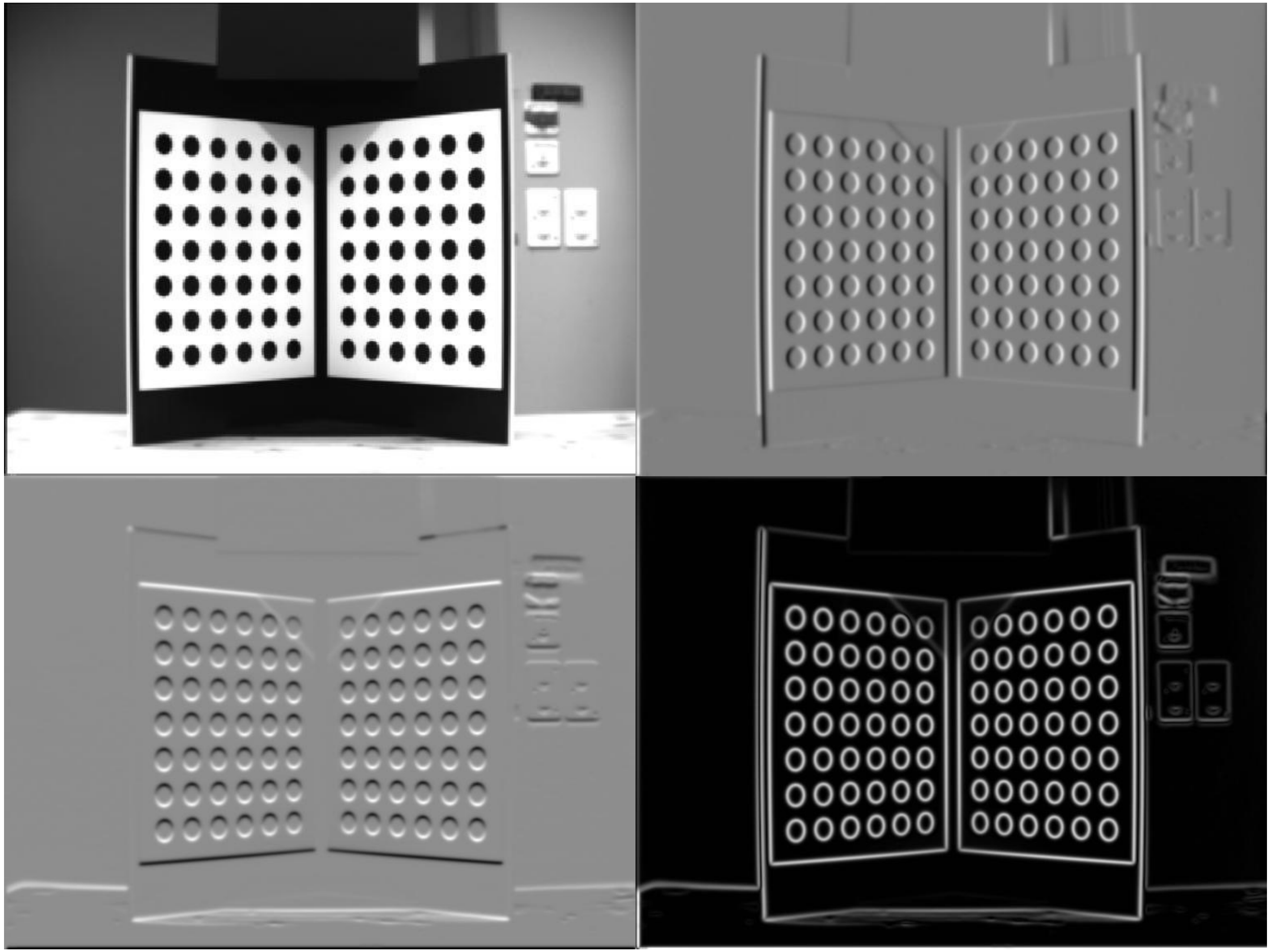
Sigma=2:

g : 0.005167 0.029735 0.103784 0.219712 0.282115 0.219712 0.103784 0.029735 0.005167

g′: 0.010334 0.044602 0.103784 0.109856 0.000000 -0.109856 -0.103784 -0.044602 -0.010334

—> Only requires 1D convolutions with relatively small masks.

Derivative Images



I

$\frac{\partial I}{\partial x}$

$\frac{\partial I}{\partial y}$

$\sqrt{\frac{\partial I}{\partial x}^2 + \frac{\partial I}{\partial y}^2}$

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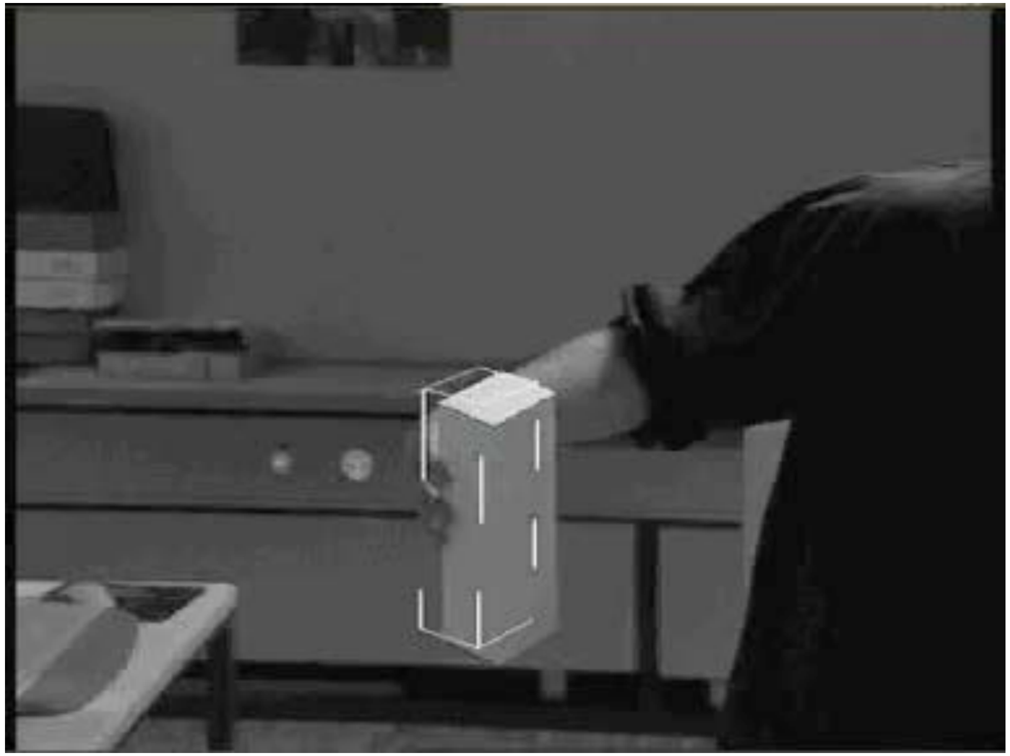
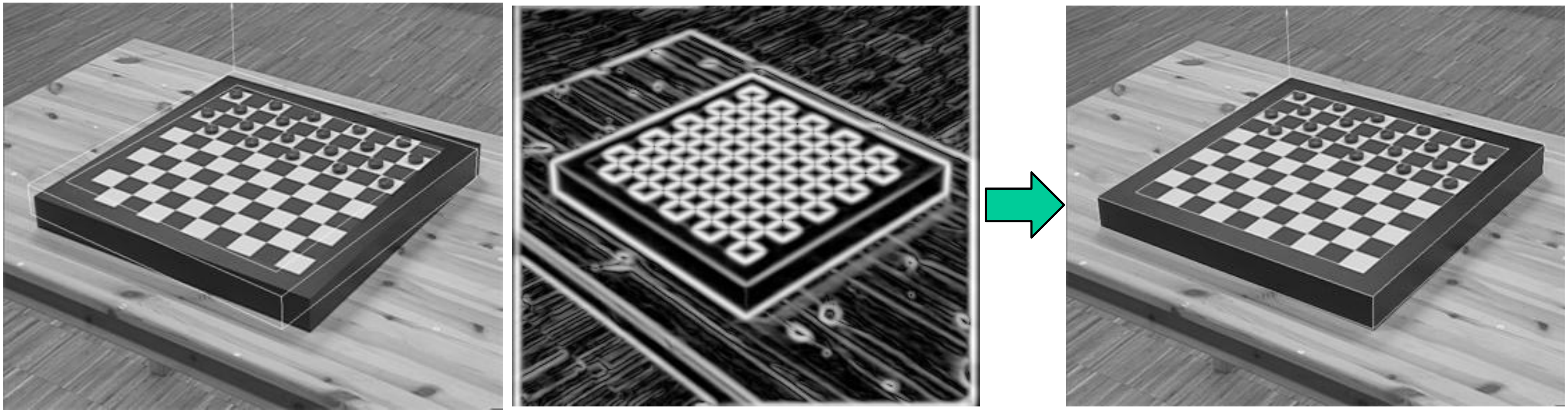
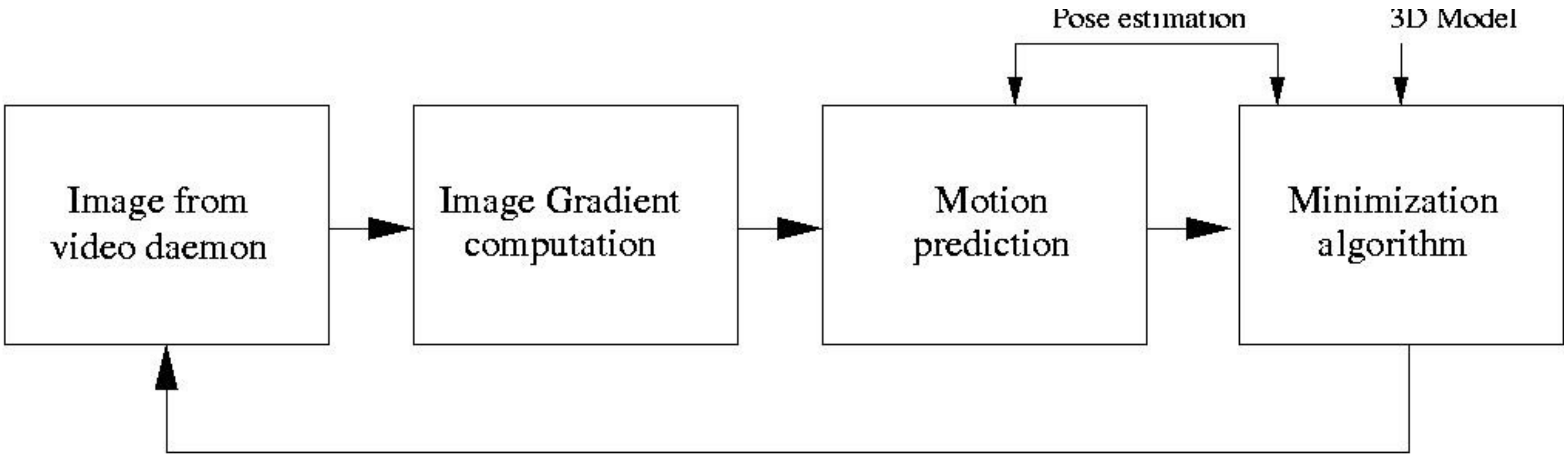
Gradient-Based Tracking



Maximize edge-strength along projection of the 3—D wireframe.

