Integrated nonlinear optics

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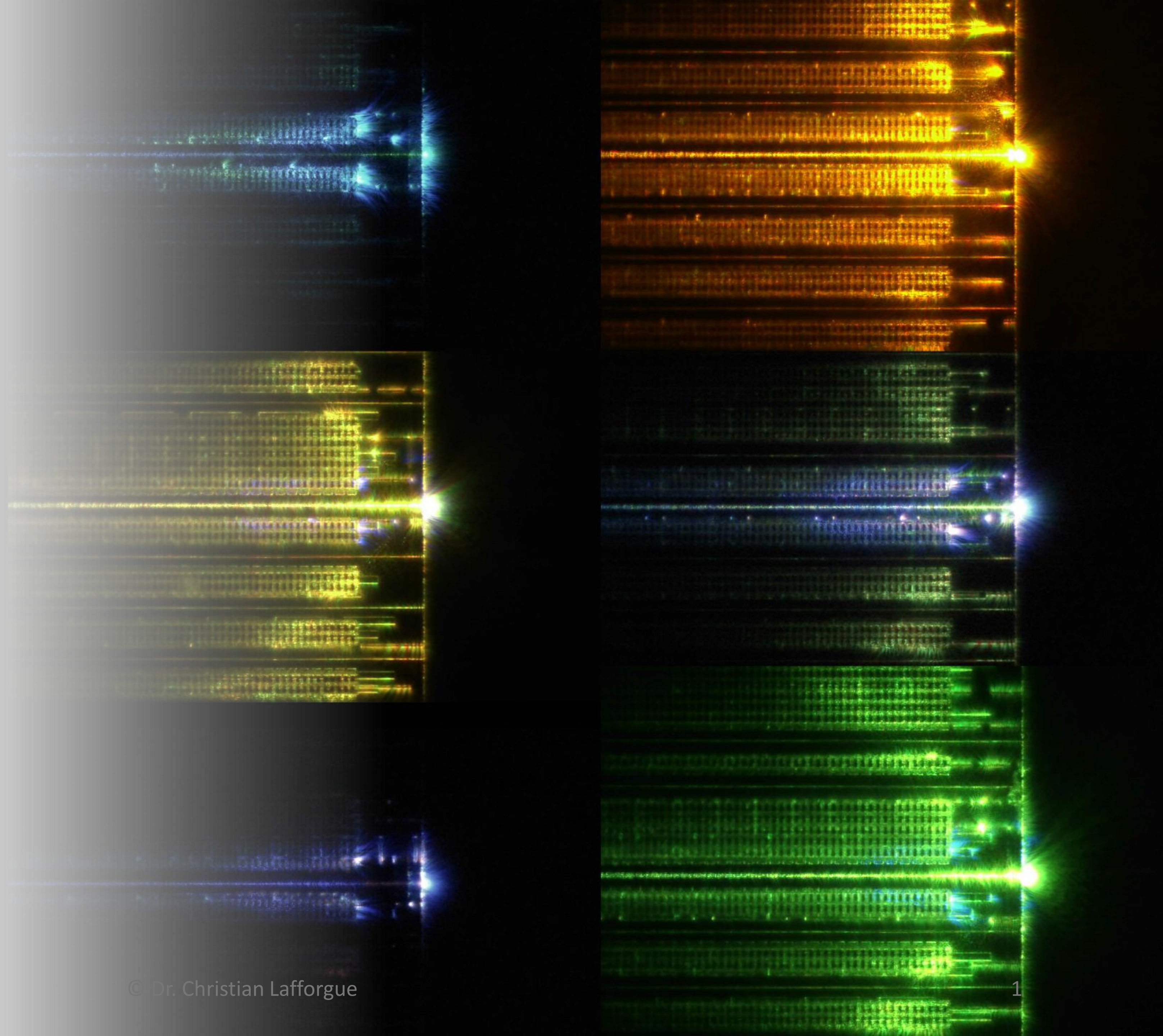


Outline

Introduction

Why integrated optics?

Application side

- Communications: High density, low power
- Computation: High density, low latency
- Imaging: compact broadband sources
- Sensing: compactness, high sensitivity

Physics side

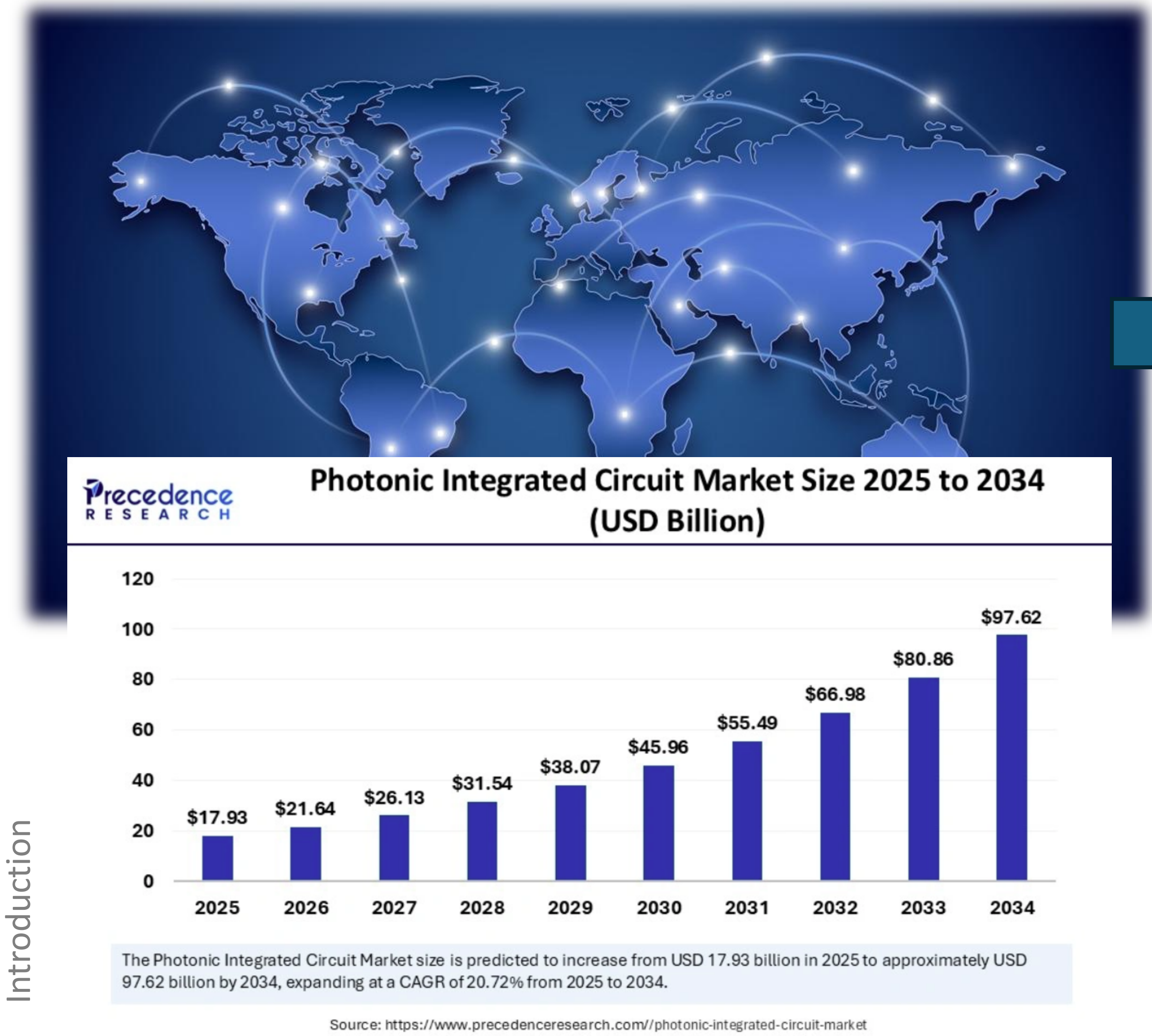
- Nonlinear effects: high intensity
- Complete functionalities: $\chi^{(2)}$ materials
- Optical cavities: high FSR

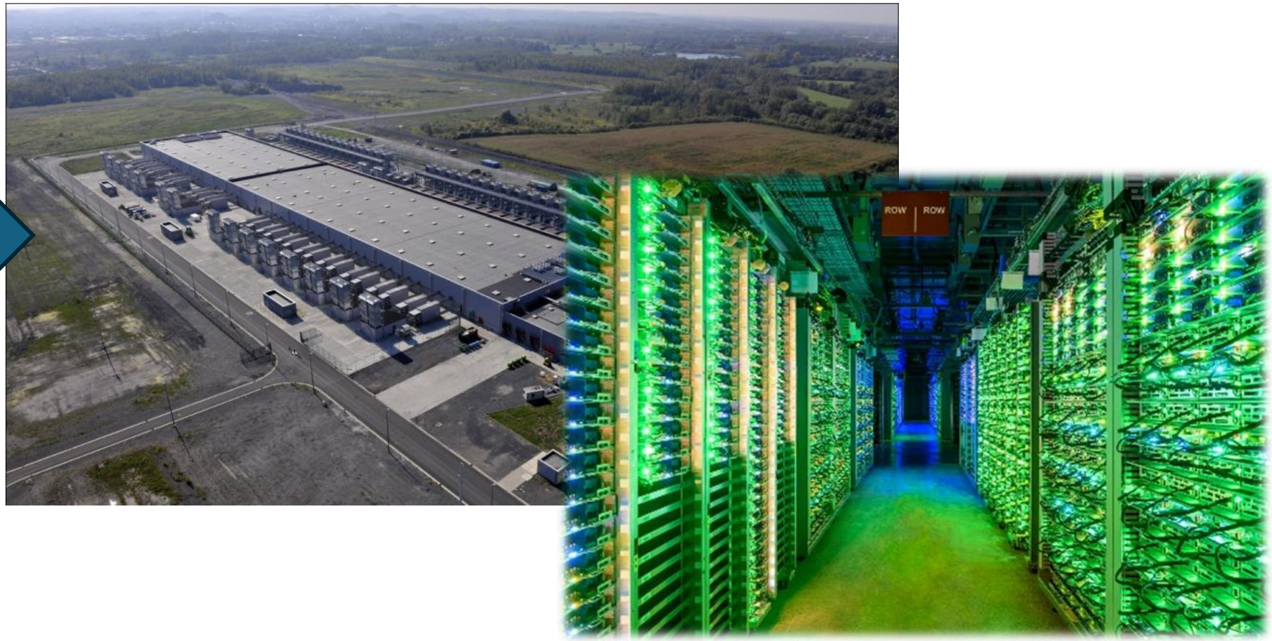
Trend

Need to shrink down optical systems: increases density, intensity, material choice, design parameter space

Applications

- Communications: short-distance data processing

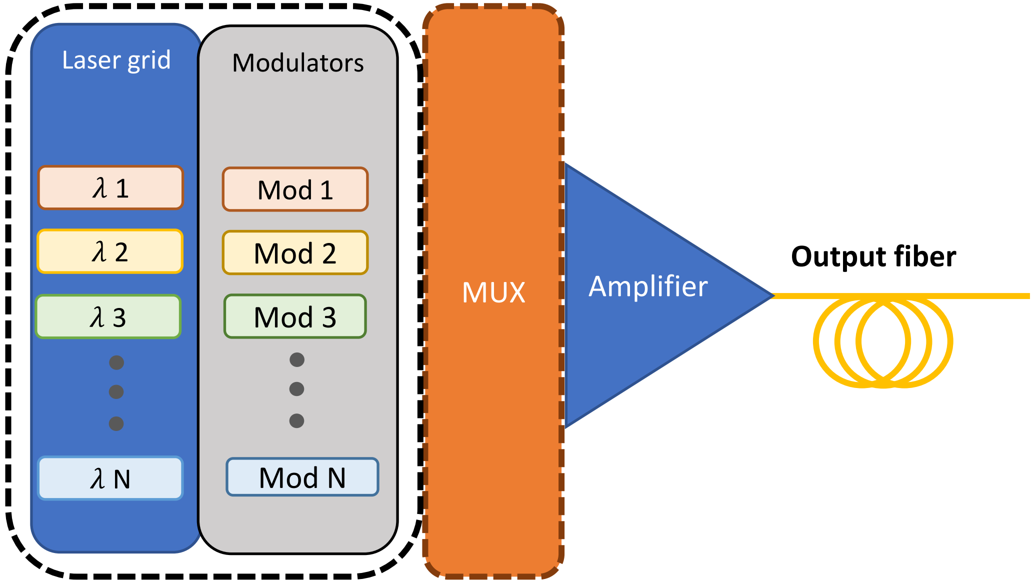




We build more and more gigantic, power-hungry facilities

Applications

- Communications: short-distance data processing
- Challenge:
  - get rid of bulky, power-hungry systems
  - Get the optical processing as close as possible to the electronic drivers



Applications

- Communications: short-distance data processing
- Challenge:
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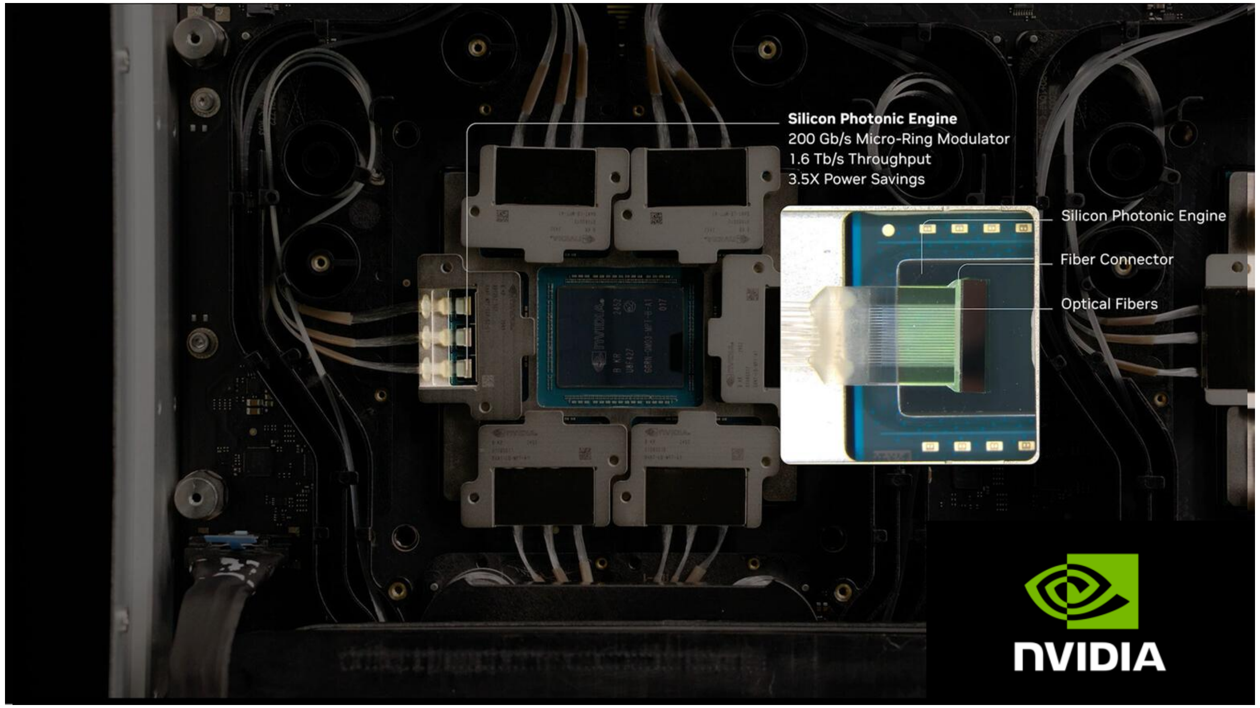
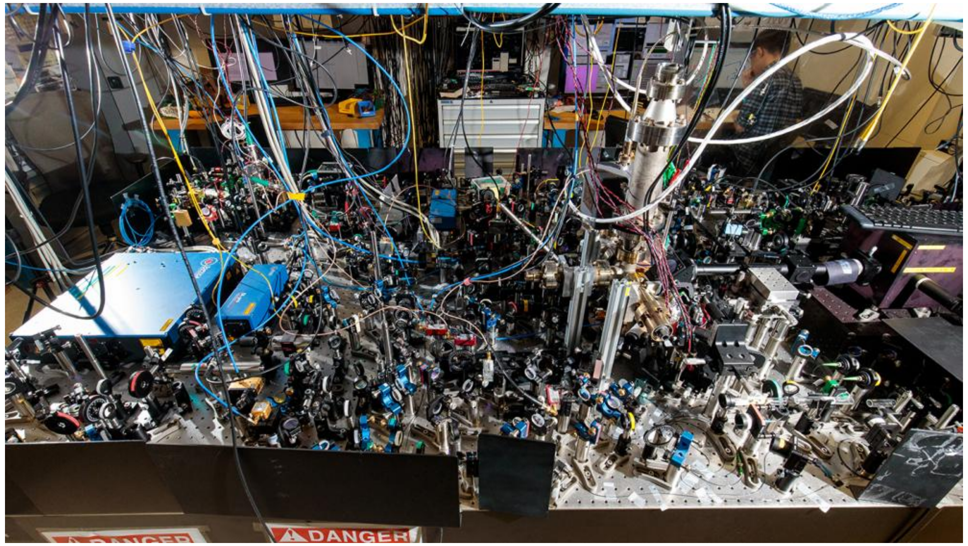
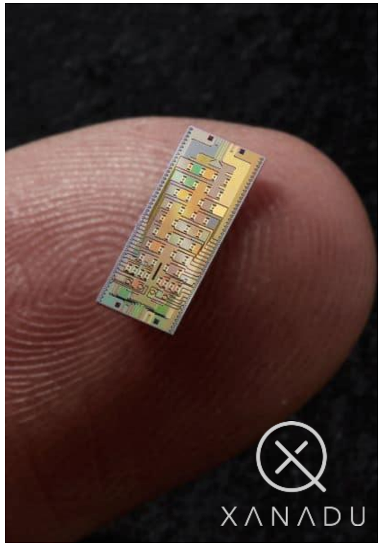


