

SOLUTION**Problem 1:**

Choose units:

Time in min; Volume in m³; mass of AC in kg; mass of contaminant in µg.

$$Q = 2 \text{ m}^3/\text{min}$$

$$C_{in} = 300 \frac{\mu\text{g}}{\text{L}} * \frac{1000\text{L}}{\text{m}^3} = 3 * 10^5 \frac{\mu\text{g}}{\text{m}^3}$$

$$C_{out} = 3 \frac{\mu\text{g}}{\text{L}} * \frac{1000\text{L}}{\text{m}^3} = 3 * 10^3 \frac{\mu\text{g}}{\text{m}^3}$$

$$L = 15 \text{ m}$$

$$v_f = 5 \frac{\text{m}}{\text{h}} = 5 \frac{\text{m}}{\text{h}} * \frac{1\text{h}}{60 \text{ min}} = 0.083 \frac{\text{m}}{\text{min}}$$

$$\rho_b = 425 \text{ kg/m}^3$$

a – calculate EBCT

$$\text{EBCT} = \frac{V_b}{Q} \text{ and } V_b = A_b * L$$

$$\text{EBCT} = \frac{L}{v_f} = \frac{15 \text{ m}}{0.083 \frac{\text{m}}{\text{min}}} = 180.7 \text{ min}$$

b- calculate A_b (cross section of the bed) and the bed diameter

$$V_b = Q * \text{EBCT} = 2 \frac{\text{m}^3}{\text{min}} * 180.7 \text{ min} = 361.4 \text{ m}^3$$

$$A_b = \frac{V_b}{L} = \frac{361.4 \text{ m}^3}{15 \text{ m}} = 24 \text{ m}^2$$

$$d = 2 * \sqrt{r^2} = 2 * \sqrt{\frac{A_b}{\pi}} = 5.5 \text{ m}$$

c - what is t_b?

$$Q_{in} * (C_{in} - C_{out}) * t_b = C_s^e * \rho_b * V_{bed,used}$$

$$t_b = \frac{C_s^e * \rho_b * V_{bed,used}}{Q_{in} * (C_{in} - C_{out})}$$

The adsorption zone is S-shaped and symmetrical, we assume an inefficiency coefficient of 0.75 (%_used,b = 75%):

$$V_{bed,used} = V_b \cdot 0.75 = 271 \text{ m}^3$$

$$\text{Need to calculate: } C_s^e = 250 \cdot C_{in}^{0.003} = 254.315 \frac{\mu g_{atrazine}}{g_{AC}} \cdot \frac{1000 g_{AC}}{kg_{AC}} = 254'315 \frac{\mu g_{atrazine}}{kg_{AC}}$$

$$\text{So, } t_b = \frac{254315 \frac{\mu g_{atrazine}}{kg_{AC}} \cdot 425 \frac{kg_{of AC}}{m^3_{of AC}} \cdot 271 m^3_{of AC}}{2 \frac{m^3_{water}}{min} \cdot (3 \cdot 10^5 - 3 \cdot 10^3 \frac{\mu g_{atrazine}}{m^3_{water}})} = 49'063 \text{ min} = \sim 34 \text{ days}$$

To calculate bed volumes, need EBCT:

$$\frac{t_b}{EBCT} = \theta_b = \frac{49'063 \text{ min}}{180.7 \text{ min}} \sim 272 \text{ bed volumes}$$

d- We are looking for: $\frac{\text{kilogram of AC}}{m^3 \text{ of water treated until regeneration/disposal}}$

$$rate_{AC} = \frac{M_{AC}}{Q \cdot t_b}$$

$$M_{AC} = V_b \cdot \rho_b = 361.4 m^3 \cdot 425 \frac{kg}{m^3} = 153,614 \text{ kg AC}$$

$$Q \cdot t_b = 2 \frac{m^3_{water}}{min} \cdot 50353 \text{ min} = 100'706 m^3 \text{ water}$$

$$rate_{AC} = \frac{153'614 \text{ kg AC}}{100'706 m^3 \text{ water}} = 1.53 \frac{kg AC}{m^3 \text{ water}}$$

Problem 2:

We can choose consistent units: time in days; contaminant in g; volume in m³; mass of AC in kg

$$Q = 500 \frac{m^3}{d}$$

$$C_{in} = 12 \frac{mg}{L} \cdot \frac{1000 L}{m^3} \cdot \frac{1 g}{1000 mg} = 12 \frac{g}{m^3}$$

$$C_s^e = 80 \frac{mg}{g} \cdot \frac{1000 g}{1 kg} \cdot \frac{1 g}{1000 mg} = 80 \frac{g}{kg}$$

$$\rho_b = 450 \frac{kg}{m^3}$$

1. $V_b = EBCT \cdot Q = 0.01042d \cdot 500 \frac{m^3}{d} = 5.2m^3$
2. $M_{AC} = V_b \cdot \rho_b = 5.2 m^3 \cdot 450 \frac{kg}{m^3} = 2340 kg$
3. $t_b = \frac{C_s^e \cdot \rho_b \cdot V_{b,used}}{Q \cdot C_{in}} = \frac{80 \frac{g}{kg} \cdot 450 \frac{kg}{m^3} \cdot 5.2m^3}{500 \frac{m^3}{d} \cdot 12 \frac{g}{m^3}} = 31.2 days$

Problem 3:

Q = 5 L/min

possible sorbents: SIA or Actinomycetes 50 kg of sorbent

Waste stream 1: pH 4

influent concentration: 2 mg/L Cu²⁺ 2 mg/L Cd²⁺ 2 mg/L Zn²⁺

desired effluent concentration: 600 µg/L

Waste stream 2: pH 6

influent concentration: 1.2 mg/L Cd²⁺ 1.5 mg/L Zn²⁺

desired effluent concentration: 600 µg/L

Mass balance (for each sorbent/contaminant):

$$M_{AC} \cdot C_s^e = Q \cdot C_{in} \cdot t_b$$

M_{Sorbent} = mass of sorbent available (g)

C_s^e = sorbed mass at equilibrium (µg metal/g sorbent)

Q = volume of liquid treated per time (L/d)

C_{in} = influent concentration (µg metal/L)

t_b = time at breakthrough

Example calculation: Copper SIA vs Actino at pH 4 (waste stream 1)

Looking at the isotherms, we see that for SIA:

At Cu²⁺ = 600 µg/L => C_s = 6 mg/g sorbent (read on graph)

But at Cu²⁺ = 2,000 µg/L => C_s = 15 mg/g sorbent (read on graph)

For Actinomycetes:

At $Cu^{2+} = 600 \mu g/L \Rightarrow C_s = 6.5 \text{ mg/g sorbent}$ (read on graph)

But at $Cu^{2+} = 2,000 \mu g/L \Rightarrow C_s = 10 \text{ mg/g sorbent}$ (read on graph)

We see that at equilibrium, at high concentrations, more copper is adsorbed onto SIA but at low concentrations, Actinomycetes do better.

The isotherms are obtained for batch systems, so the initial concentration decreases over time until it reaches the equilibrium concentration. So, if this exercise was about a batch system, it would make sense to select Actinomycetes.

However, we are considering a packed system (like activated carbon but with a different sorbent). Therefore, at breakthrough, most of the bed will have a porewater with a concentration of C_{in} .

Therefore, the solid phase concentration at exhaustion (which means that the solid phase is exhausted, it cannot bind any more compound) should be calculated in equilibrium with the influent concentration. This is because (unlike batch), more contaminant is continuously coming in, reaching the maximum load given the influent concentration.

We can calculate

$$Cu^{2+} \text{ on SIA} \Rightarrow t_{b_{sorbent}} = \frac{C_s^e M_{sorbent}}{Q \cdot C_{aq}^i} = \frac{14.5 \frac{mg}{g} \cdot 10^3 \frac{g}{kg} \cdot 50 \text{ kg}}{5 \frac{L}{min} \cdot 2 \frac{mg}{L}} = 50 \text{ days}$$

$$Cu^{2+} \text{ on Actino} \Rightarrow t_{b_{sorbent}} = \frac{C_s^e M_{sorbent}}{Q \cdot C_{aq}^i} = \frac{10.5 \frac{mg}{g} \cdot 10^3 \frac{g}{kg} \cdot 50 \text{ kg}}{5 \frac{L}{min} \cdot 2 \frac{mg}{L}} = 37 \text{ days}$$

Then, do the same for all conditions:

		SIA			Actinomycetes		
		Cu. pH 4	Cd. pH 4	Zn. pH 4	Cu. pH 4	Cd. pH 4	Zn. pH 4
Mass	kg	50	50	50	50	50	50
Cs	mg/g	14.5	5.5	1	10	3.25	1
Q	L/min	5	5	5	5	5	5
Cin	mg/L	2	2	2	2	2	2
Cout	mg/L	0.6	0.6	0.6	0.6	0.6	0.6
Time	days	72	27	5	50	16	5
		SIA		Actinomycetes			
		Cd. pH 6	Zn. pH 6	Cd. pH 6	Zn. pH 6		
Mass	kg	50	50	50	50		
Cs	mg/g	15	13	3	1.5		
Q	L/min	5	5	5	5		
Cin	mg/L	2	2	2	2		
Cout	mg/L	0.6	0.6	0.6	0.6		
Time	days	74	64	15	7		

waste stream #1	pH = 4	C _s (mg/g)	days till breakthrough
SIA	Cu	14.5	50
	Cd	5.5	19
	Zn	1	3.5

Actinomycetes	Cu	10.5	37
	Cd	3.25	12
	Zn	1	3.5

waste stream #2	pH = 6	C _s	days till breakthrough
SIA	Cd	14.5	50
	Zn	13	45

Actinomycetes	Cd	3	10.5
	Zn	1.5	5

To determine which sorbent is better, we need to consider the metal that breaks through the fastest within a set but the slowest to compare the two sorbents.

ws #1 Zn^{2+} breaks through first (of the three metals) after 3.5d (SIA and Actino)=> best option is either sorbent

ws #2 the fastest breakthrough is for Zn^{2+} for Atinomycetes (5 days).

SIA is the best option here by far as the breakthrough is at 45 days for Zinc.

Problem 4:

Select units: time in hours; AC mass in kg; volume in m^3 ; mass of contaminant in g; distance in m.

$$C_{in} = 4 \frac{mg}{L} * \frac{1000L}{m^3} * \frac{1g}{1000 mg} = 4 \frac{g}{m^3}$$

$$L = 2.5 \text{ m}$$

$$v_f = 8 \frac{m}{h}$$

$$t_b = 32 \text{ h}$$

$$t_e = 48 \text{ h}$$

1. Find length of MTZ

$$L_m = L \cdot \frac{(t_e - t_b)}{t_e}$$

The length of the mass transfer zone (L_m) represents the portion of the bed where adsorption is actively occurring. It is calculated as the ratio of the time it takes the zone to move through itself ($t_e - t_b$) relative to the total time for the zone to establish and move down the bed (t_e)

$$L_m = 2.5 \text{ m} * \frac{(48 - 32)}{48} = \mathbf{0.83 \text{ m}}$$

2. Find fraction of GAC bed capacity used at breakthrough

Assuming a symmetrical S curve for the breakthrough curve, the carbon within the MTZ is on average 50% used at breakthrough. The fraction of the total capacity used at breakthrough is then given by

$$f = \frac{L - 0.5 * L_m}{L}$$

$$f = \frac{2.5m - 0.5 * 0.833m}{2.5m} = 0.833 = 83.3\%$$

Another way of looking at it is that equivalent time fraction of the GAC bed used is equal to the fully used bed (t_b) plus half of the MTZ zone.

$$f = \frac{(t_b + 0.5 * (t_e - t_b))}{t_e}$$

What is the equivalent time duration of the MTZ, Δt_{MTZ} ?

$$\Delta t_{MTZ} = t_e - t_b$$

$$f = \frac{32 \text{ h} + 0.5 * 16 \text{ h}}{48 \text{ h}} = 0.833 = \mathbf{83.3\%}$$

3. Total bed length with a 120 h breakthrough time

The bed length that is saturated is proportional to the time to reach breakthrough (t_b).

Therefore, for a saturated bed of $L_b = (L - L_m) = 2.5 - 0.83 = 1.67 \text{ m}$ and a corresponding $t_b=32\text{h}$, we expect the bed corresponding to a $t_b=120 \text{ h}$ to be:

$$L_b = 1.67\text{m} * \frac{120\text{h}}{32\text{h}} = 6.26\text{m}$$

For the total bed length, we need to add the length of the MTZ

The size of the MTZ is dependent on the linear velocity of the fluid and that doesn't change. Therefore, MTZ has a constant length $L_m=0.83 \text{ m}$.

The total length of the bed is then: $6.26+0.83=\mathbf{7.09 \text{ m}}$