Gradient Maximization





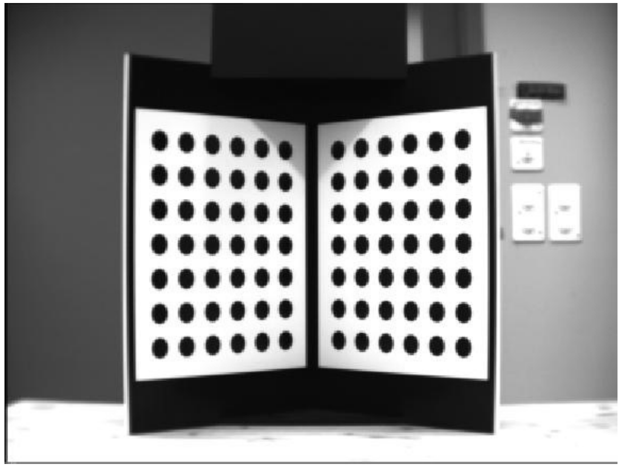
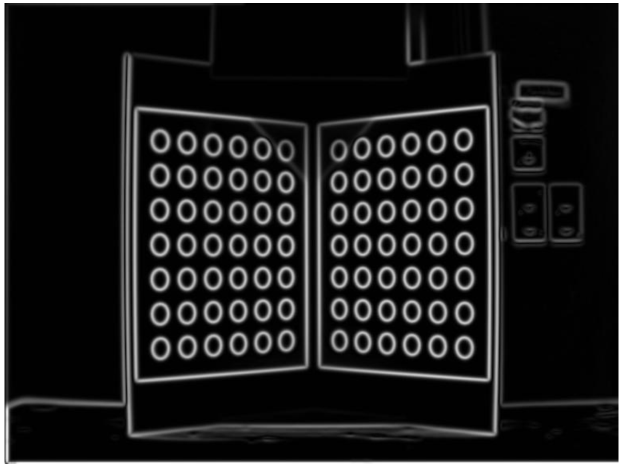
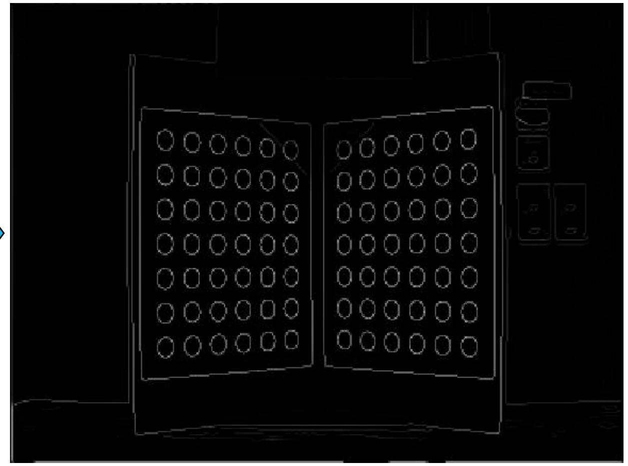
Real-Time Tracking

Canny Edge Detector

I

$$\sqrt{\frac{\partial I}{\partial x}^2 + \frac{\partial I}{\partial y}^2}$$

Thinned gradient images


Canny Edge Detector

Convolution

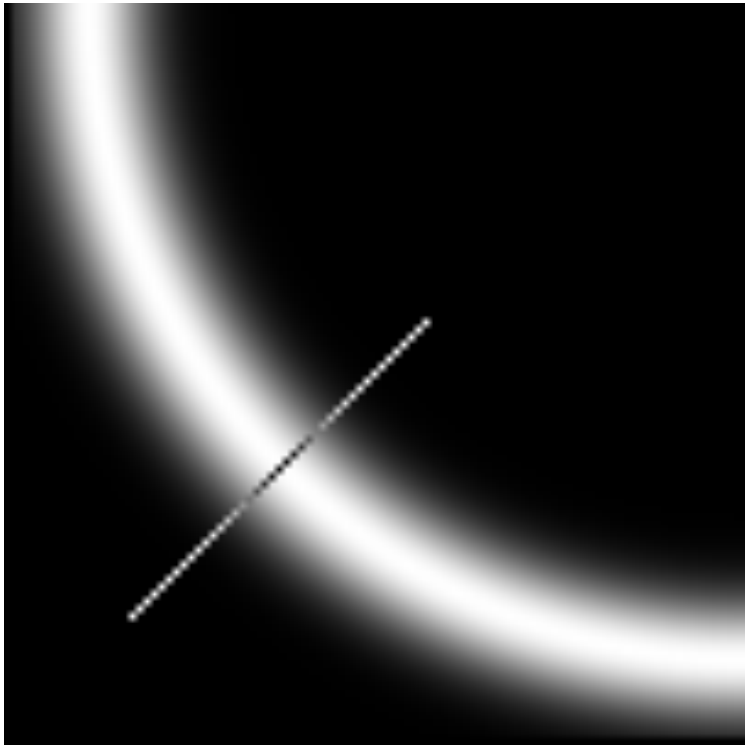
- Gradient strength
- Gradient direction

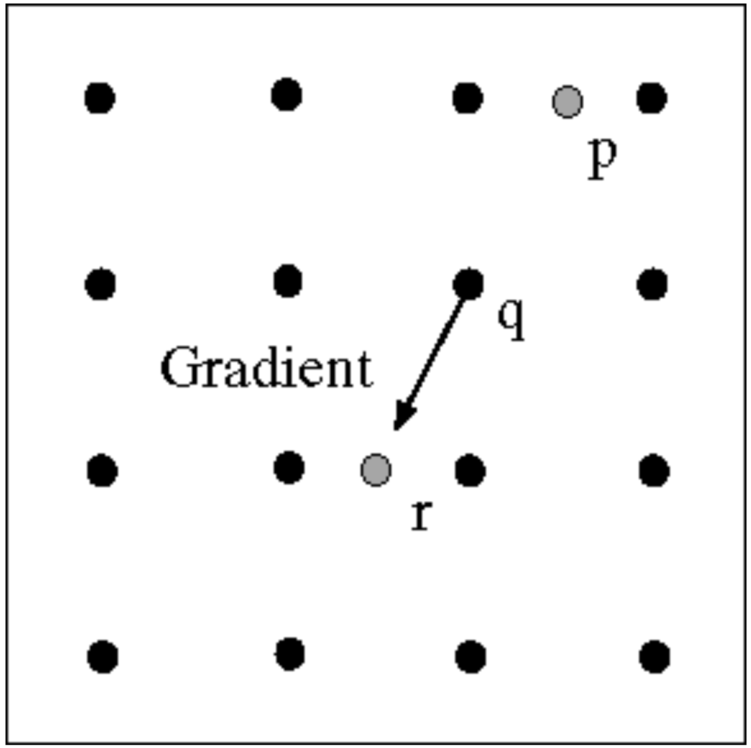
Thresholding

Non Maxima Suppression

Hysteresis Thresholding

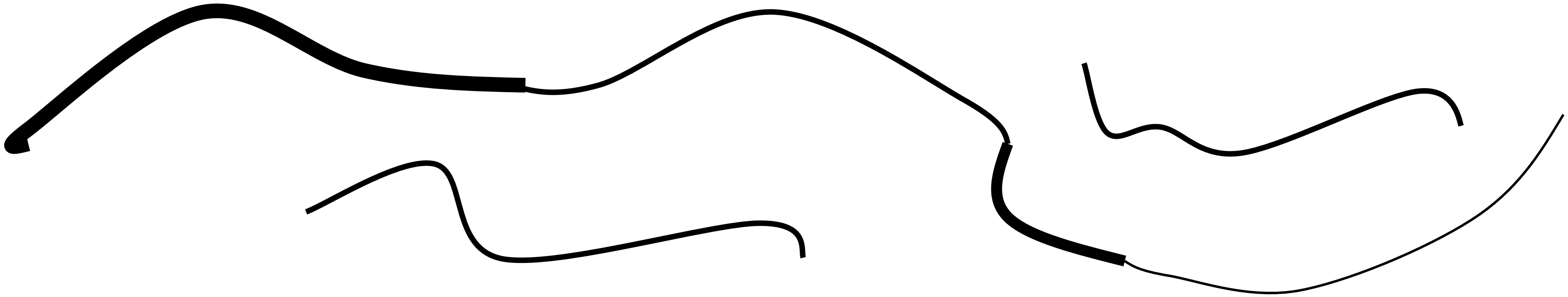
Non-Maxima Suppression





Check if pixel is local maximum along gradient direction, which requires checking interpolated pixels p and r.

Hysteresis Thresholding



- Algorithm takes two thresholds: high & low
  - A pixel with edge strength above high threshold is an edge.
  - Any pixel with edge strength below low threshold is not.
  - Any pixel above the low threshold and next to an edge is an edge.

- Iteratively label edges
  - Edges grow out from ‘strong edges’
  - Iterate until no change in image.