Applications

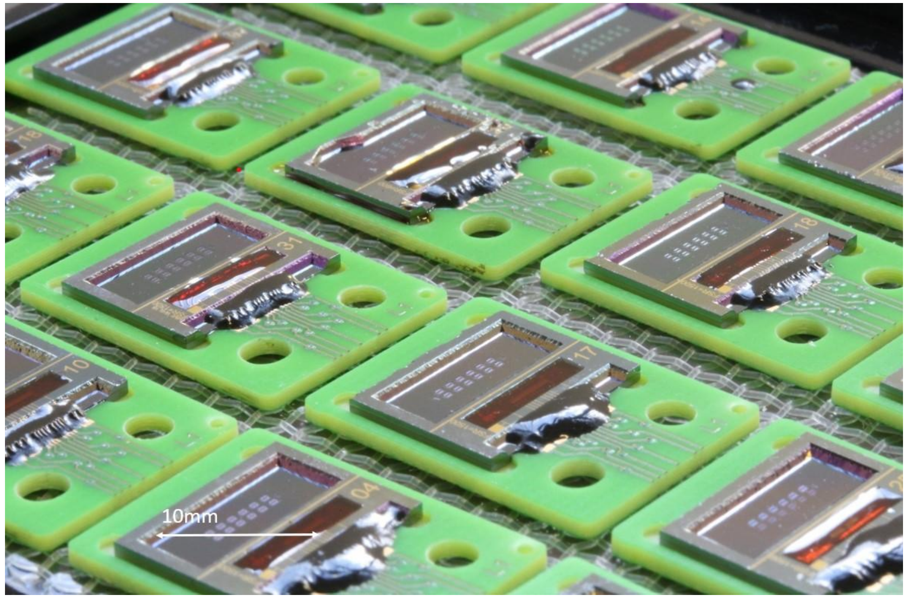
- Computing: PICs for AI and quantum computing
  - Small footprint
  - Low latency
  - Scalable for mass-production

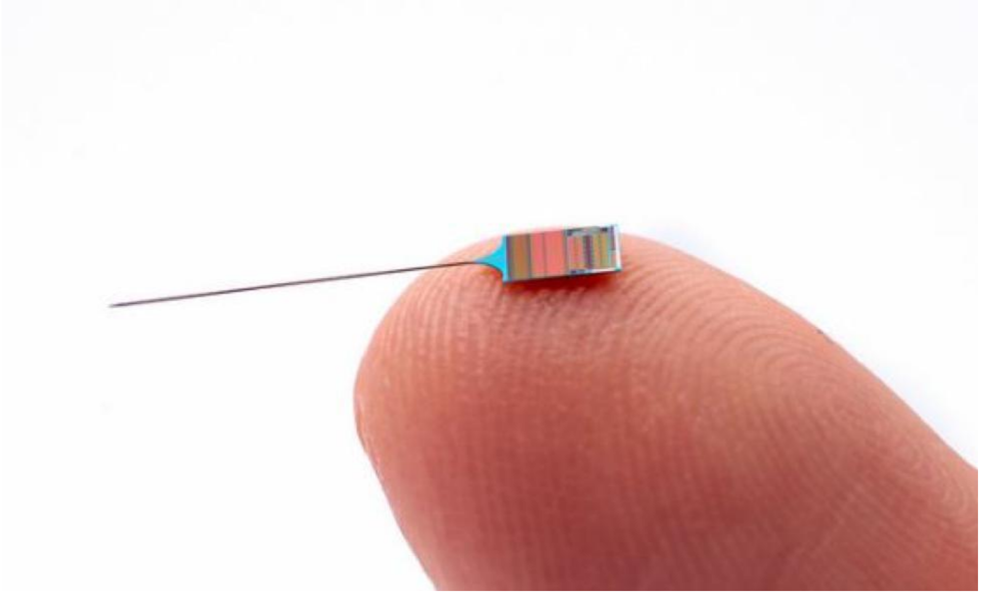




Applications

- Sensing: biomedical technologies, environment monitoring, lidar…
  - Small footprint
  - Low power
  - High sensitivity





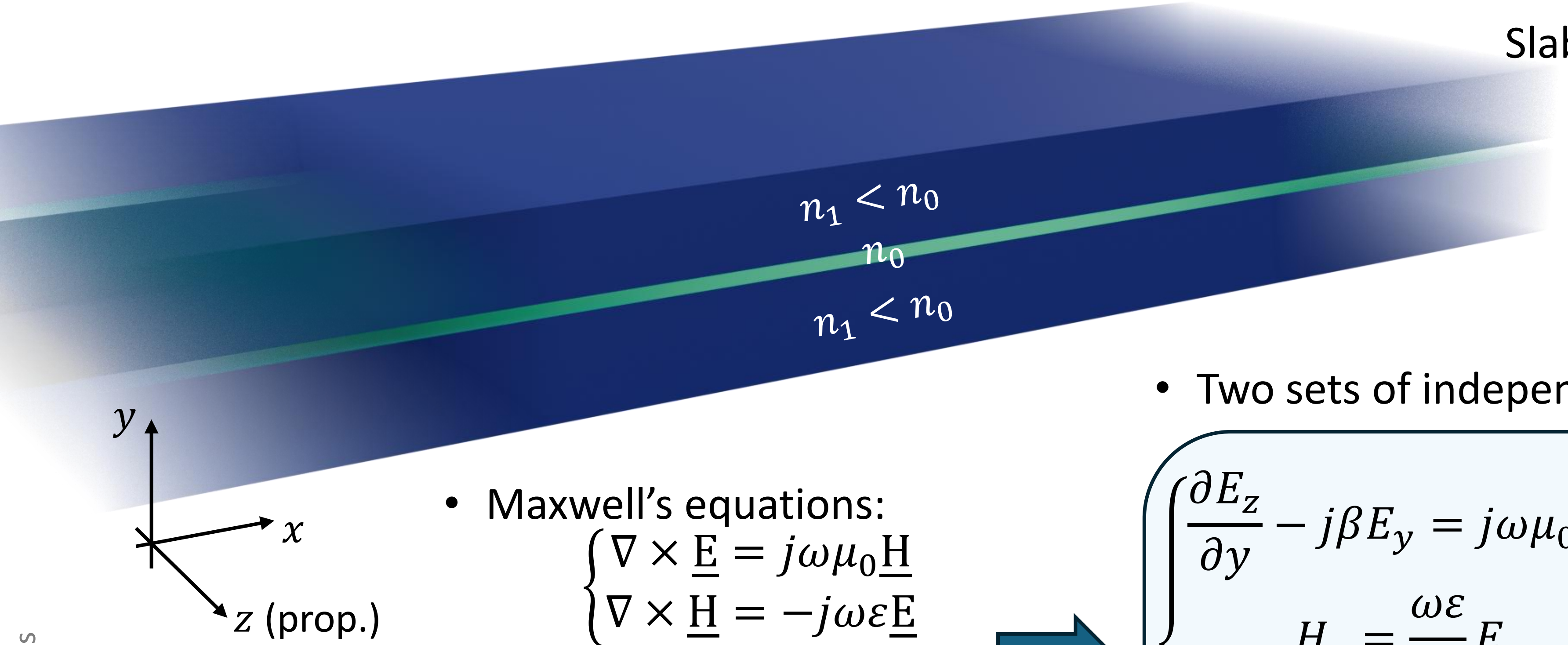
Applications

- To go further: roadmap of integrated photonics

https://photonicsmanufacturing.org/2023-ipsr-i-integrated-photonic-systems-roadmap/

Photonic integrated circuits (PIC)

Waveguides



Slab waveguide: infinite along x-axis

$$\underline{\mathrm{E}}(x,y,z) = \mathrm{E}(y)e^{j(\beta z - \omega t)}$$
$$\underline{\mathrm{H}}(x,y,z) = \mathrm{H}(y)e^{j(\beta z - \omega t)}$$

- Maxwell's equations:

$$\begin{cases} \nabla \times \underline{\mathrm{E}} = j\omega\mu_0 \underline{\mathrm{H}} \\ \nabla \times \underline{\mathrm{H}} = -j\omega\varepsilon \underline{\mathrm{E}} \end{cases}$$

- Invariance on x-axis:

$$\frac{\partial g}{\partial x} = 0$$

- Two sets of independent equations:

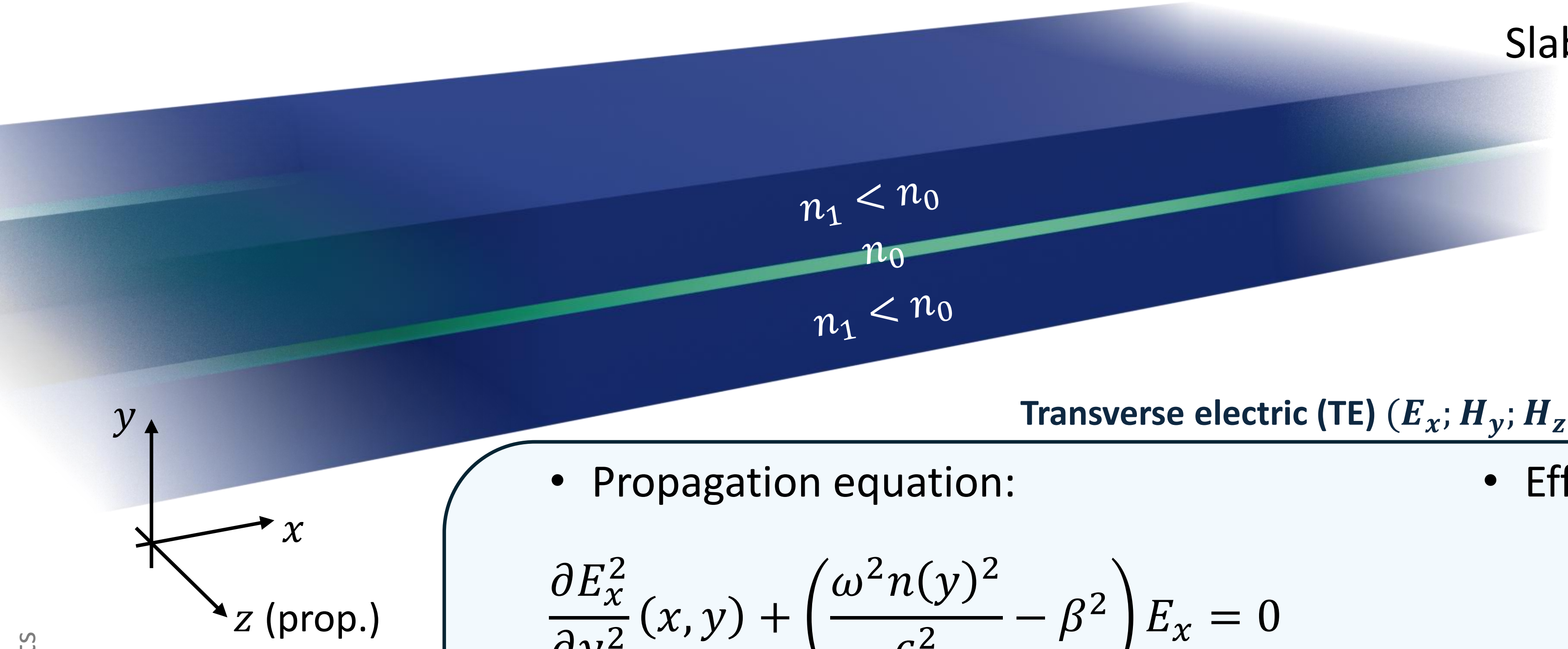
$$\begin{cases} \dfrac{\partial E_z}{\partial y} - j\beta E_y = j\omega\mu_0 H_x \\ H_x = \dfrac{\omega\varepsilon}{\beta} E_y \\ \dfrac{\partial H_x}{\partial y} = -j\omega\varepsilon E_z \end{cases}$$

Transverse magnetic (TM)
$(\boldsymbol{E_y}; \boldsymbol{E_z}; \boldsymbol{H_x})$

$$\begin{cases} \dfrac{\partial H_z}{\partial y} - j\beta H_y = -j\omega\varepsilon E_x \\ E_x = -\dfrac{\omega\mu_0}{\beta} H_y \\ \dfrac{\partial E_x}{\partial y} = j\omega\mu_0 H_z \end{cases}$$

Transverse electric (TE)
$(\boldsymbol{E_x}; \boldsymbol{H_y}; \boldsymbol{H_z})$

Waveguides



Slab waveguide: infinite along x-axis

$$\underline{\mathrm{E}}(x,y,z) = \mathrm{E}(y)e^{j(\beta z - \omega t)}$$

$$\underline{\mathrm{H}}(x,y,z) = \mathrm{H}(y)e^{j(\beta z - \omega t)}$$

Transverse electric (TE) $(\boldsymbol{E_x}; \boldsymbol{H_y}; \boldsymbol{H_z})$

- Propagation equation:

$$\frac{\partial E_x^2}{\partial y^2}(x,y) + \left(\frac{\omega^2 n(y)^2}{c^2} - \beta^2\right) E_x = 0$$

- Boundary conditions: continuity relations at interfaces
  - Problem is fully defined

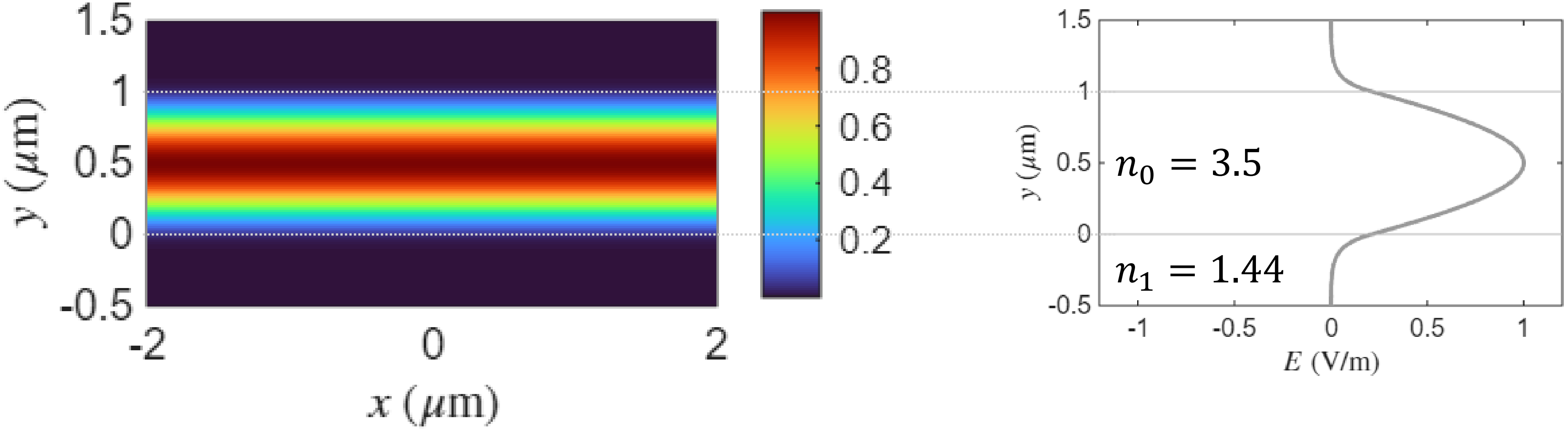
- Effective index:

$$\beta = \frac{\omega n_{\mathrm{eff}}}{c} = \frac{2\pi n_{\mathrm{eff}}}{\lambda}$$