Canny Results

σ=1, T2=255, T1=1

'Y' or 'T' junction problem with Canny operator

Canny Results

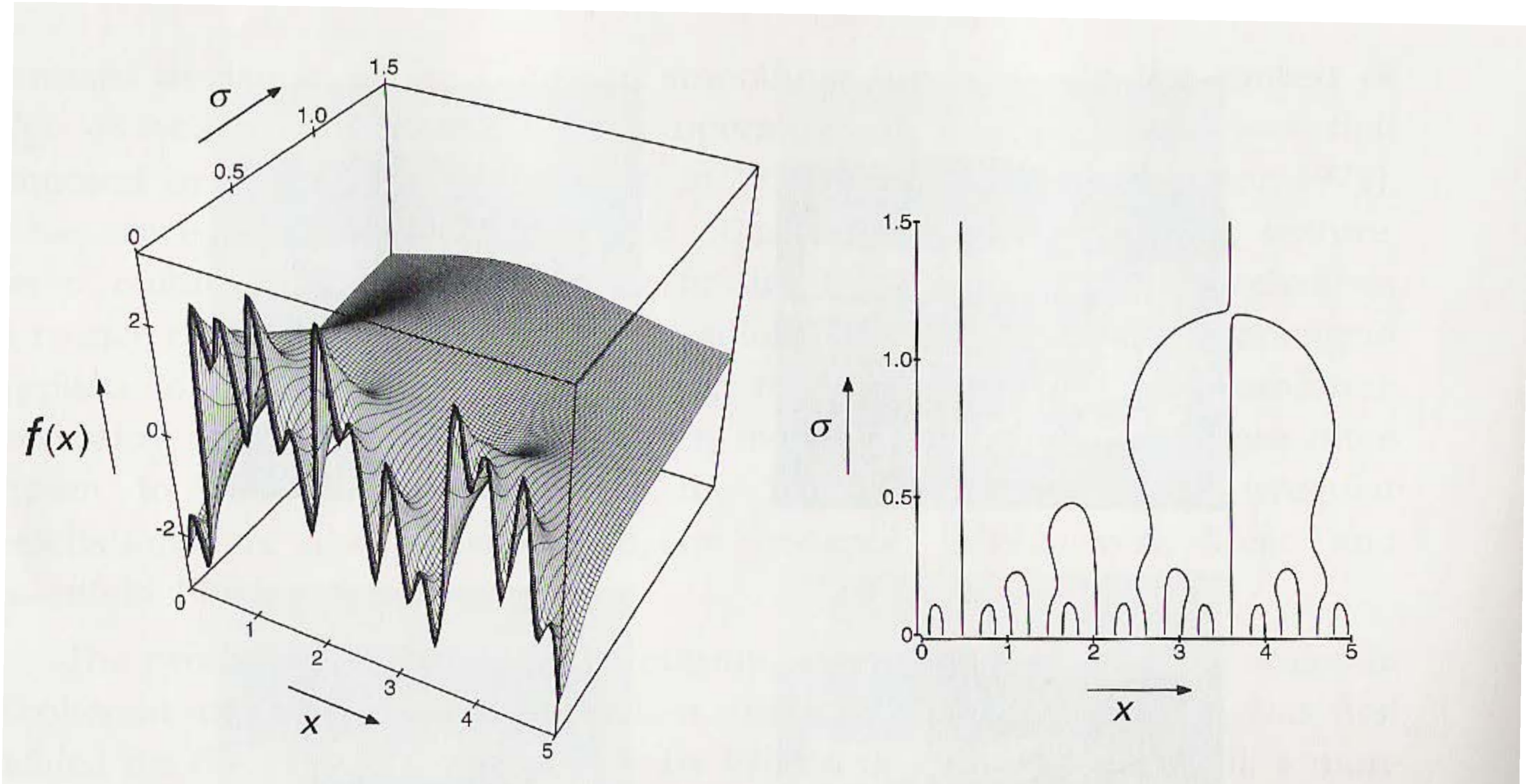
σ=1, T2=255, T1=220

σ=1, T2=128, T1=1

σ=2, T2=128, T1=1

Heath et al., PAMI'97

Scale Space Revisited



Increasing scale (σ) removes details but never adds new ones:

- Edge position may shift.
- Two edges may merge.
- An edge may **not** split into two.

Multiple Scales

$\sigma = 1$ $\sigma = 2$ $\sigma = 4$

→Choosing the right scale is a difficult semantic problem.

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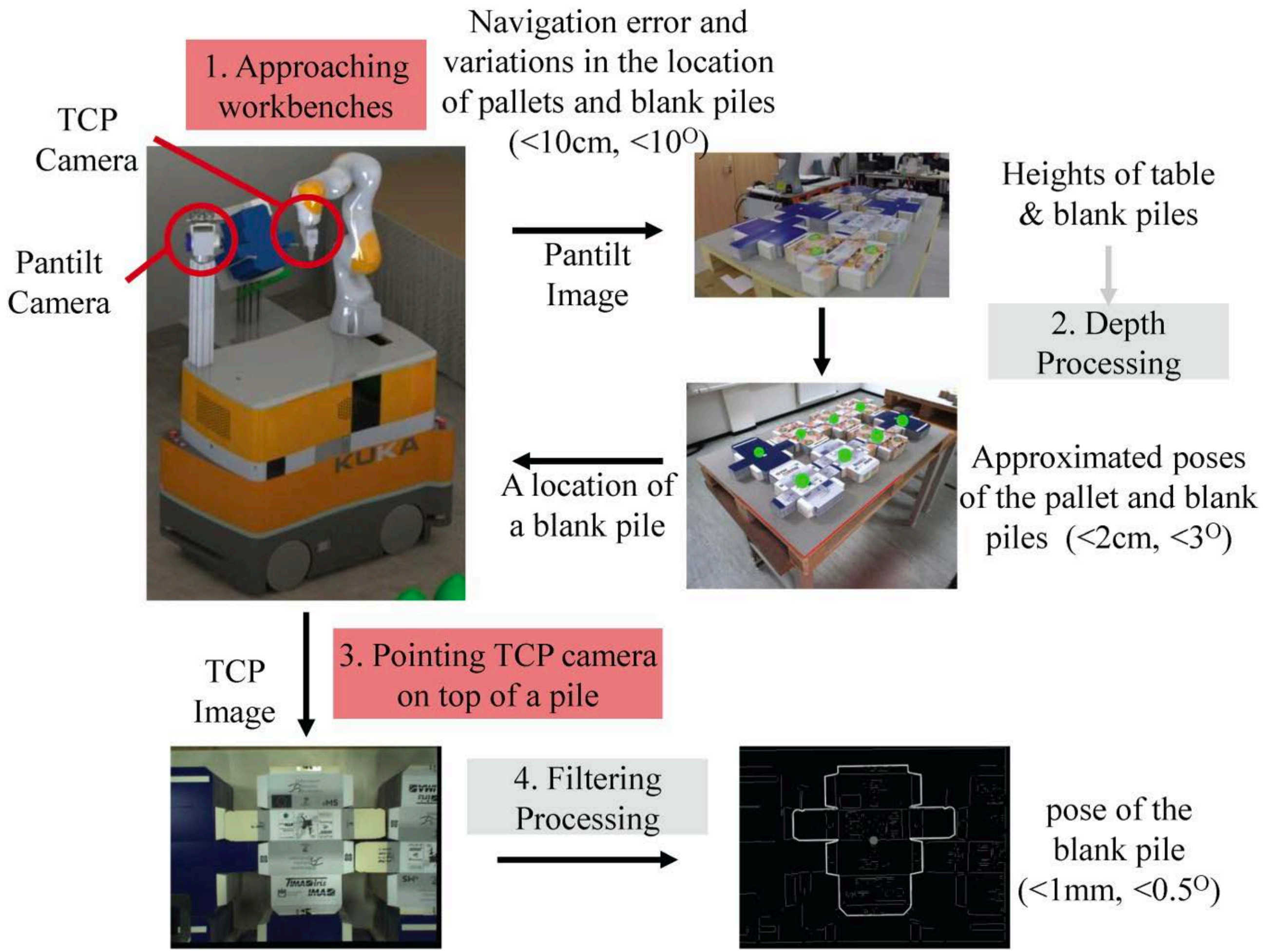
Scale vs Threshold

Fine scale
High threshold

Coarse scale
High threshold

Coarse scale
Low threshold

Industrial Application



In industrial environments where the Canny parameters can be properly adjusted:

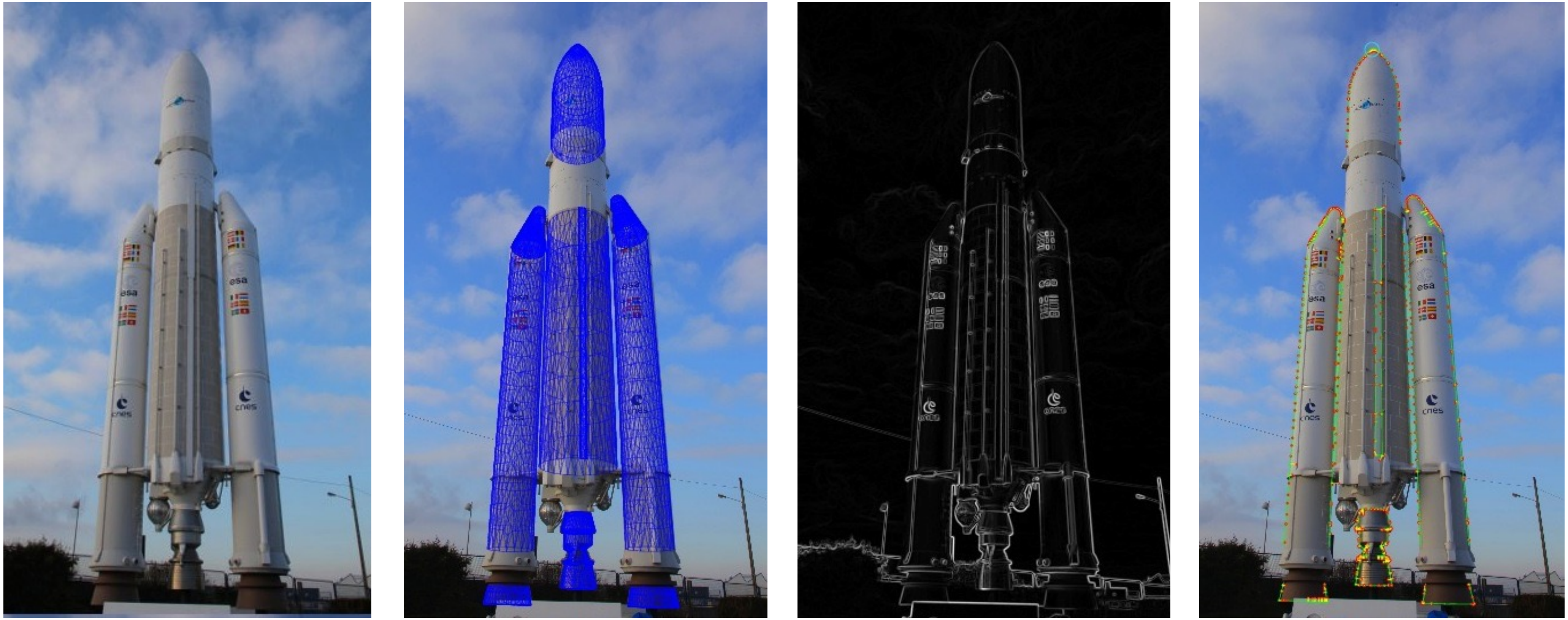
- It is fast.
- Does not require training data.

Lim et al. , Advanced Robotics 2019.

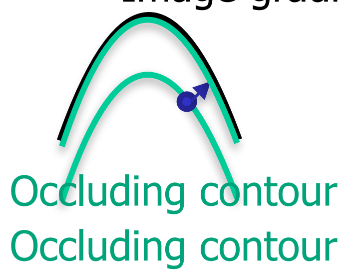
Visual Servoing

—> A useful tool in our toolbox.

Tracking a Rocket

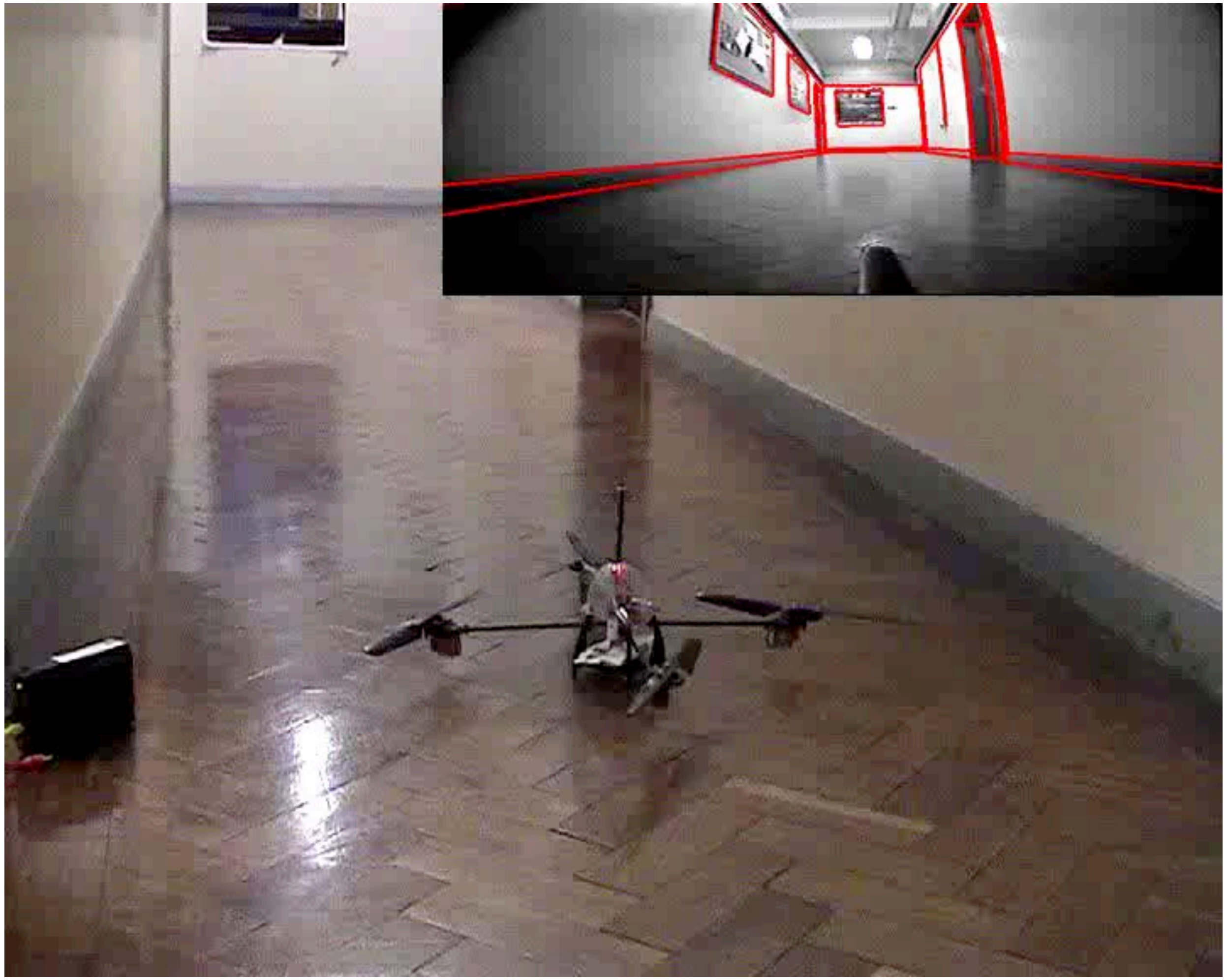


Given an initial pose estimate:

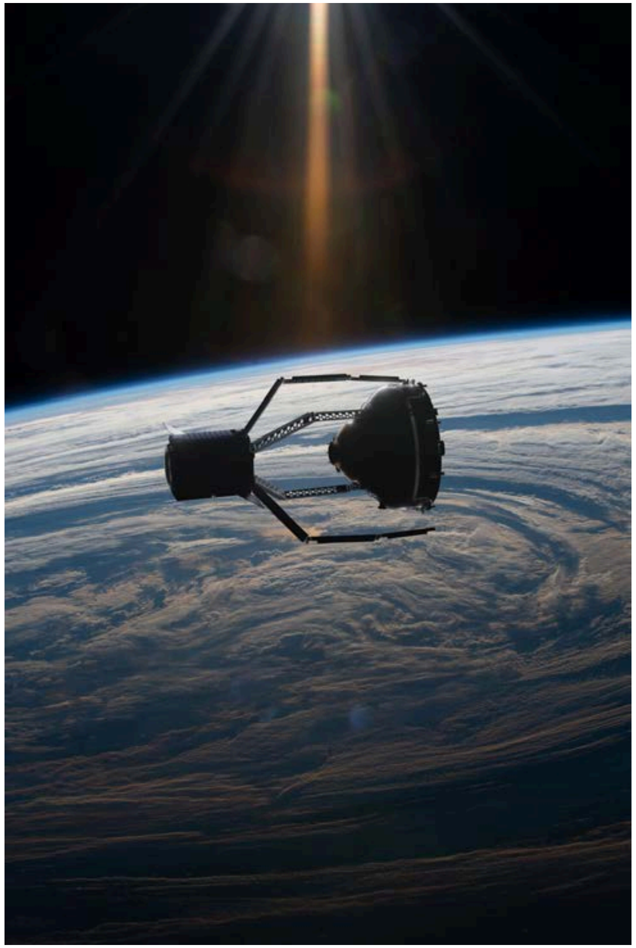


- Find the occluding contours.
- Find closest edge points in the normal direction.
- Re-estimate pose to minimize sum of square distances.
- Iterate until convergence.

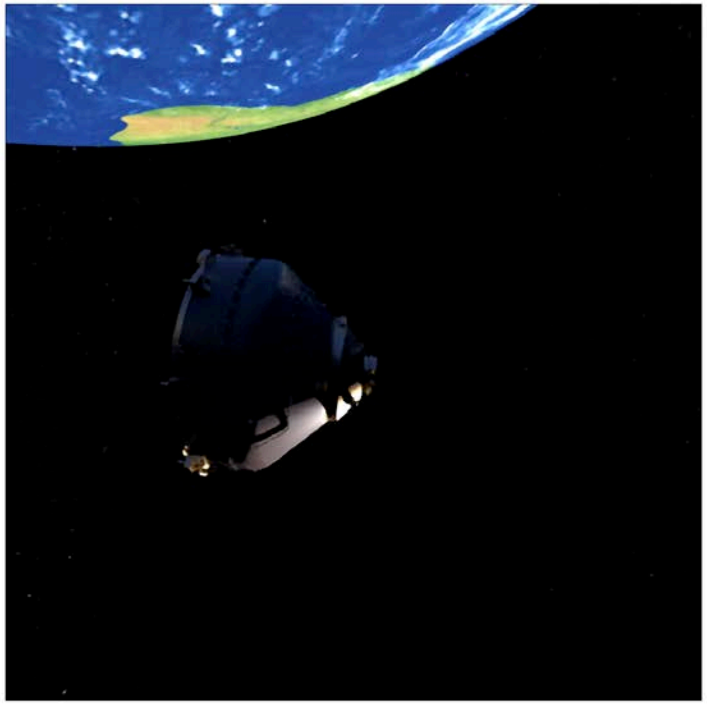
Visual Servoing

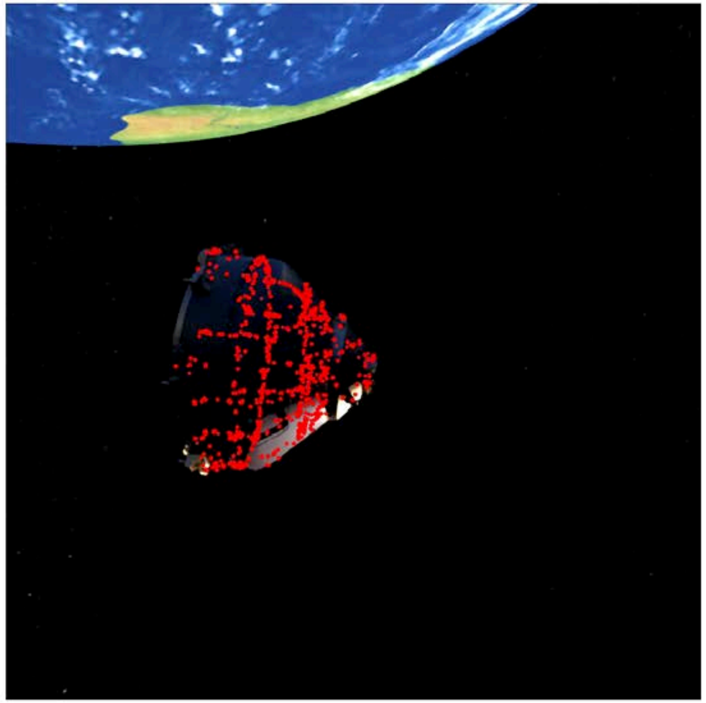


Space Cleaning



Capturing and deorbiting a dead satellite.





- A more sophisticated version of this old algorithm will blast off in 2026!
- ESA does not yet trust neural nets for such a mission.