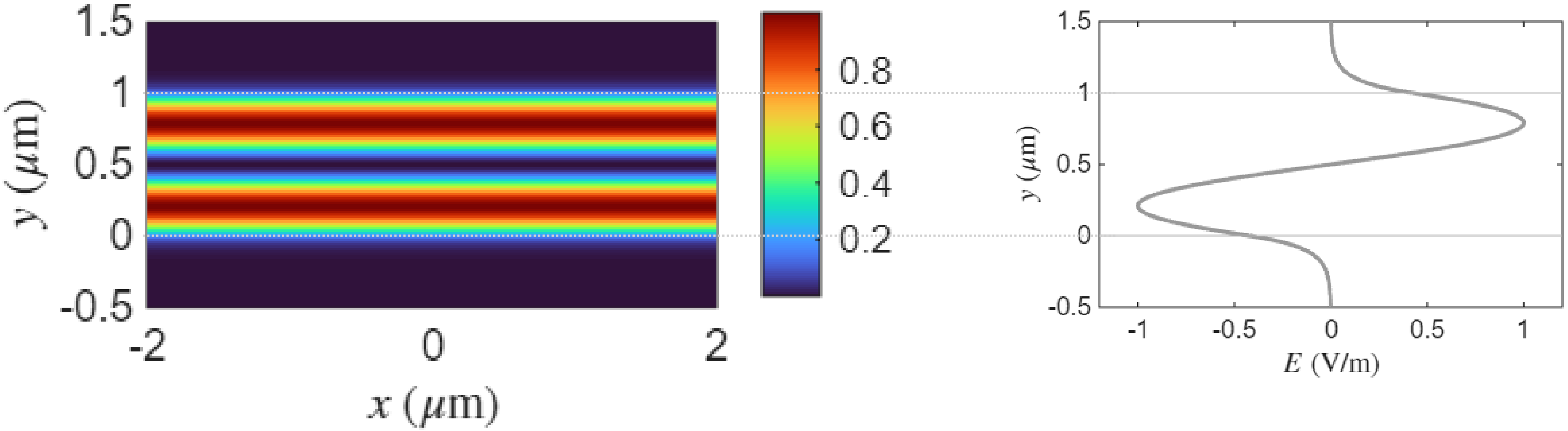
$$n_0 < n_{\mathrm{eff}} < n_1$$

Waveguides

TE modes, $\lambda = 1550$ nm, field intensity map ($|E|^2$)



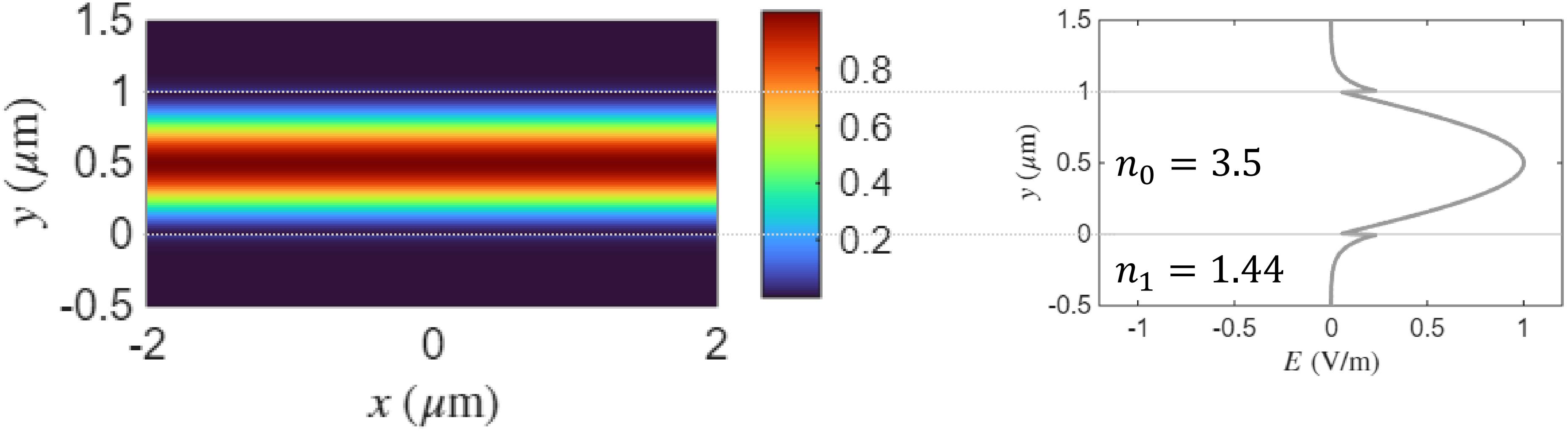
TE 0
$n_{\text{eff}} = 3.41$



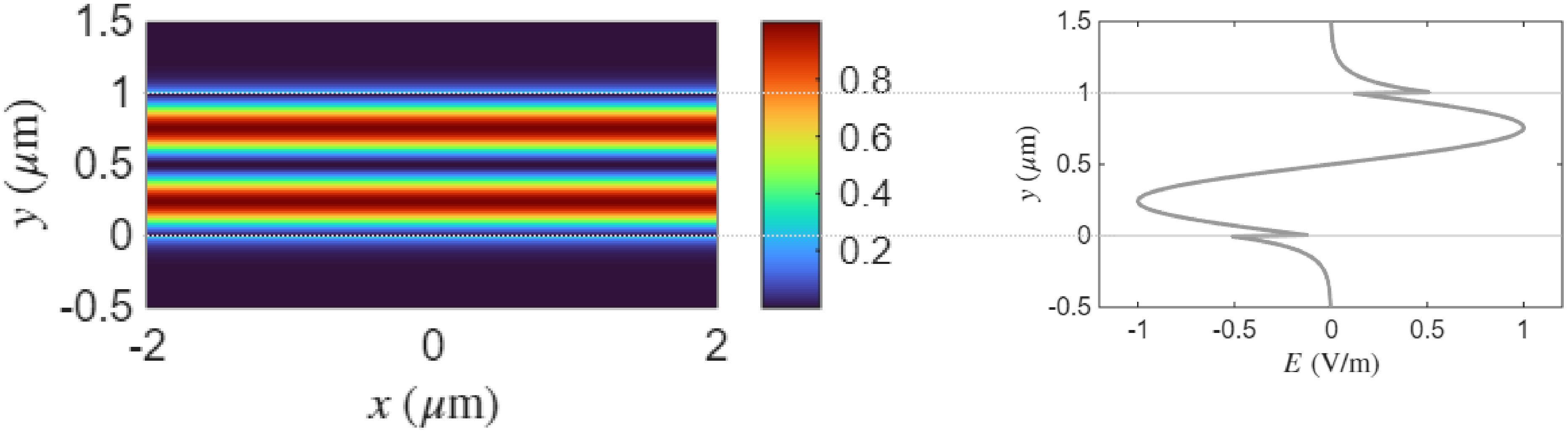
TE 1
$n_{\text{eff}} = 3.21$

Waveguides

TM modes, $\lambda = 1550$ nm, field intensity map ($|E|^2$)

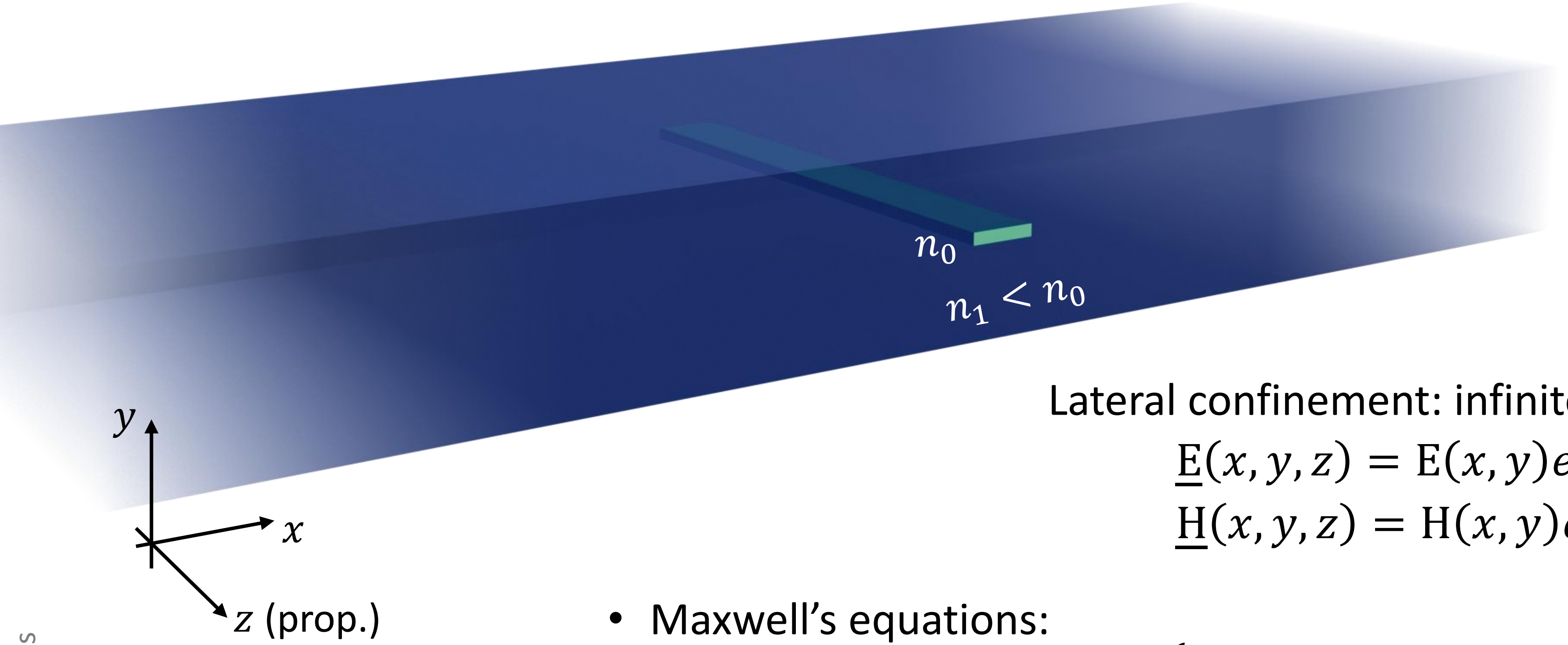


TM 0
$n_{\text{eff}} = 3.39$



TM 1
$n_{\text{eff}} = 3.13$

Waveguides



Lateral confinement: infinite along x-axis

$$\underline{\mathrm{E}}(x,y,z) = \mathrm{E}(x,y)e^{j(\beta z-\omega t)}$$
$$\underline{\mathrm{H}}(x,y,z) = \mathrm{H}(x,y)e^{j(\beta z-\omega t)}$$

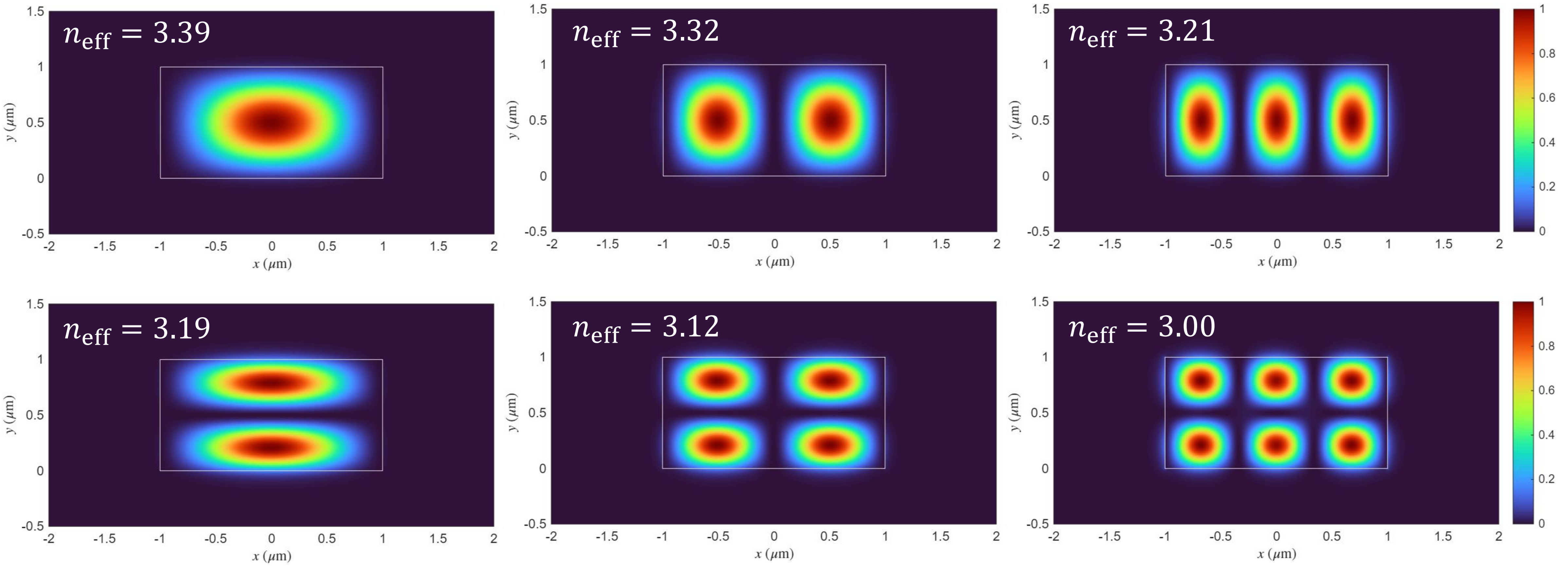
- Maxwell's equations:

$$\begin{cases} \nabla \times \underline{\mathrm{E}} = j\omega\mu_0\underline{\mathrm{H}} \\ \nabla \times \underline{\mathrm{H}} = -j\omega\varepsilon\underline{\mathrm{E}} \end{cases}$$

- No invariance on transverse axis: quasi-TE and quasi-TM modes
- $\beta = \frac{\omega n_{\mathrm{eff}}}{c} = \frac{2\pi n_{\mathrm{eff}}}{\lambda}$
- $n_0 < n_{\mathrm{eff}} < n_1$

Waveguides

TE modes, $\lambda = 1550$ nm, field intensity map ($|E|^2$)

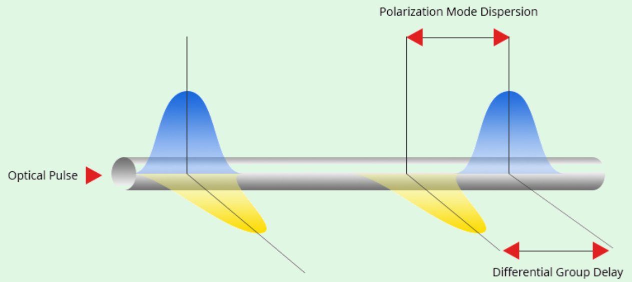


Waveguides

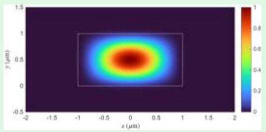
Properties (n_{eff}, area) depend on different factors

Inter-modal dispersion

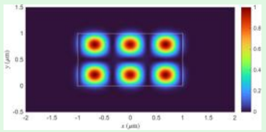
Dependence on:
- Polarization



- Mode



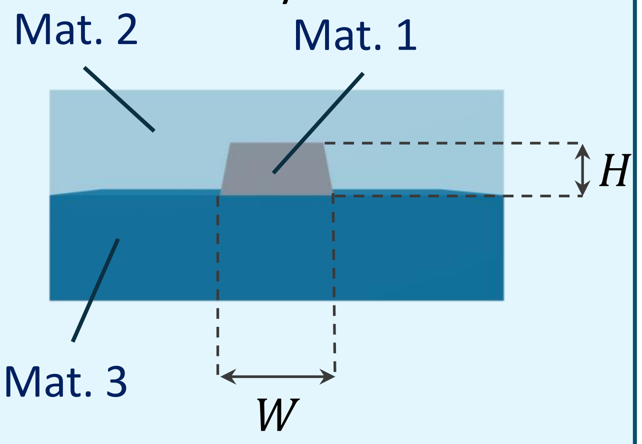
$n_{\text{eff}} = 3.39$



$n_{\text{eff}} = 3.00$

Intra-modal dispersion

Dependence on:
- Materials
- Geometry



- Temperature

Chromatic dispersion

Dependence on:
- Wavelength

$$n(\lambda) = \sqrt{1 + \sum_j \frac{B_j \lambda^2}{\lambda^2 - C_j}}$$

Waveguide dispersion

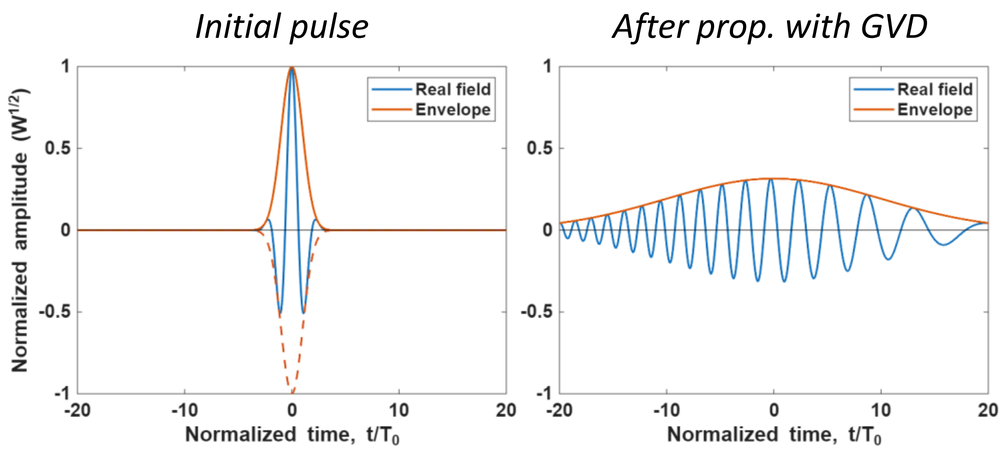
- All this leads to a strong dependence of β with ω:

$$\beta(\omega) = \beta_0 + \beta_1(\omega - \omega_0) + \frac{1}{2}\beta_2(\omega - \omega_0)^2 + \cdots$$

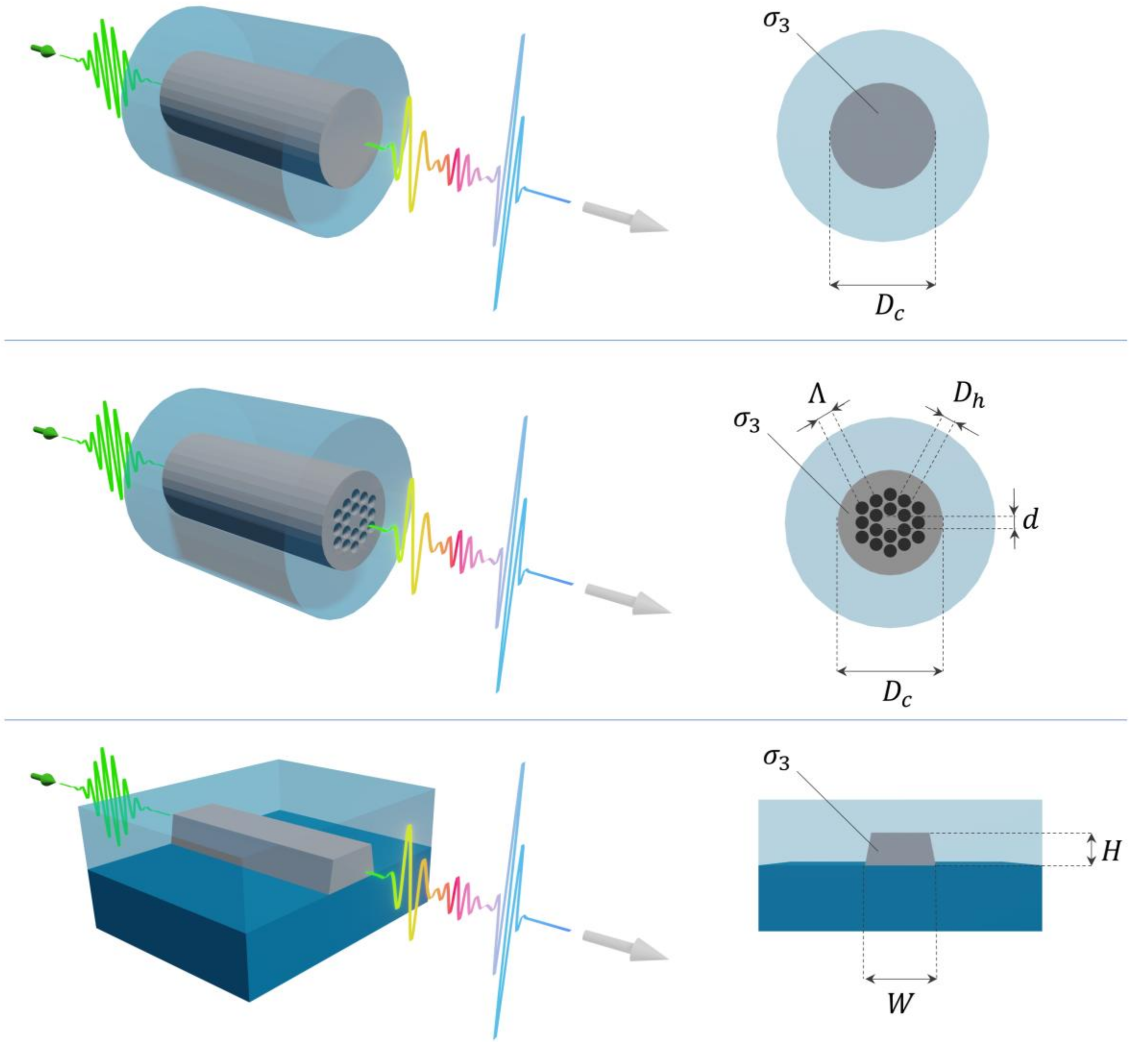
$$\beta_k = \frac{\partial \beta}{\partial \omega}\bigg|_{\omega_0}$$

- Rely on numerical solvers

- $\beta_1 \rightarrow$ group velocity
- $\beta_2 \rightarrow$ group velocity dispersion (GVD)



Waveguide dispersion



- Standard fibers
  - Material (but limited)
  - Diameter
- Photonic crystal fibers
  - Material (but limited)
  - Core diameter
  - Rod diameter
  - Spacing
- Integrated waveguides
  - Materials (wide choice)
  - Width
  - Height
  - Shape

Materials

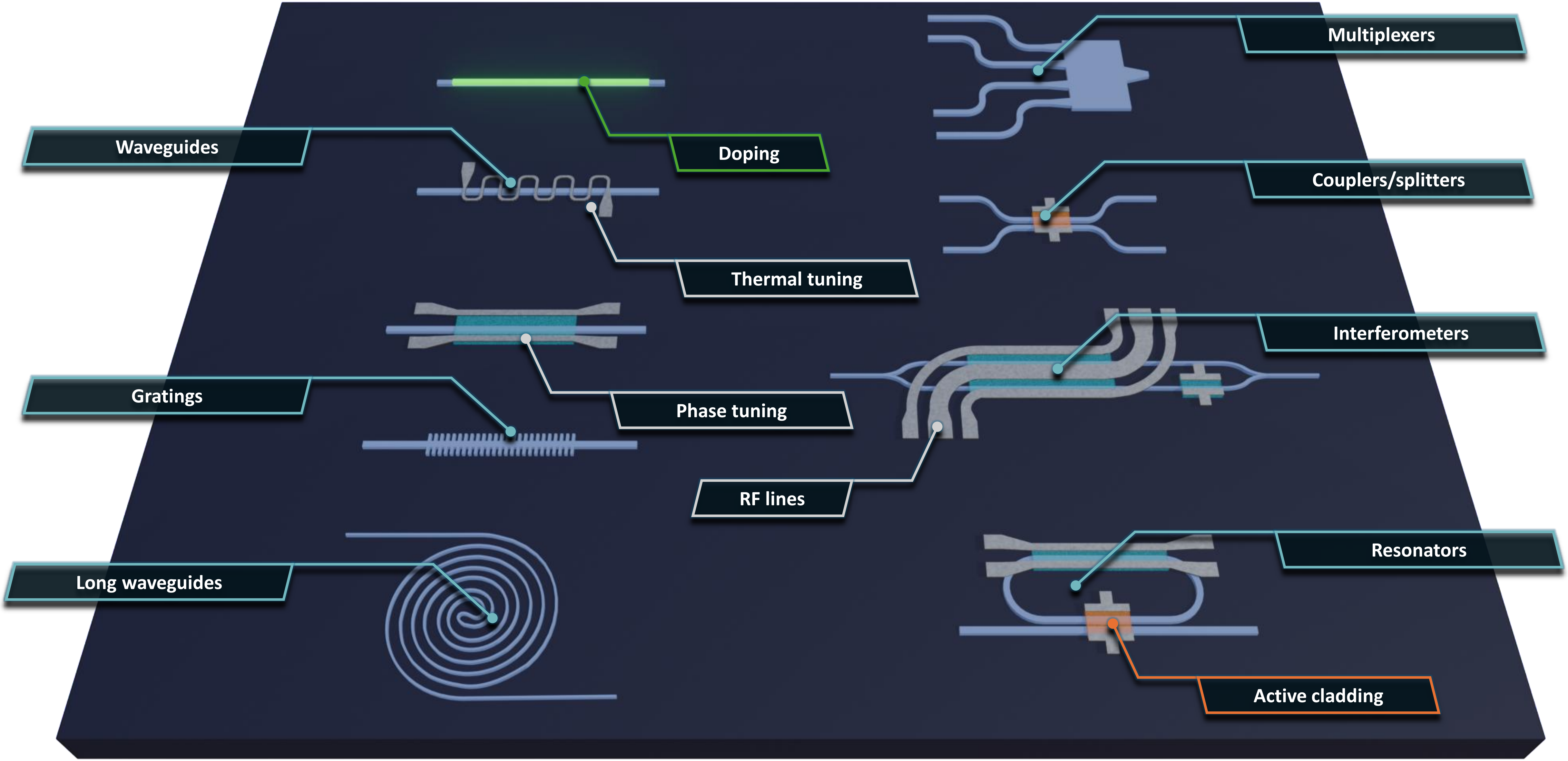
Material	Trans. Window (μm)	Bandgap	n
SiO_2	0.13 – 3.5	9 eV	1.46
Si	1.1 – 9	1.12 eV	3.48
Si_3N_4	0.35 – 7	5 eV	2
$Si_{1\text{-}x}Ge_x$	1.5 – 11	~1.1 eV	3.6 (at 4 μm)
Ge	2 – 14	0.7 eV	4 (at 4 μm)
III-V	0.57/0.87 – >6.5	1.42 – 2.16 eV	2.86 -3.5
AlN	0.2 – >5.5	6.2 eV	2.21 (*o*), 2.26 (*e*)
TFLN	0.4 – 5	~4 eV	2.21 (*o*), 2.13 (*e*)
SiC	0.5 – MIR	2.36 – 3.23 eV	2.6
Diamond	0.22 – MIR	5.5 eV	2.38
Ta_2O_5	0.3 – 8	3.8 eV	2

LIGENTEC
imec
leti ceatech
csem
LioniX INTERNATIONAL

Typical propagation loss: $1\ \mathrm{dB \cdot m^{-1}} \rightarrow 10\ \mathrm{dB \cdot cm^{-1}}$
Typical propagation loss of commercial fibers: $\sim 0{,}1\ \mathrm{dB \cdot km^{-1}}$

Factors playing on loss:
- Material (absorption)
- Geometry: smaller waveguide → higher interaction with sidewall roughness
- Wavelength: scattering $\propto 1/\lambda^4$

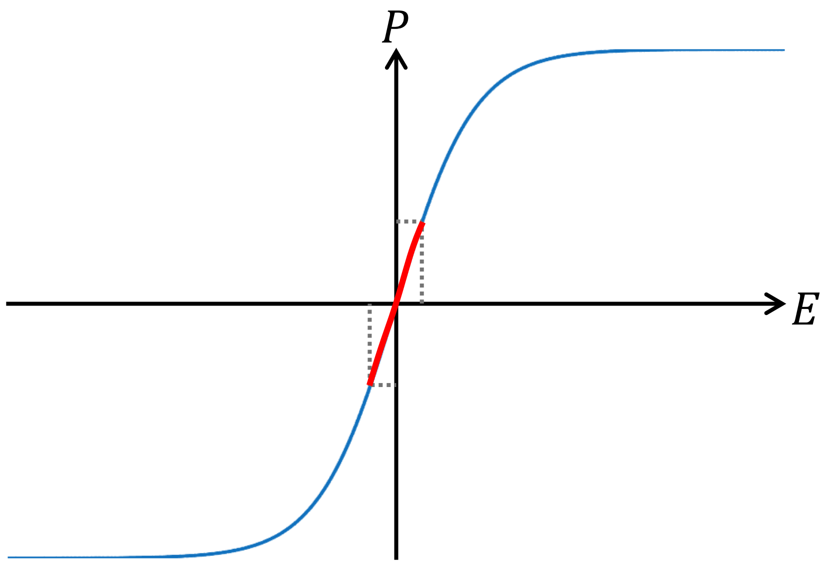
Integrated devices



Linear optics

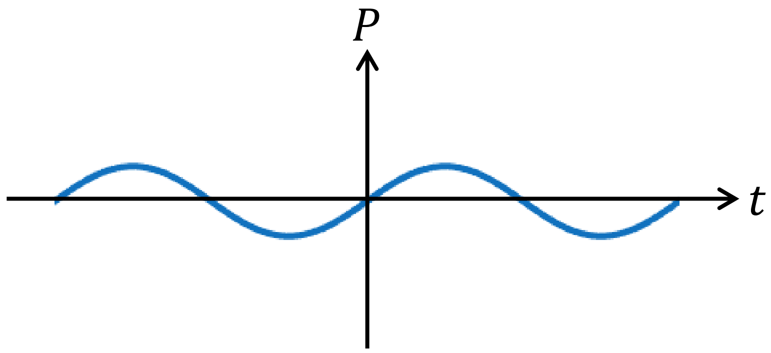
Materials submitted to a field E → dipoles → dielectric polarization P → total field