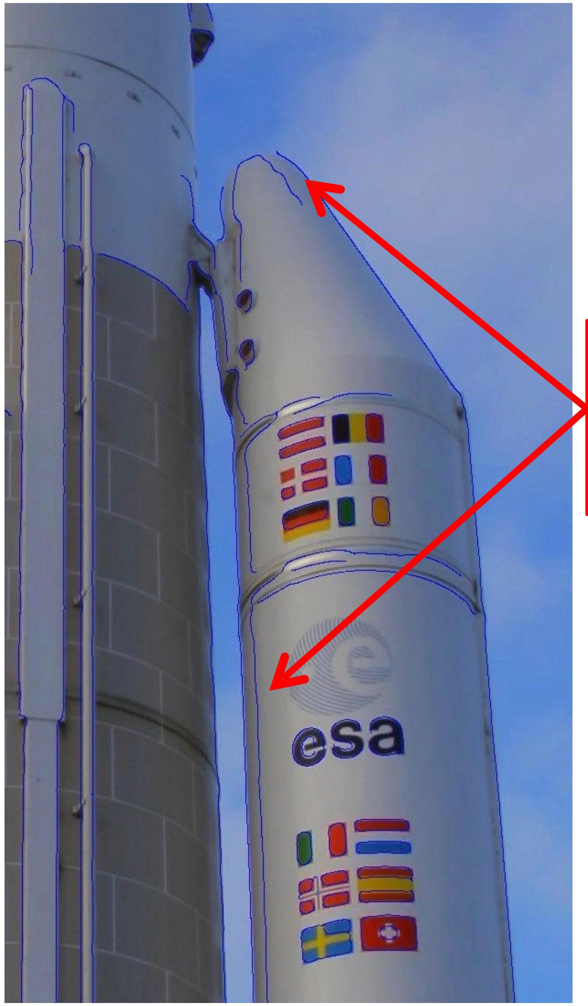
https://clearspace.today/

Limitations of the Canny Algorithm

There is no ideal value of σ!

EPFL

Steep Smooth Shading



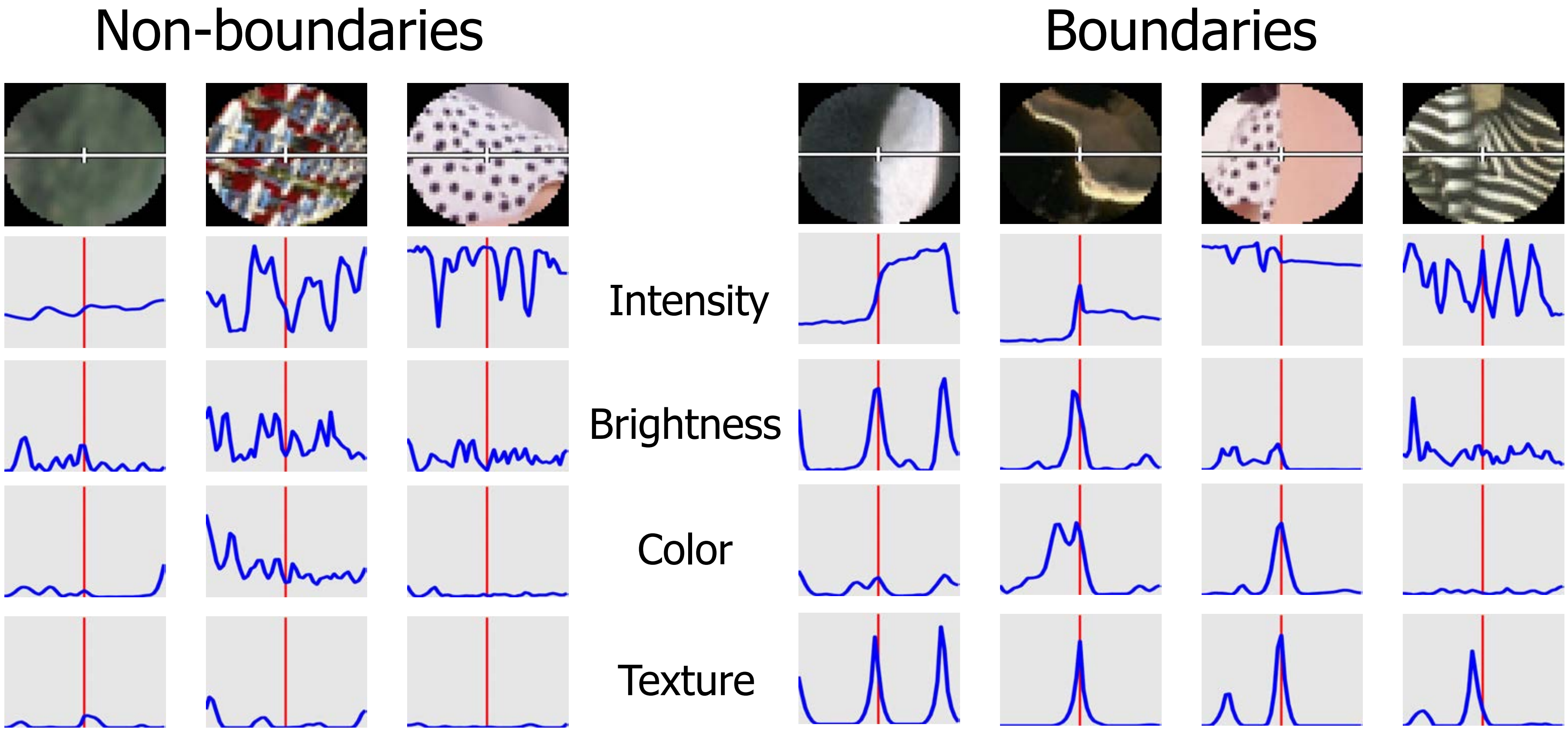
- Rapidly varying gray levels.
- Large gradients.

→ Shading can produce spurious edges.

Texture Boundaries

- Not all image contours are characterized by strong contrast.
- Sometimes, textural changes are just as significant.