Material response



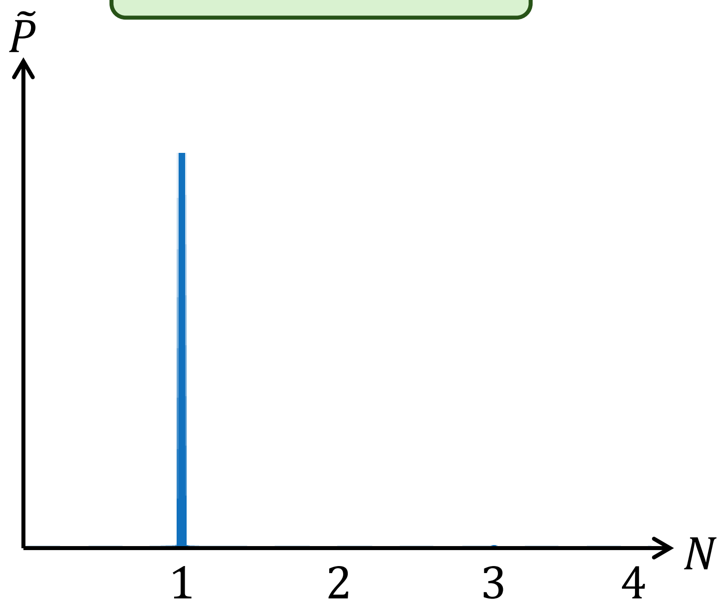
$P \simeq \varepsilon_0 \chi^{(1)} E$

Time trace



Sine wave

Fourier transform

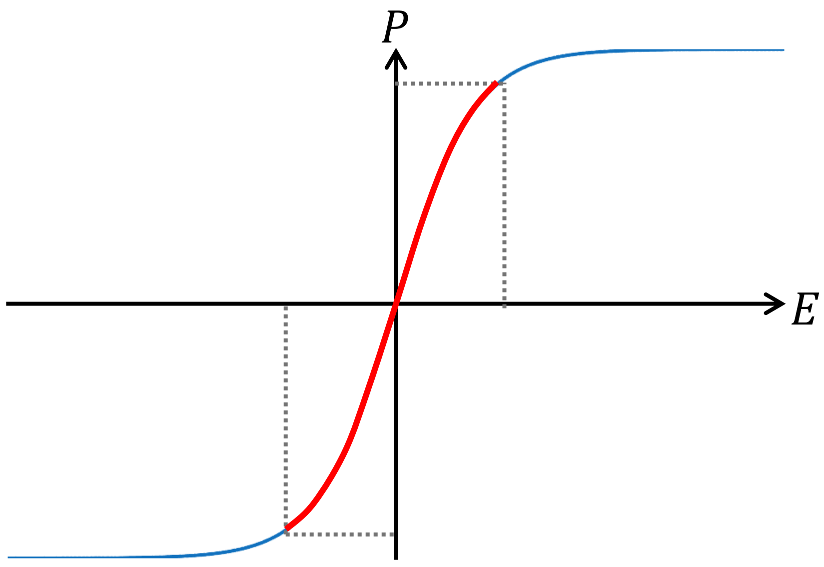


Only fundamental harmonic

Nonlinear optics – 3rd order

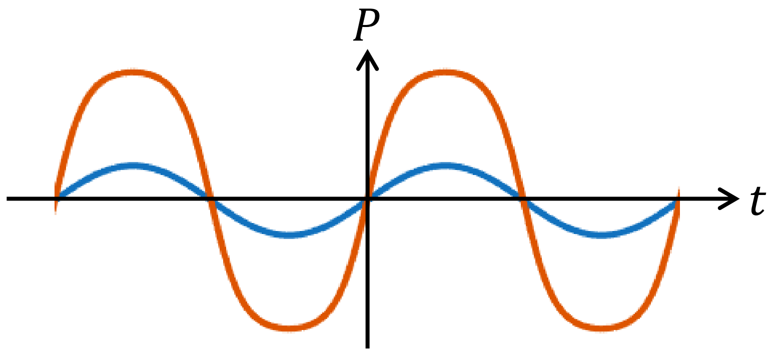
Materials submitted to a field E → dipoles → dielectric polarization P → total field

Material response



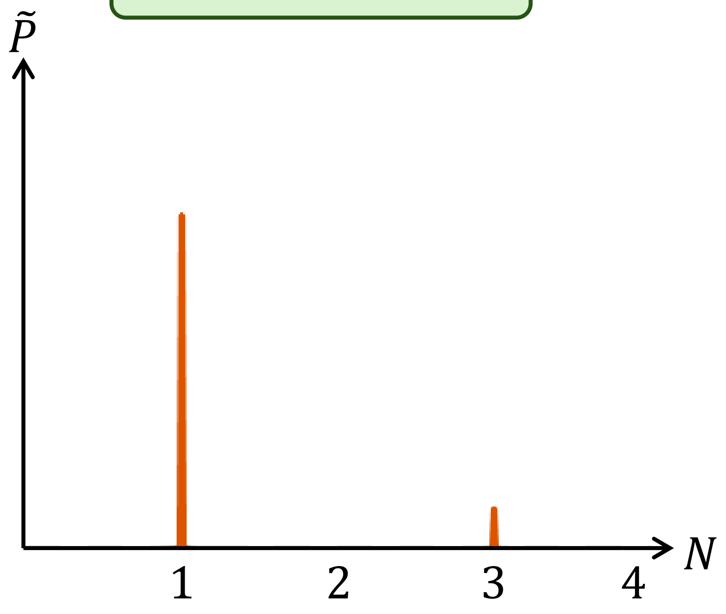
$$P \simeq \varepsilon_0\left(\chi^{(1)}E + \chi^{(3)}E^3\right)$$

Time trace



Distorted sine wave,
But centro-symmetric!

Fourier transform

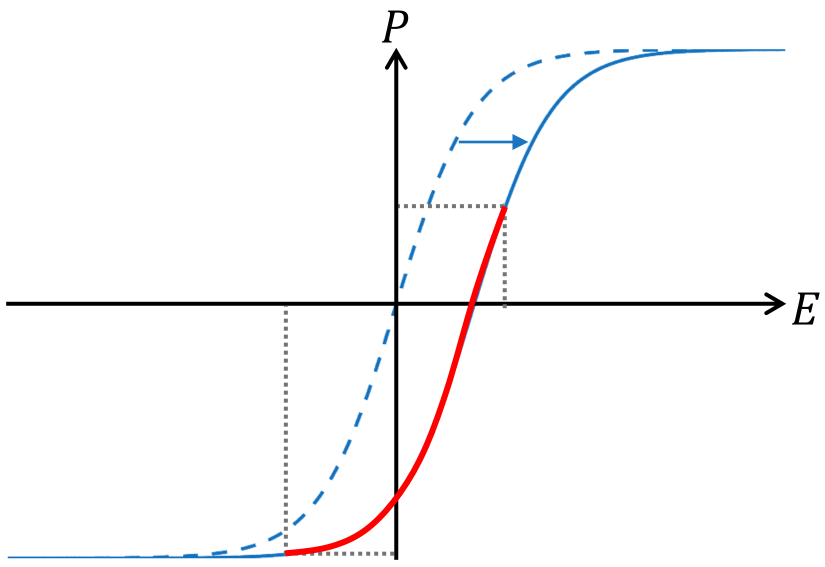


Fundamental + 3rd harmonic

Nonlinear optics – 2nd and 3rd order

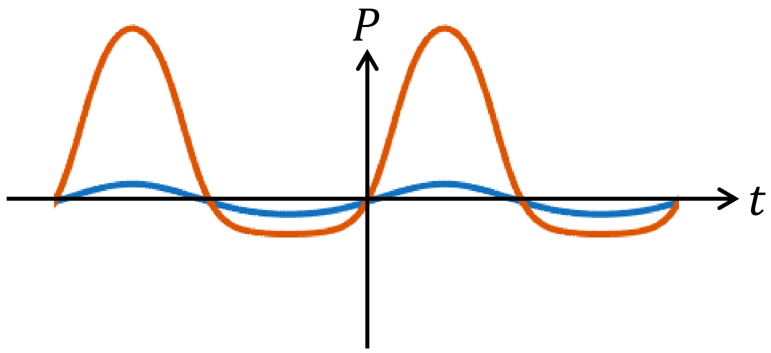
Materials submitted to a field E → dipoles → dielectric polarization P → total field

Material response



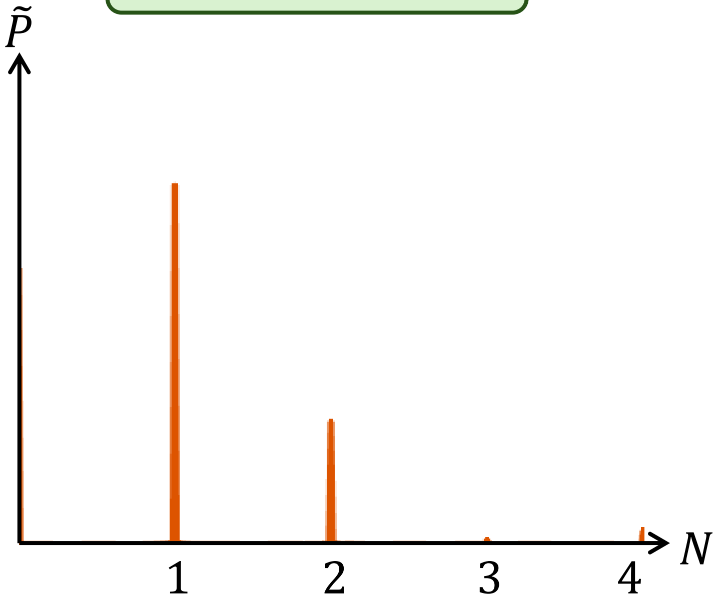
$$P \simeq \varepsilon_0\left(\chi^{(1)}E + \chi^{(2)}E^2 + \chi^{(3)}E^3\right)$$

Time trace



Distorted sine wave,
But not symmetric!

Fourier transform



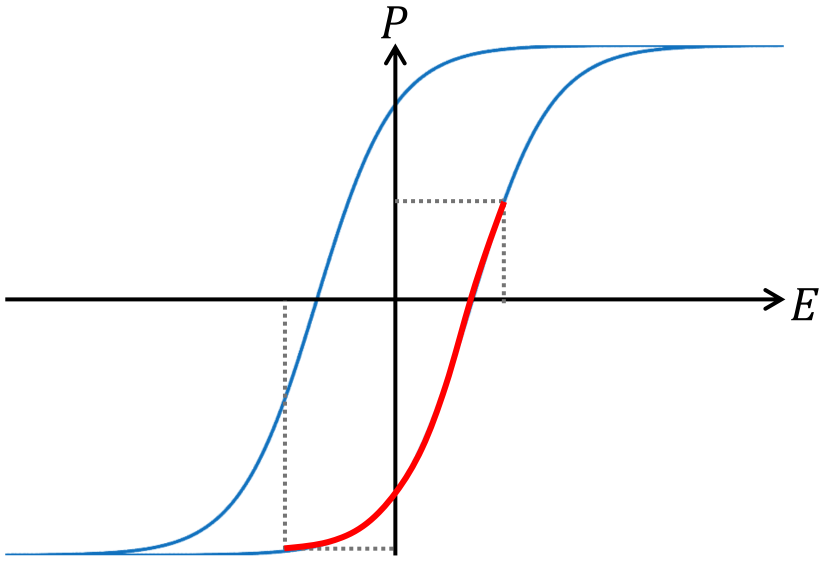
Fundamental +2nd (+3rd) harmonic

Nonlinear optics – 2^{nd} and 3^{rd} order

Materials submitted to a field E → dipoles → dielectric polarization P → total field

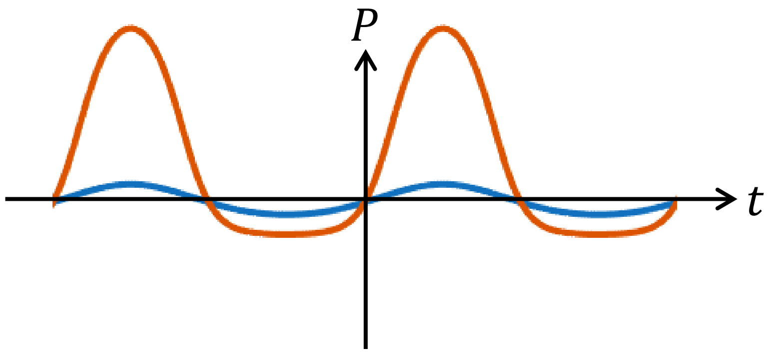
Ferroelectric material: possess a spontaneous polarization

Material response



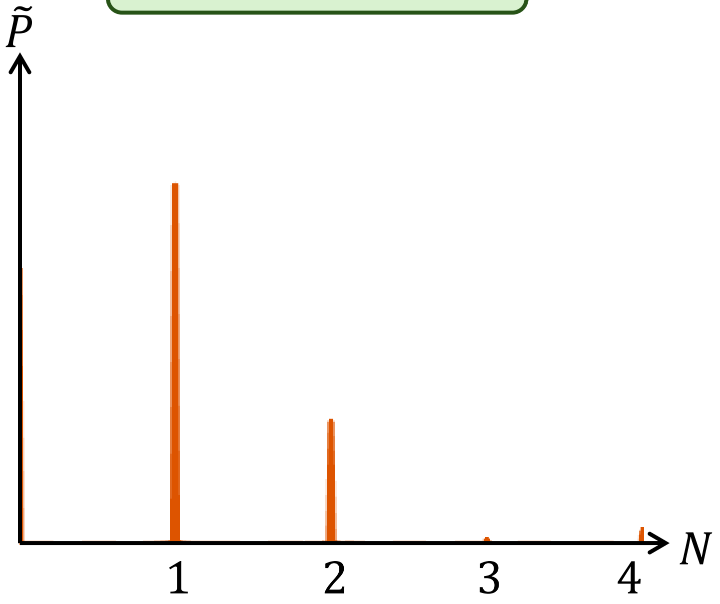
$$P \simeq \varepsilon_0\left(\chi^{(1)}E + \chi^{(2)}E^2 + \chi^{(3)}E^3\right)$$

Time trace



Distorted sine wave,
But not symmetric!

Fourier transform



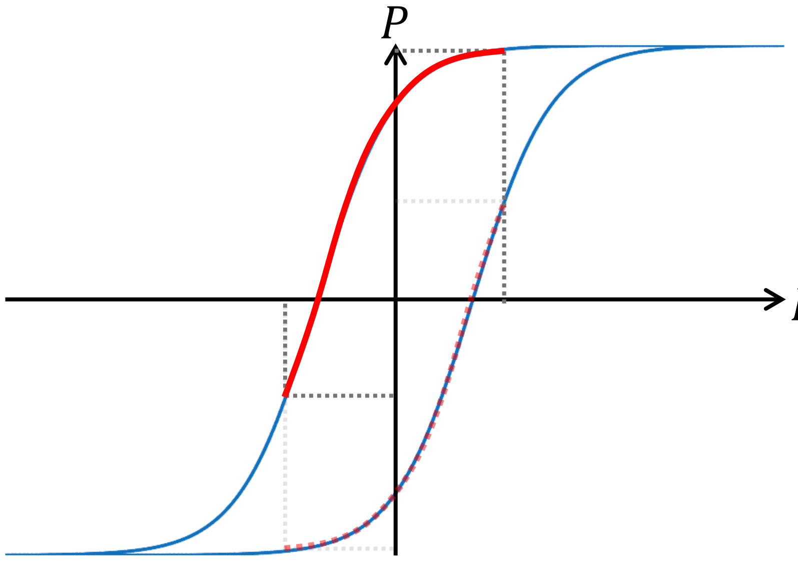
Fundamental +2^{nd} (+3^{rd}) harmonic

Nonlinear optics – 2nd and 3rd order

Materials submitted to a field E → dipoles → dielectric polarization P → total field

Ferroelectric material: 2 possible paths (hysteresis)

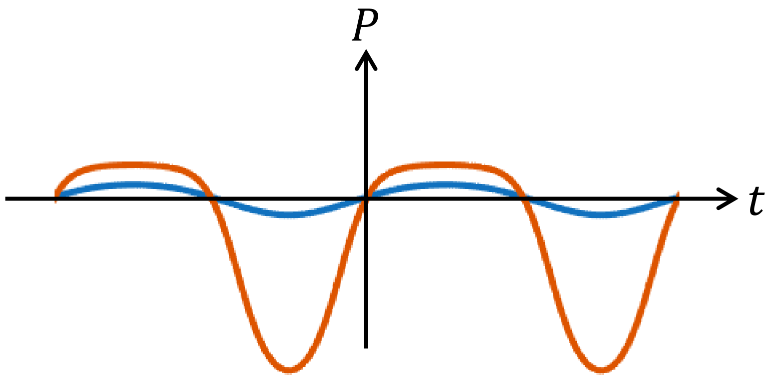
Material response



$$P \simeq \varepsilon_0\left(\chi^{(1)}E + \chi^{(2)}E^2 + \chi^{(3)}E^3\right)$$

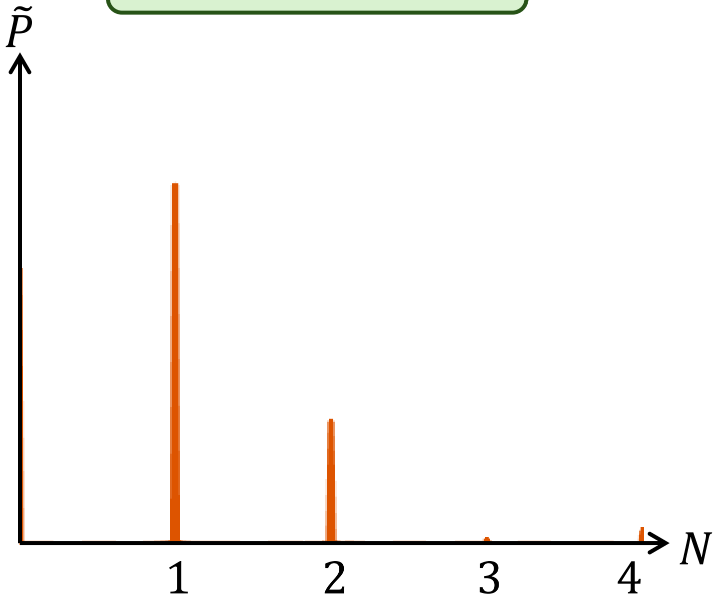
Opposite curvature → opposite $\chi^{(2)}$

Time trace



Distorted sine wave,
But not symmetric!

Fourier transform



Fundamental +2nd (+3rd) harmonic

Photonic integrated circuits

Nonlinear optics

$\chi^{(2)}$

EO modulator Optical switch Sensor	**Pockels**
OPO OPA Frequency comb	**DFG**
Quantum optics	**SPDC**
VIS/UV source Microscopy Spectroscopy	**SHG**

$\chi^{(3)}$

Kerr	Solitons Modulation Supercontinuum
SA	Lasers Thresholder
THG	VIS sources Imaging
FWM	Quantum optics Signal processing Frequency combs

$\chi^{(n)}$

HHG

Attoscience — UV sources — Strong-field physics

Nonlinear optics: what integration can do?

- Example: supercontinuum generation
  - SPM: the refractive index depends on the light intensity: $n = n_0 + n_2 I$
    - n_2 depends on the material

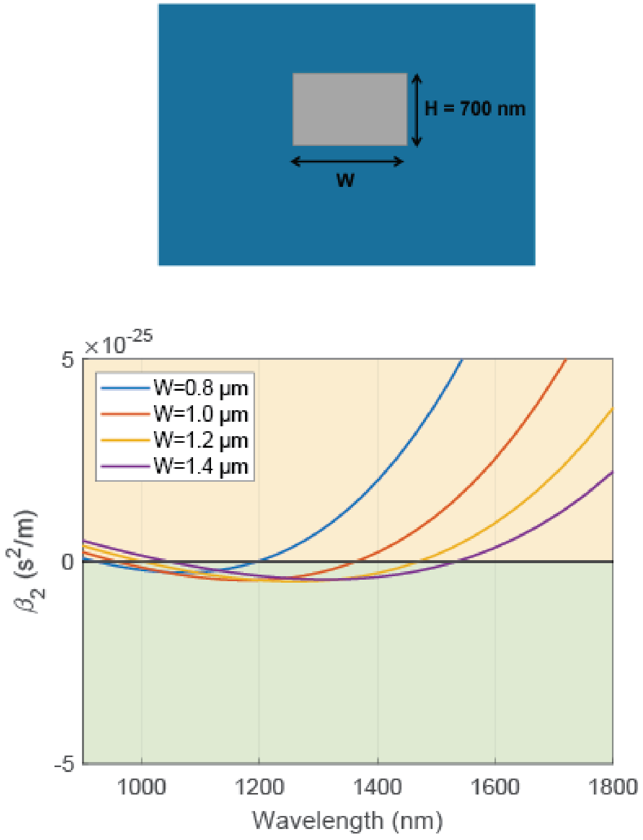
$$n_2 = \frac{3\chi^{(3)}}{4\varepsilon_0 c_0 n_0^2}$$

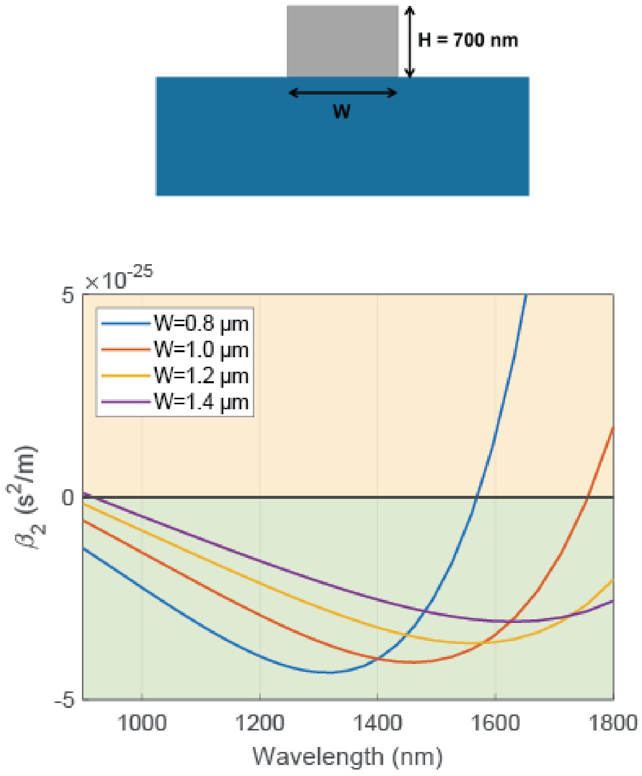
    - The total effect scales with the nonlinear coefficient $\gamma = \frac{\omega_0 n_2}{c A_{\text{eff}}}$
    - A_{eff} depends on the confinement of light in the waveguide:

$$A_{\text{eff}} = \frac{\left(\iint_{-\infty}^{\infty} |\mathbf{F}(x,y)|^2\, dxdy\right)^2}{\iint_{\sigma_3} |\mathbf{F}(x,y)|^4 dxdy}$$

GVD: the pulse propagation depends on the strength of the GVD and its wavelength dependence

Nonlinear optics: what integration can do?





- Large core-cladding index contrast + small dimensions
- Reduces A_{eff} drastically
  - Higher nonlinear effect for a given input power
- GVD is very sensitive to the geometry
  - Lithographic control of dispersion
  - Can also be an issue: sensitivity to fabrication tolerances and errors

Materials

Material	Trans. Window (μm)	Bandgap	n	n_2 (cm²/W)	$\chi^{(2)}$ (pm/V)
SiO_2	0.13 – 3.5	9 eV	1.46	$\sim 2.7 \cdot 10^{-16}$	x
Si	1.1 – 9	1.12 eV	3.48	$\sim 6 \cdot 10^{-14}$	x
Si_3N_4	0.35 – 7	5 eV	2	$\sim 2.4 \cdot 10^{-15}$	x
$Si_{1-x}Ge_x$	1.5 – 11	~1.1 eV	3.6 (at 4 μm)	$\sim 4 \cdot 10^{-14}$ (at 4 μm)	n.a.
Ge	2 – 14	0.7 eV	4 (at 4 μm)	$0.5 – 1 \cdot 10^{-13}$ (at 4 μm)	x
III-V	0.57/0.87 – >6.5	1.42 – 2.16 eV	2.86 -3.5	$\sim 10^{-13}$	High
AlN	0.2 – >5.5	6.2 eV	2.21 (*o*), 2.26 (*e*)	$\sim 2.3 \cdot 10^{-15}$	Moderate
TFLN	0.4 – 5	~4 eV	2.21 (*o*), 2.13 (*e*)	$\sim 1.8 \cdot 10^{-15}$	High
SiC	0.5 – MIR	2.36 – 3.23 eV	2.6	$\sim 1 \cdot 10^{-14}$	Moderate
Diamond	0.22 – MIR	5.5 eV	2.38	$\sim 0.8 \cdot 10^{-15}$	x
Ta_2O_5	0.3 – 8	3.8 eV	2	$\sim 2 \cdot 10^{-14}$	n.a.

Frequency conversion

Second harmonic generation

- In presence of a perturbation on the dielectric polarization, the propagation equation is modified:

$$\nabla^2 \underline{\mathrm{E}}(r,t) - \frac{n(x,y)^2}{c^2}\frac{\partial^2 \underline{\mathrm{E}}}{\partial t^2}(r,t) = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2 \underline{\mathrm{P}}^{\mathrm{NL}}}{\partial t^2}(r,t)$$

$$\underline{\mathrm{P}} = \varepsilon_0 \chi^{(1)} \underline{\mathrm{E}} + \underline{\mathrm{P}}^{\mathrm{NL}}$$

- $\underline{\mathrm{P}}^{\mathrm{NL}}$ contains harmonic terms: we need to model the propagation of each component

Second harmonic generation

- Coupled mode theory:
  - we consider the nonlinear polarization as a perturbation on the propagating modes
  - We study the « cross-Poynting vector » : the amount of power coupling between different waves with modes l and m

$$\Pi_{lm} = \mathrm{E}_l \times \mathrm{H}_m^* + \mathrm{E}_m^* \times \mathrm{H}_l$$

  - In **<u>the linear case</u>**, this results in the mode orthogonality equation:

$$\iint_{\infty} [\Pi_{lm}]_z dxdy = 4\mathcal{P}_0 \delta_{lm}$$

Power

Kronecker symbol

Second harmonic generation

- Coupled mode theory:
  - we consider the nonlinear polarization as a perturbation on the propagating modes
  - We study the « cross-Poynting vector » : the amount of power coupling between different waves with modes l and m

$$\Pi_{lm} = \mathrm{E}_l \times \mathrm{H}_m^* + \mathrm{E}_m^* \times \mathrm{H}_l$$

  - In **<u>the nonlinear case</u>**, this results in the mode orthogonality equation:

$$\frac{\partial}{\partial z} \iint_\infty [\Pi_{lm}]_z dxdy = - \iint_\infty \mathrm{E}_\mathrm{m}^* \cdot \frac{\partial \mathrm{P}^{\mathrm{NL}}}{\partial t} dxdy$$

Second harmonic generation

- We need to define the propagating fields accordingly:

$$E(r,t) = A(z)F(x,y)e^{j(\beta z - \omega t)}$$

Propagation

Field distribution in the cross-section

Envelope (normalized so that $|A|^2$ =power)

- We use this definition for the pump (fundamental harmonic) and the SH fields

Second harmonic generation

- We obtain a set of two coupled equations:

$$\begin{cases} \dfrac{\partial A_p}{\partial z}(z) = j\gamma_{sh}^* A_p^*(z) A_h(z) e^{j(\beta_h - 2\beta_p)z} \\ \dfrac{\partial A_h}{\partial z}(z) = j\gamma_{sh} A_p^2(z) e^{-j(\beta_h - 2\beta_p)z} \end{cases}$$

$\Delta\beta = \beta_h - 2\beta_p$: wave-vector mismatch between the two waves

$$\gamma_{sh} = \frac{\omega_p \chi^{(2)}}{\sqrt{2\varepsilon_0 c^3 n_p^2 n_h}} \cdot \frac{\iint_{\sigma_2} F_p^*(x,y)^2 F_h(x,y)\,dxdy}{\iint_\infty \left|F_p(x,y)\right|^2 dxdy \sqrt{\iint_\infty |F_h(x,y)|^2 dxdy}}$$

Second harmonic generation

- We obtain a set of two coupled equations:

$$\begin{cases} \frac{\partial A_p}{\partial z}(z) = j\gamma_{sh}^* A_p^*(z) A_h(z) e^{j(\beta_h - 2\beta_p)z} \\ \frac{\partial A_h}{\partial z}(z) = j\gamma_{sh} A_p^2(z) e^{-j(\beta_h - 2\beta_p)z} \end{cases}$$

$\boldsymbol{\Delta\beta = \beta_h - 2\beta_p}$: wave-vector mismatch between the two waves

$\boldsymbol{\gamma_{sh} \propto \iint_{\sigma_2} F_p^*(x,y)^2 F_h(x,y)\,dxdy}$: overlap of pump and SH modes

Second harmonic generation

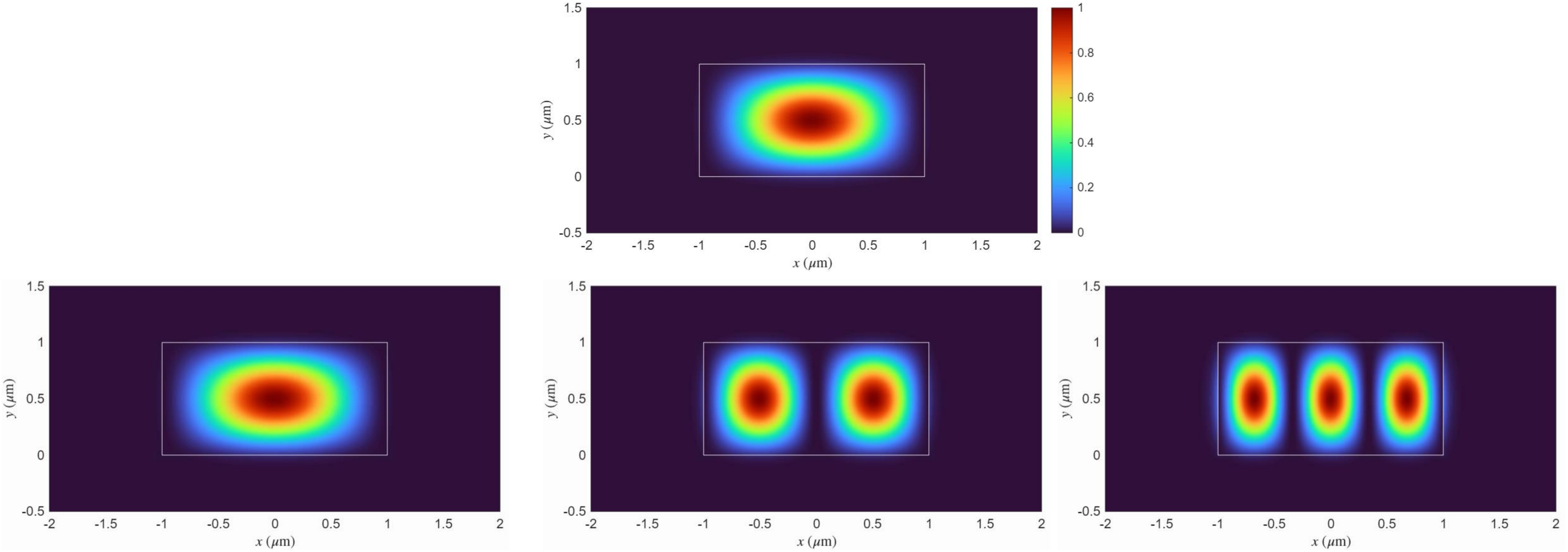
- Hypothesis: the conversion process is weak enough to neglect changes in pump amplitude: $A_p(z) = A_p(0) = \sqrt{\mathcal{P}_p}$

$$\mathcal{P}_h(z) = |\gamma_{sh}|^2 \mathcal{P}_p^2 z^2 \cdot \mathrm{sinc}^2\left(\frac{\Delta\beta}{2} z\right)$$

$\boldsymbol{\Delta\beta = \beta_h - 2\beta_p}$: wave-vector mismatch between the two waves

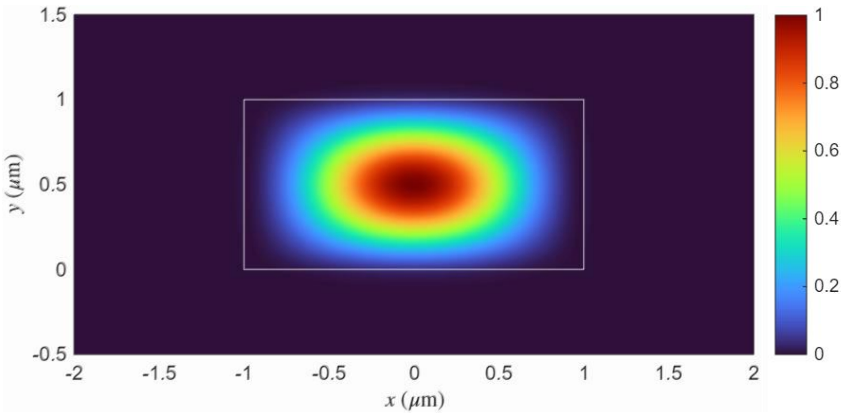
$\boldsymbol{\gamma_{sh} \propto \iint_{\sigma_2} F_p^*(x,y)^2 F_h(x,y) dxdy}$: overlap of pump and SH modes