Different Boundary Types

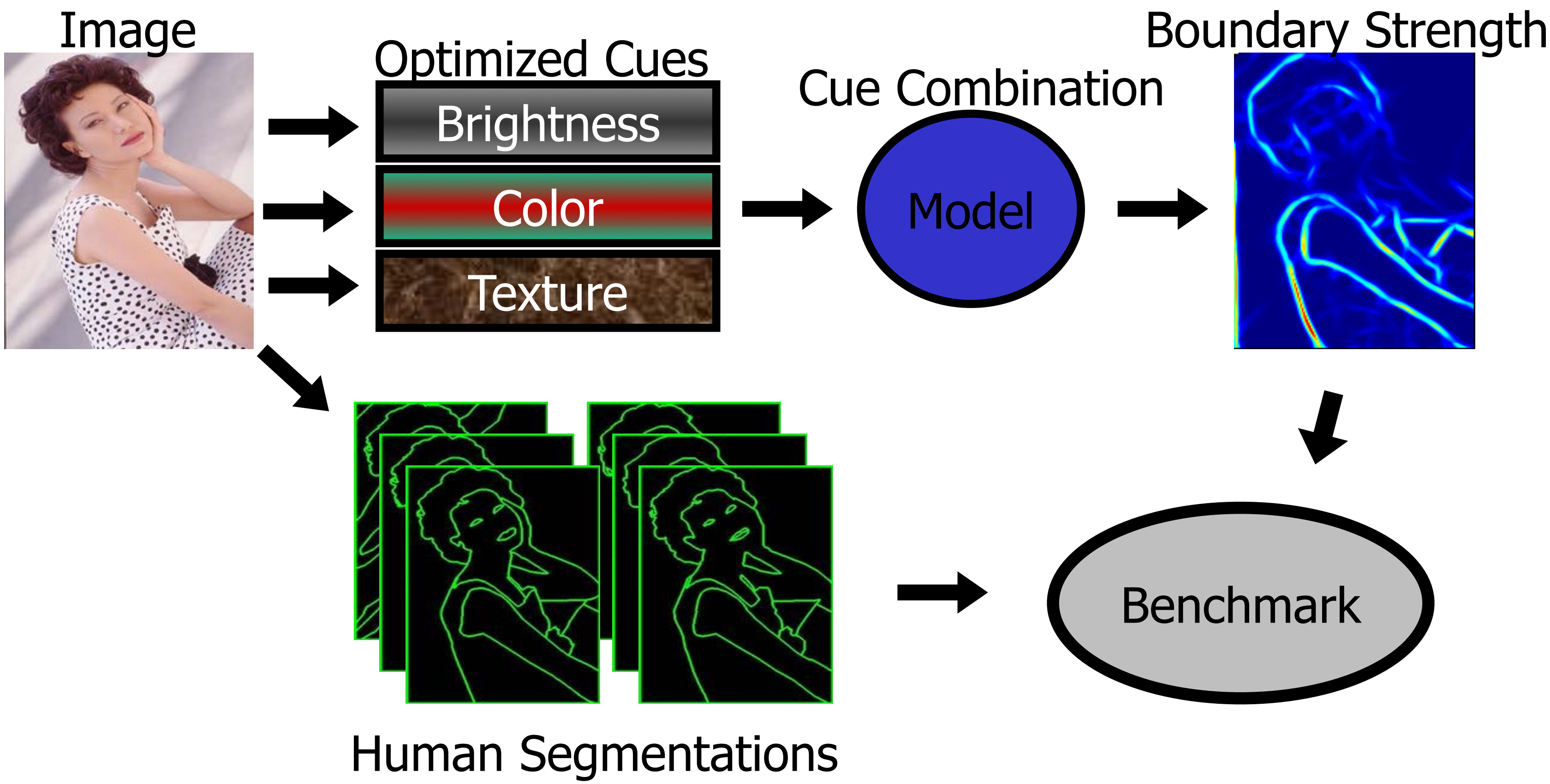


Martin et al. , PAMI'04

Training Database

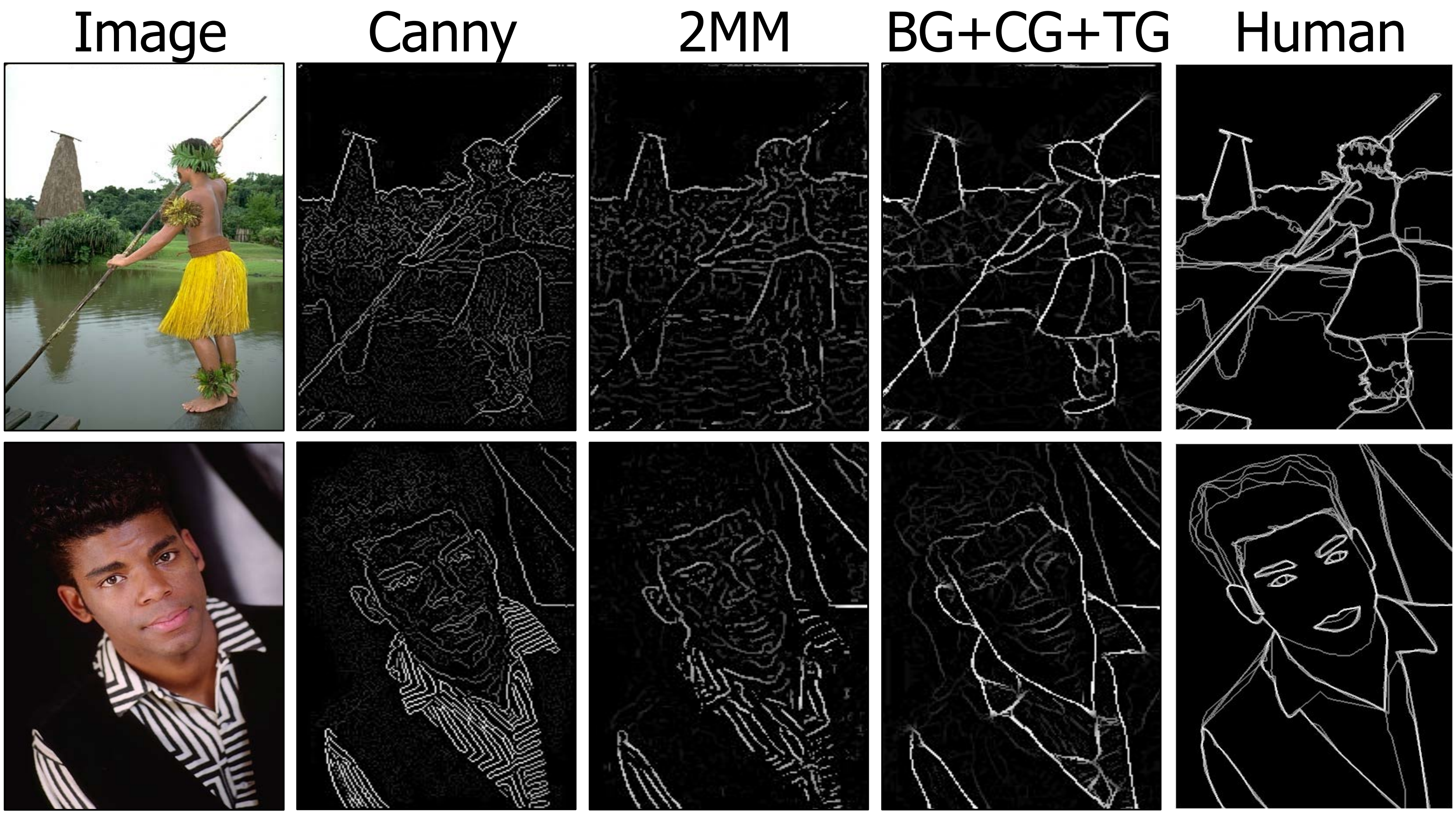
1000 images with 5 to 10 segmentations each.

Machine Learning



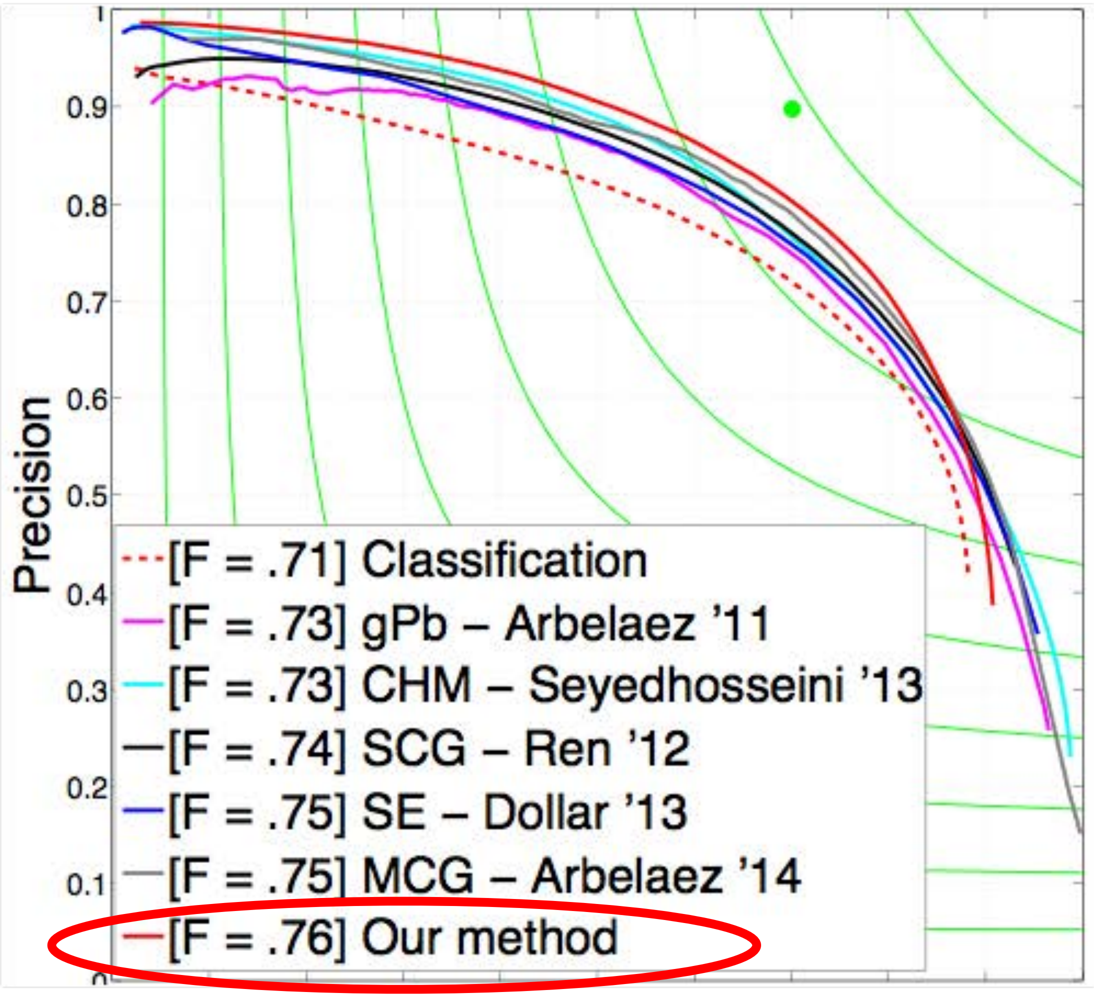
Learn the probability of being a boundary pixel on the basis of a set of features.