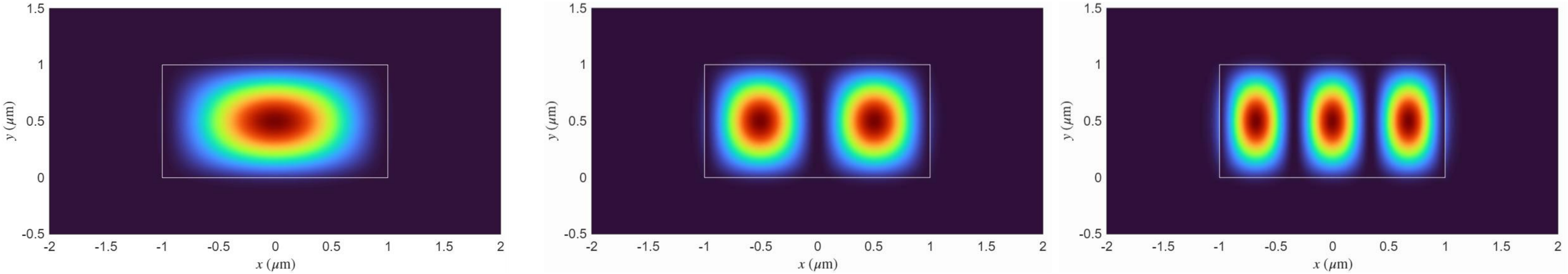
Second harmonic generation



Second harmonic generation

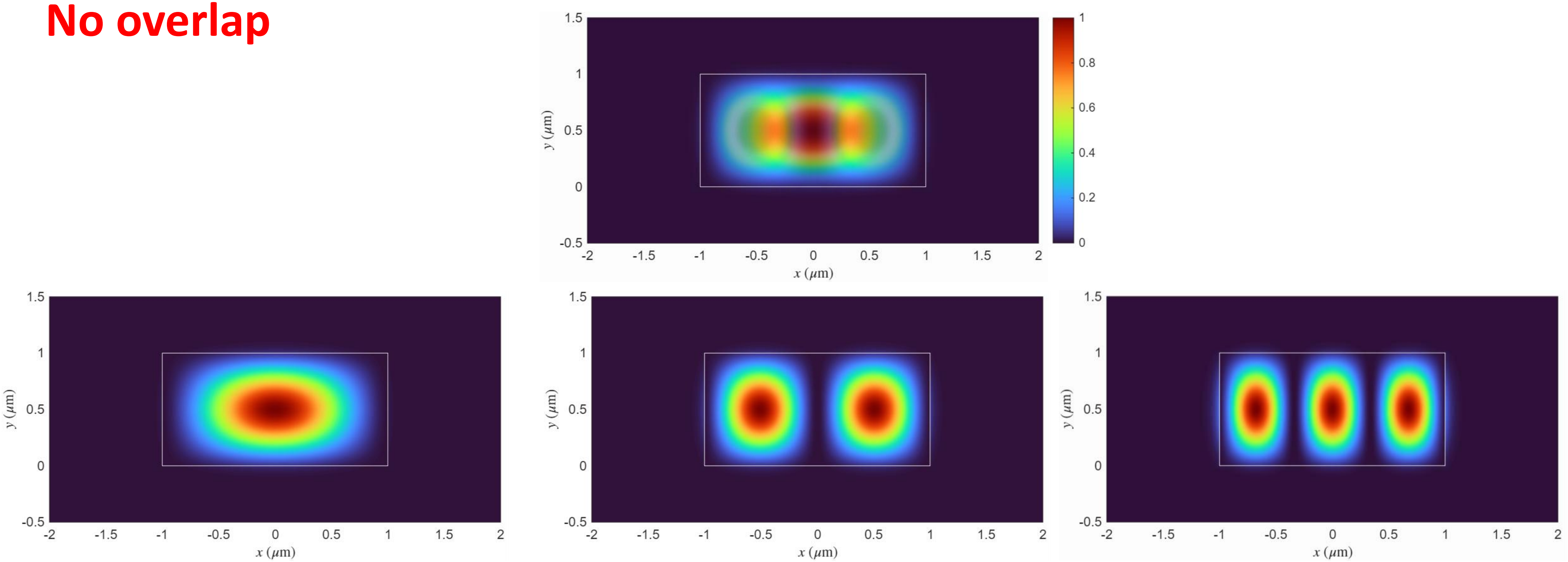
Highest overlap





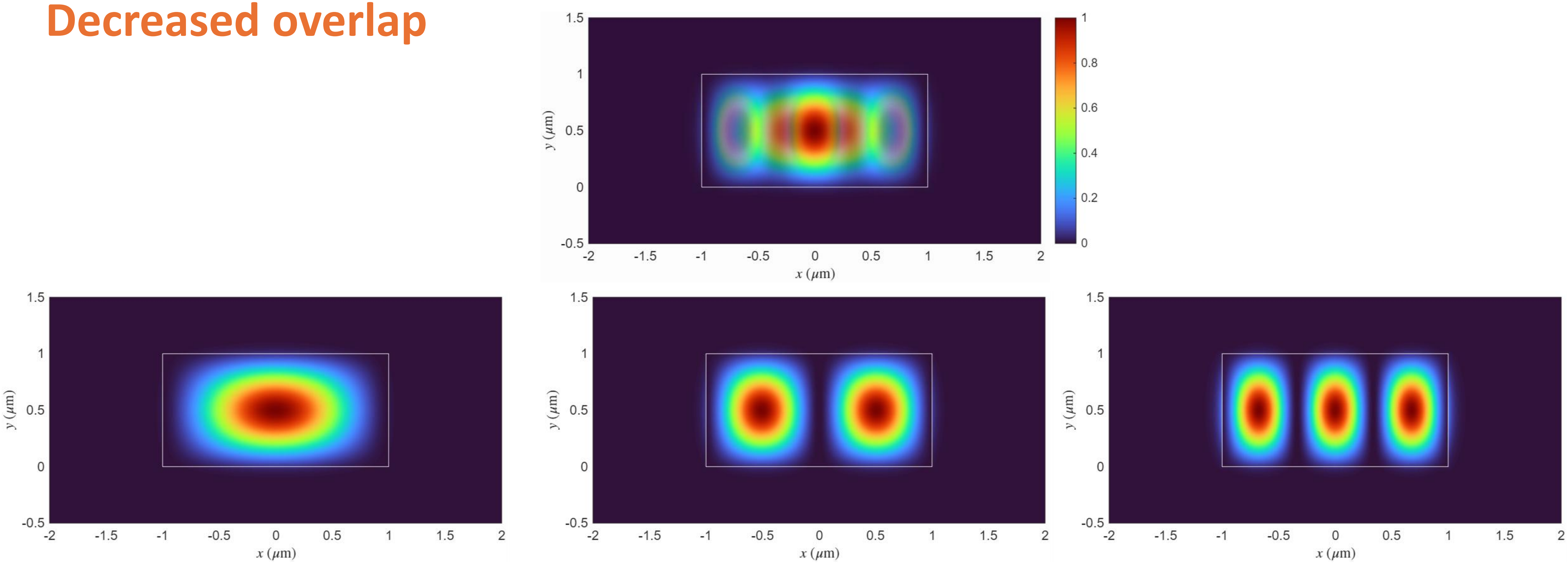
Second harmonic generation

No overlap



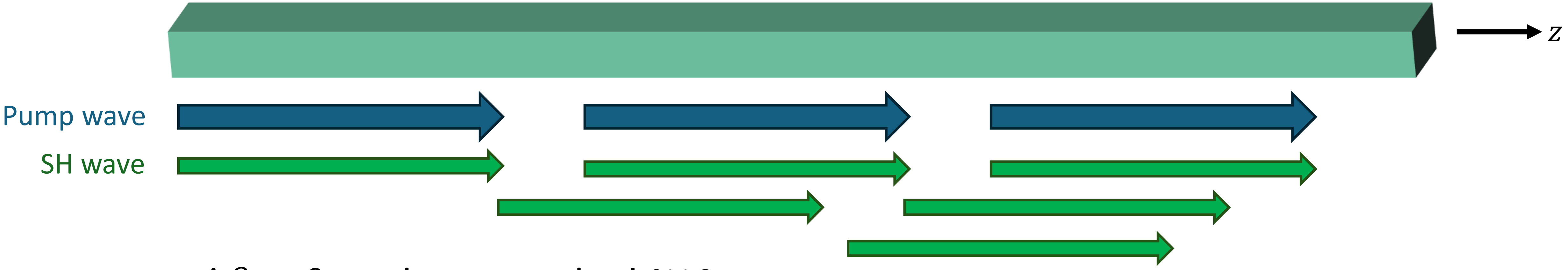
Second harmonic generation

Decreased overlap

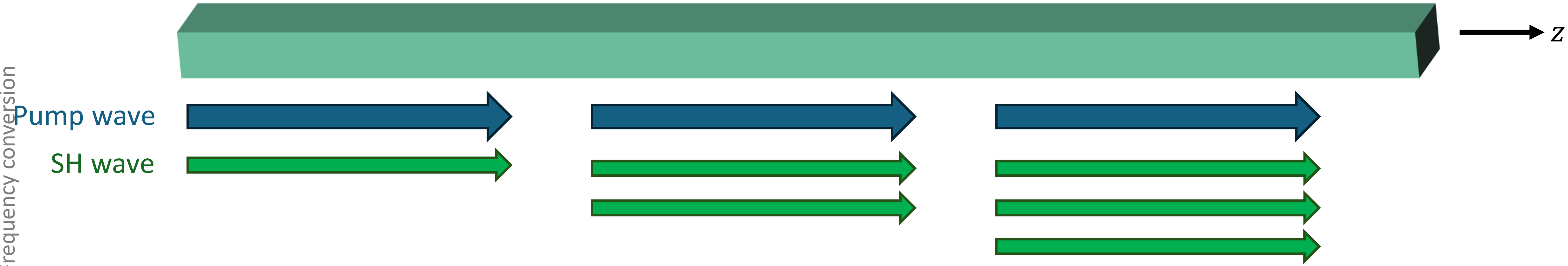


Second harmonic generation

- Two cases:
  - $\Delta\beta \neq 0 \rightarrow$ phase-mismatched SHG

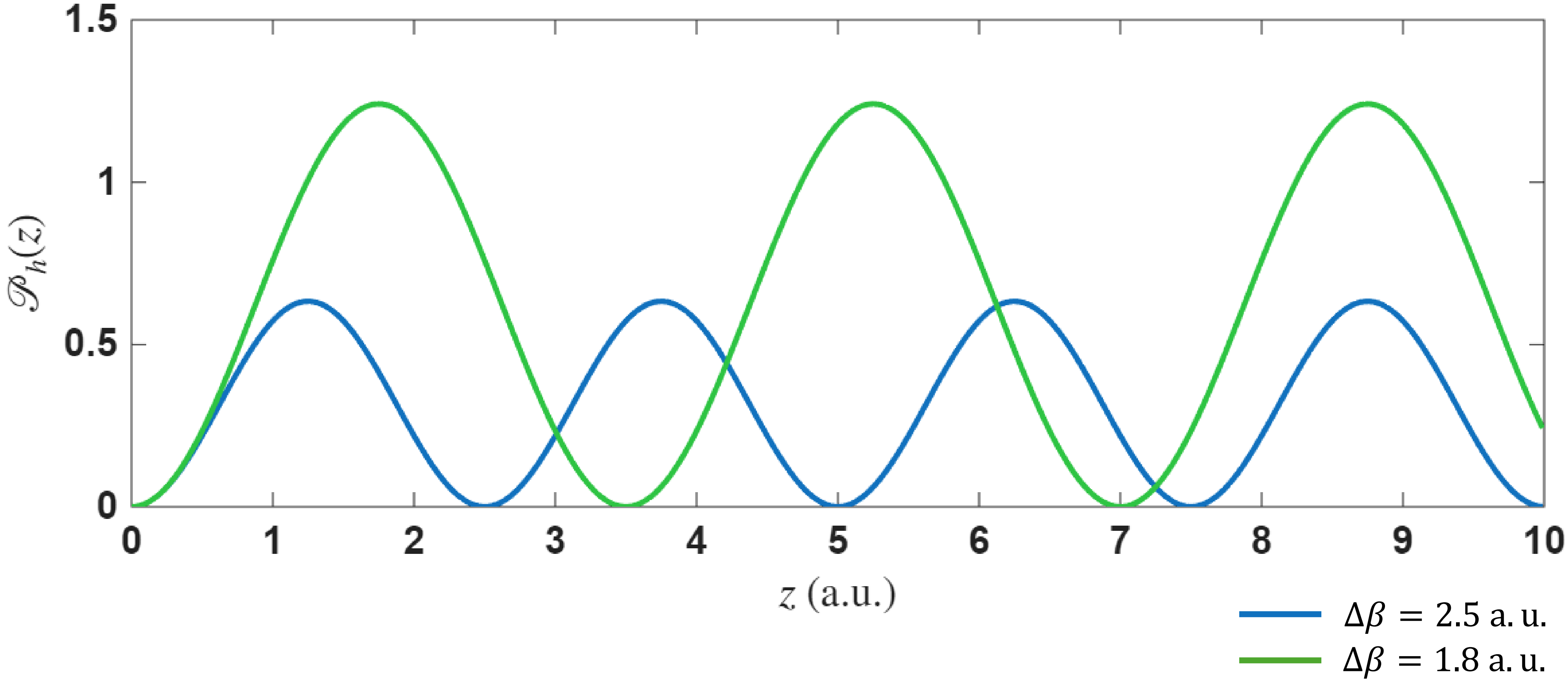


  - $\Delta\beta = 0 \rightarrow$ phase-matched SHG



Second harmonic generation

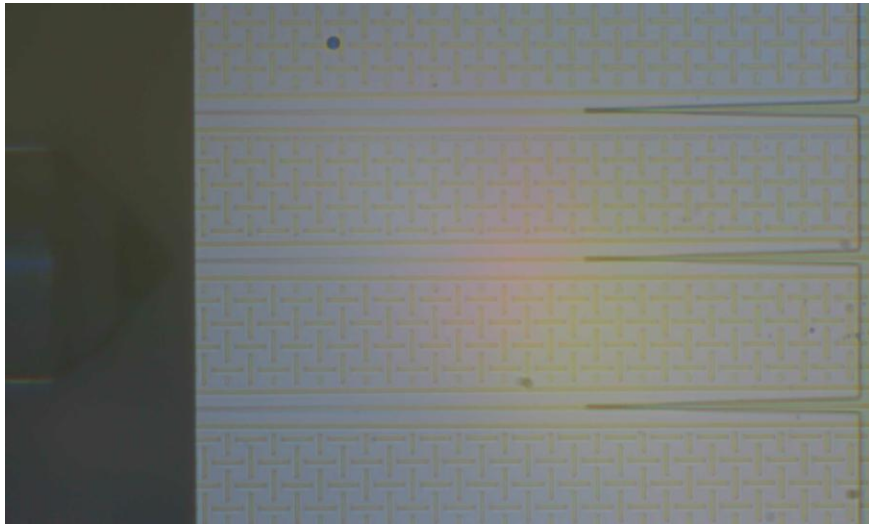
- $\Delta\beta \neq 0 \rightarrow$ the SH and pump waves are not in phase during propagation (not the same velocity), so the SH cannot grow, $\mathcal{P}_h(z) = |\gamma_{sh}|^2 \mathcal{P}_p^2 \sin^2\left(\frac{\Delta\beta z}{2}\right) \frac{4}{\Delta\beta^2}$