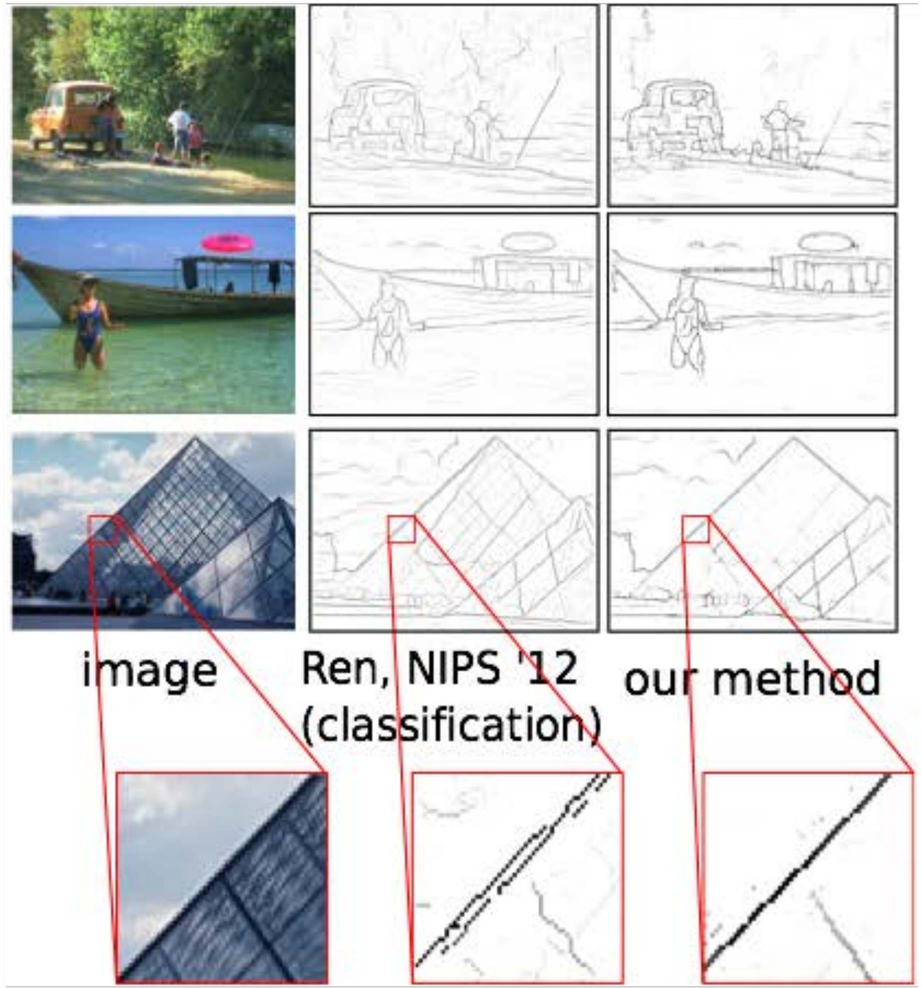
Comparative Results



Martin et al. , PAMI'04

Classification vs Regression

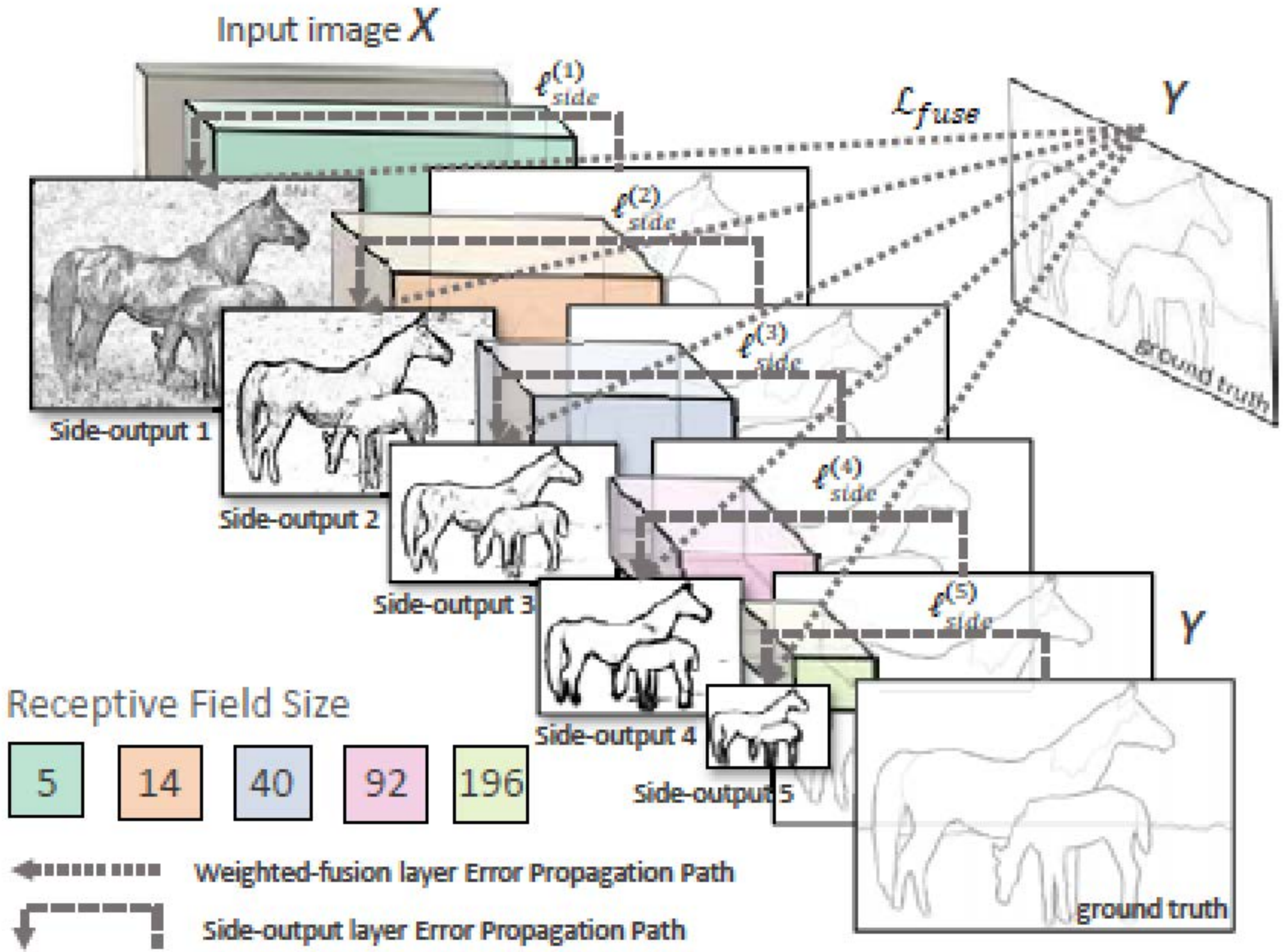




Yes!

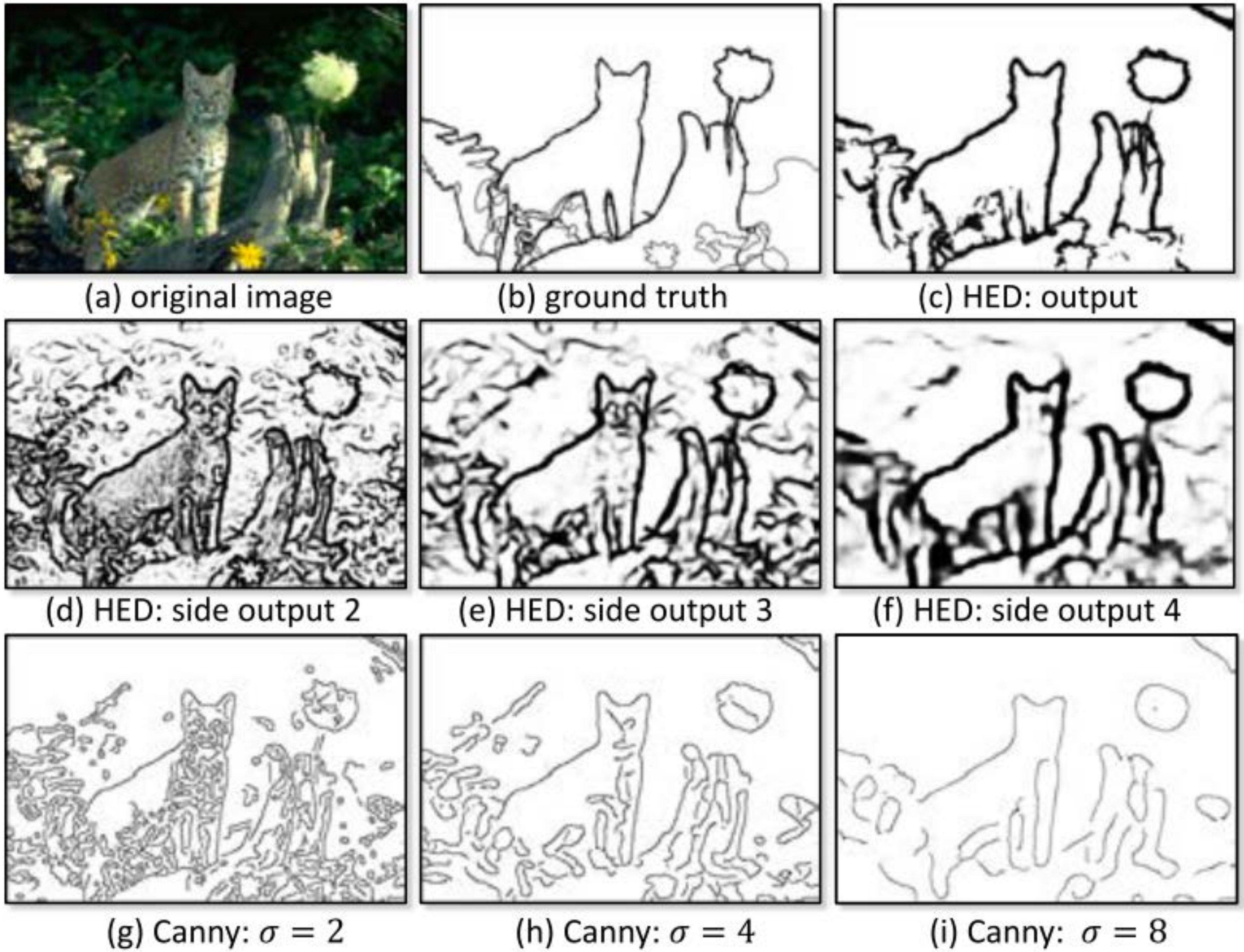
Sironi, et al. PAMI'15

Deep Learning

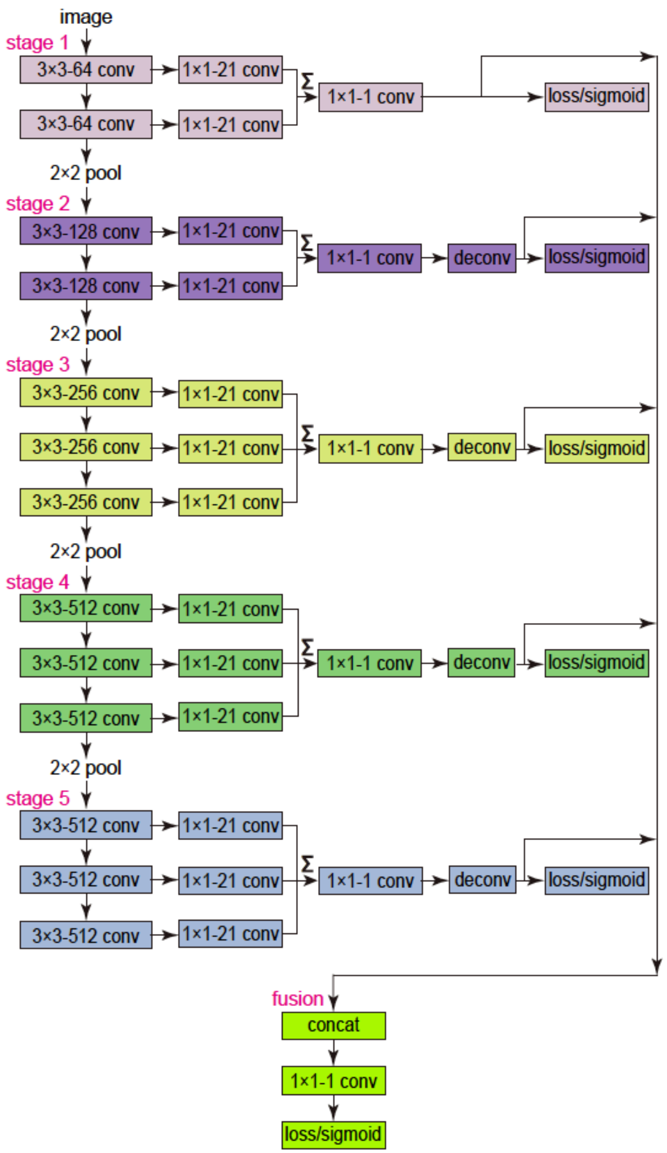


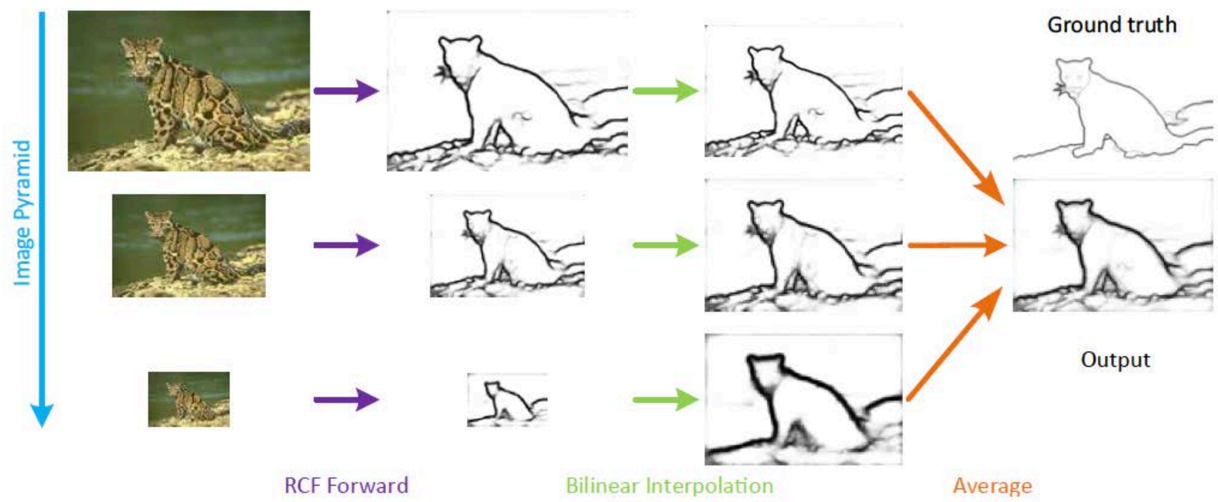
Xie and Tu, ICCV'15

Deep Learning Vs Canny



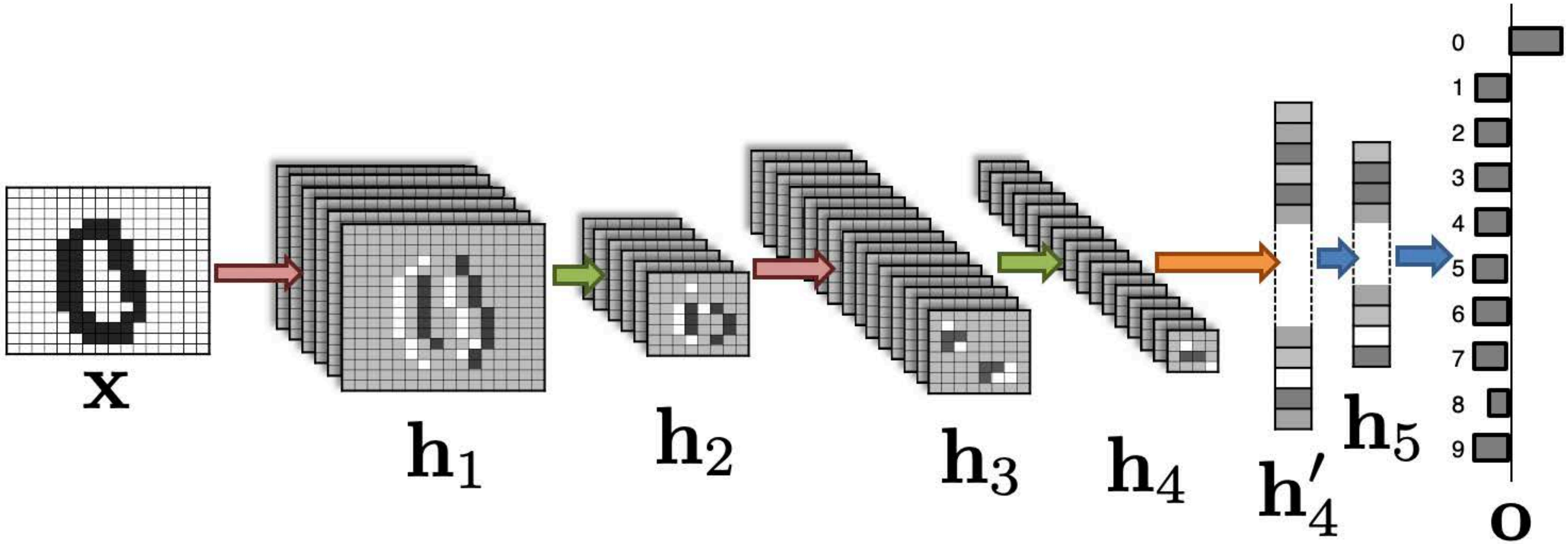
Deeper Learning





Liu et al. , CVPR’17

Convolutional Neural Network

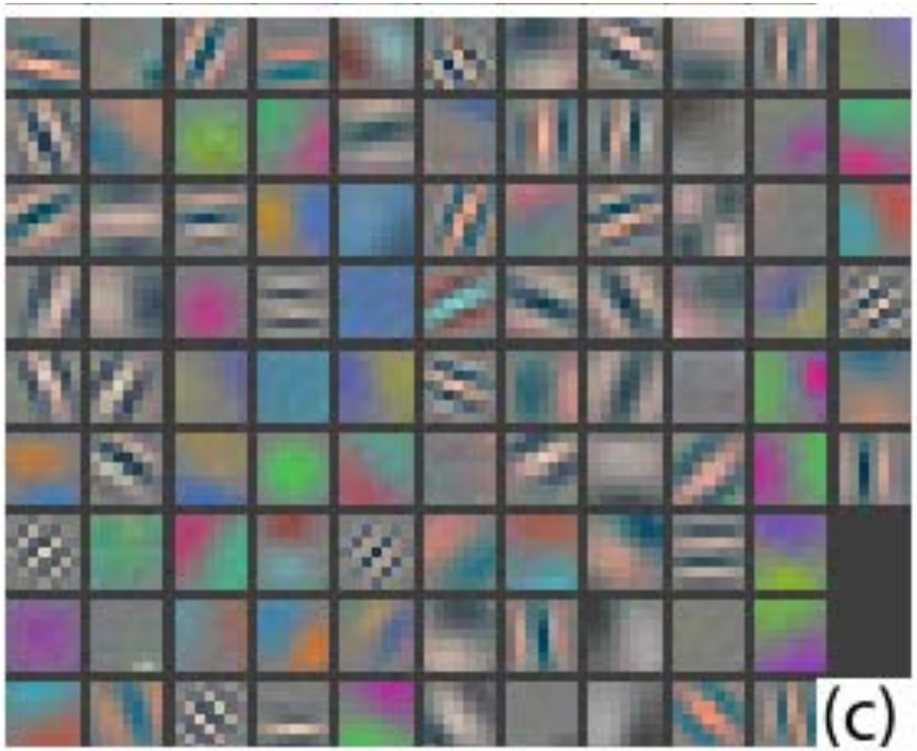


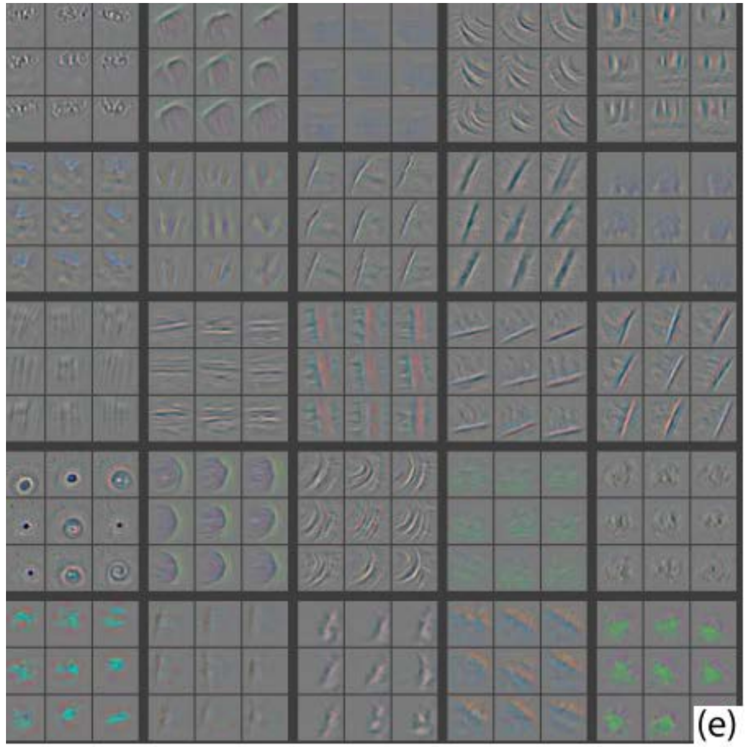
- Succession of convolutional and pooling layers.
- Fully connected layers at the end.

—> Will be discussed in more detail in the next lecture.

EPFL

A Partial Explanation?

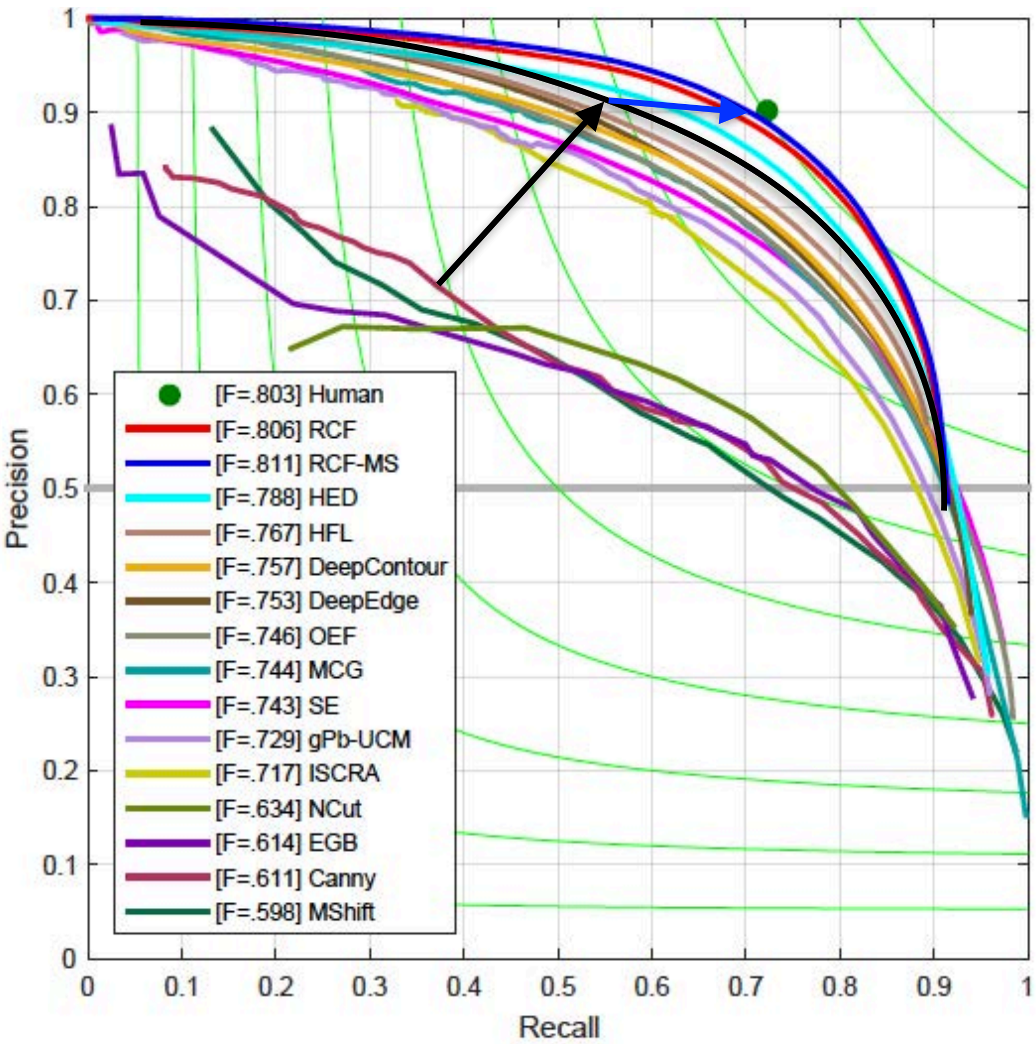




First and second layer features of a Convolutional Neural Net:

- They can be understood as performing multiscale filtering.
- The weights and thresholds are chosen by the optimization procedure.

50 Years Of Edge Detection



- Convolution operators respond to steep smooth shading.
- Parametric matchers tend to reject non ideal edges.
- Arbitrary thresholds and scale sizes are required.
- Learning-based methods need exhaustive databases.

- There still is work to go from contours to objects.

Canny, PAMI'86 —> Sironi et al. PAMI'15 Sironi et al. PAMI'15 —> Liu et al. , CVPR'17

Let us talk about deep networks.