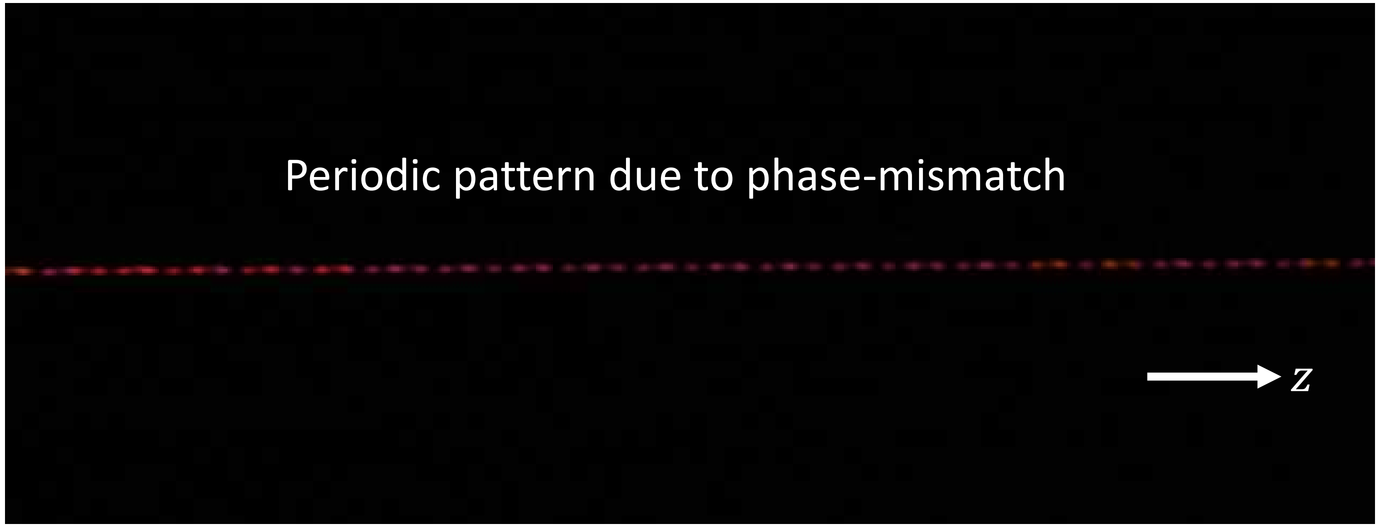
Second harmonic generation

- $\Delta\beta \neq 0 \rightarrow$ the SH and pump waves are not in phase during propagation (not the same velocity), so the SH cannot grow

- Example: lithium niobate waveguide pumped at 1550 nm



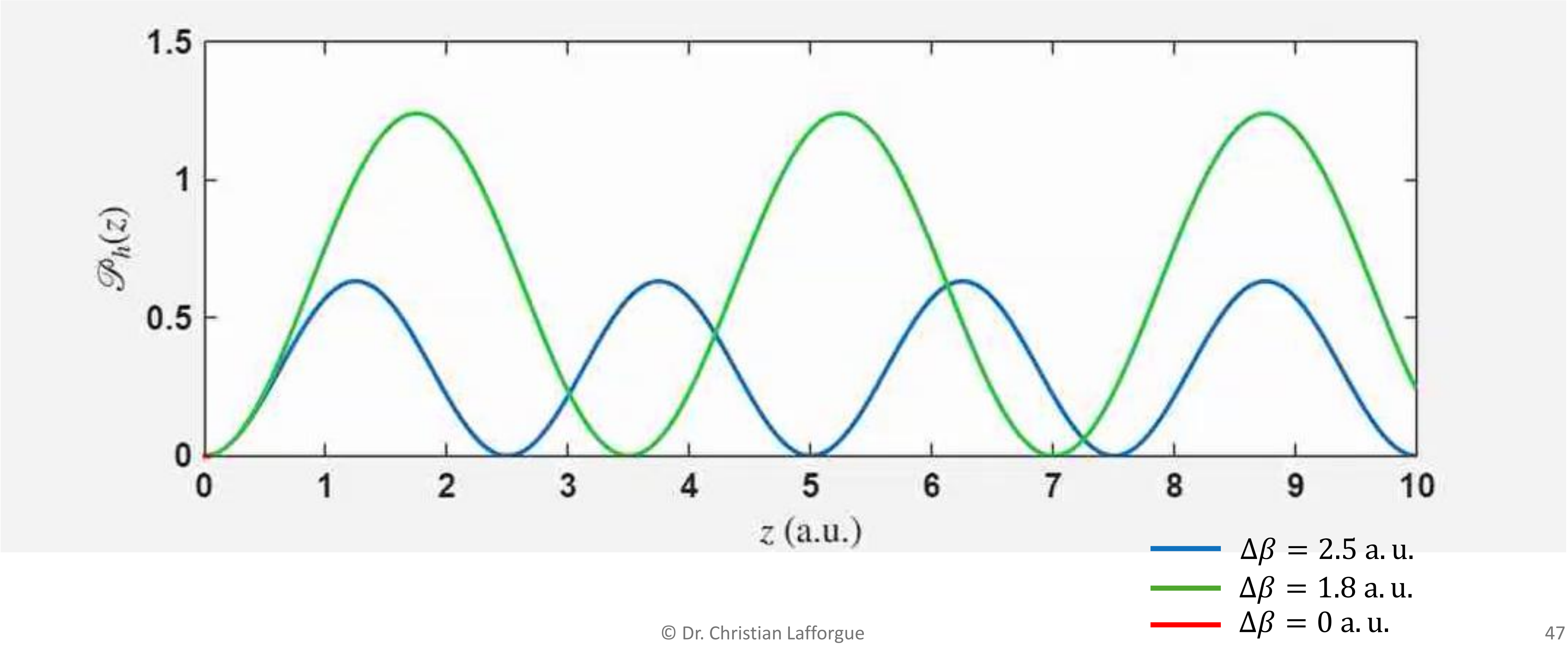
Top view (illuminated)



Top view (scattering)

Second harmonic generation

- $\Delta\beta = 0 \rightarrow$ the SH and pump waves are in phase during propagation, so the SH can grow, $\mathcal{P}_h(z) = |\gamma_{sh}|^2 \mathcal{P}_p^2 z^2$



Second harmonic generation

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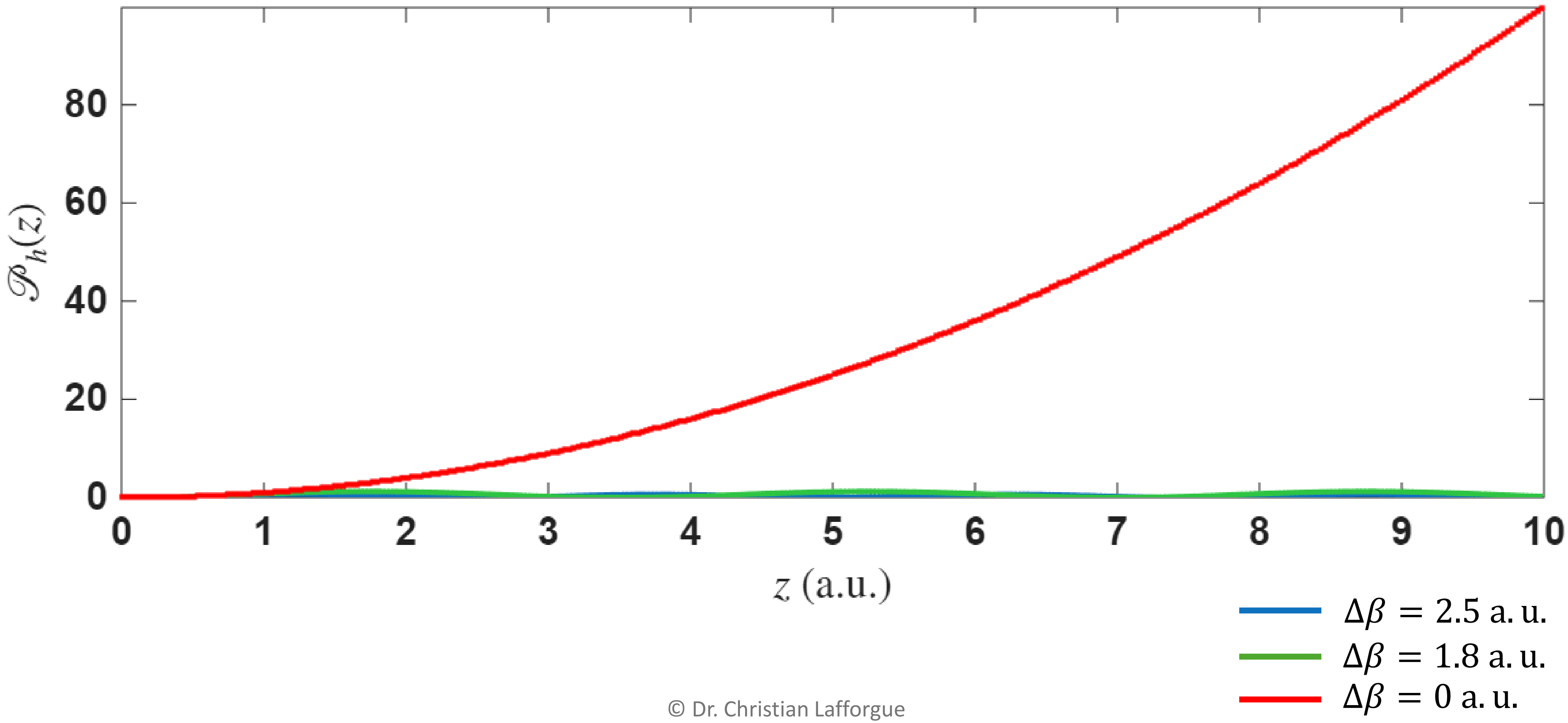


Second harmonic generation

- Hypothesis: the conversion process is weak enough to neglect changes in pump amplitude: $A_p(z) = A_p(0) = \sqrt{\mathcal{P}_p}$

$$\mathcal{P}_h(z) = |\gamma_{sh}|^2 \mathcal{P}_p^2 z^2$$

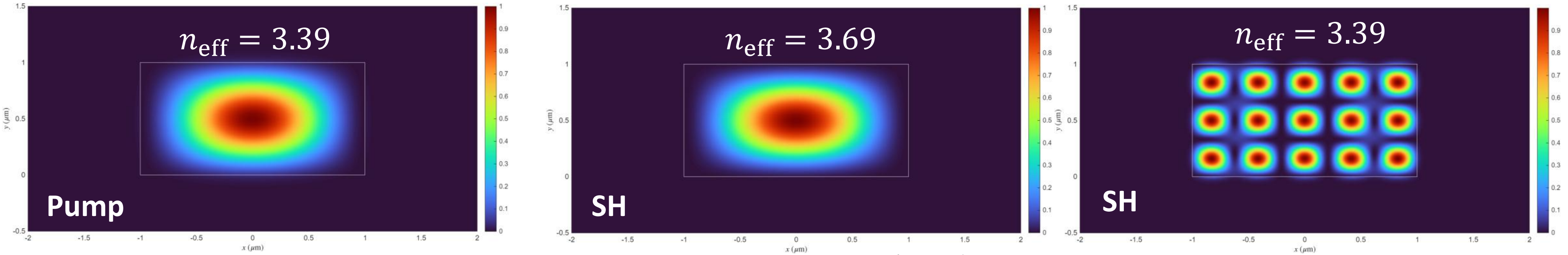
Conversion efficiency:

$$\eta(z) = \frac{\mathcal{P}_h(z)}{\mathcal{P}_p^2} = |\gamma_{sh}|^2 z^2$$

Unit: W^{-1} or $\% \cdot \mathrm{W}^{-1}$ (you can also find $\% \cdot \mathrm{W}^{-1} \cdot \mathrm{cm}^{-2}$)

Second harmonic generation

- How to get $\Delta\beta = 0$?
- $\Delta\beta = \frac{4\pi}{\lambda_p}\left(n_{\text{eff}}^{(s)} - n_{\text{eff}}^{(p)}\right)$ → we need to match the effective indices
  - We need to match effective indices of pump (λ_p) and SH ($\lambda_p/2$)
  - Due to dispersion, this is usually not possible for fundamental modes
  - We can use higher order modes: **modal phase-matching**
  - It decreases efficiency (less modal overlap)



$Overlap \sim 1 \cdot 10^6\ \text{m}^{-1}$
$\Delta\beta \sim 2.4 \cdot 10^6\ \text{m}^{-1}$

$Overlap \sim 2.5 \cdot 10^3\ \text{m}^{-1}$
$\Delta\beta \sim 0\ \text{m}^{-1}$

Second harmonic generation

- How to get $\Delta\beta = 0$?
- $\Delta\beta = \frac{4\pi}{\lambda_p}\left(n_{\text{eff}}^{(s)} - n_{\text{eff}}^{(p)}\right)$ → we need to match the effective indices
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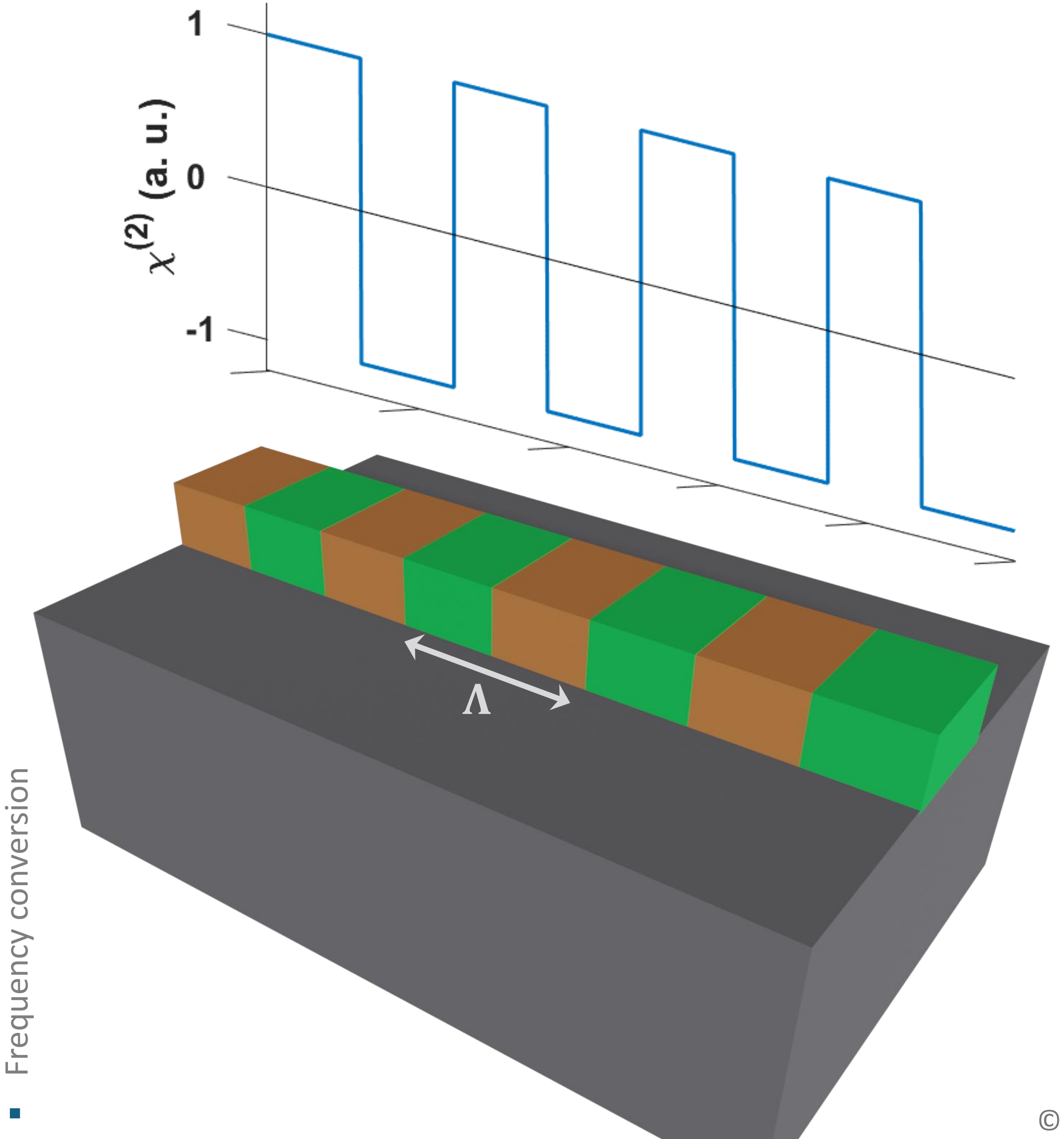
- We can introduce a periodicity (Λ) along the waveguide to compensate the phase-mismatch
  - $\Delta\beta = 2\pi/\Lambda$: Quasi-phase-matching

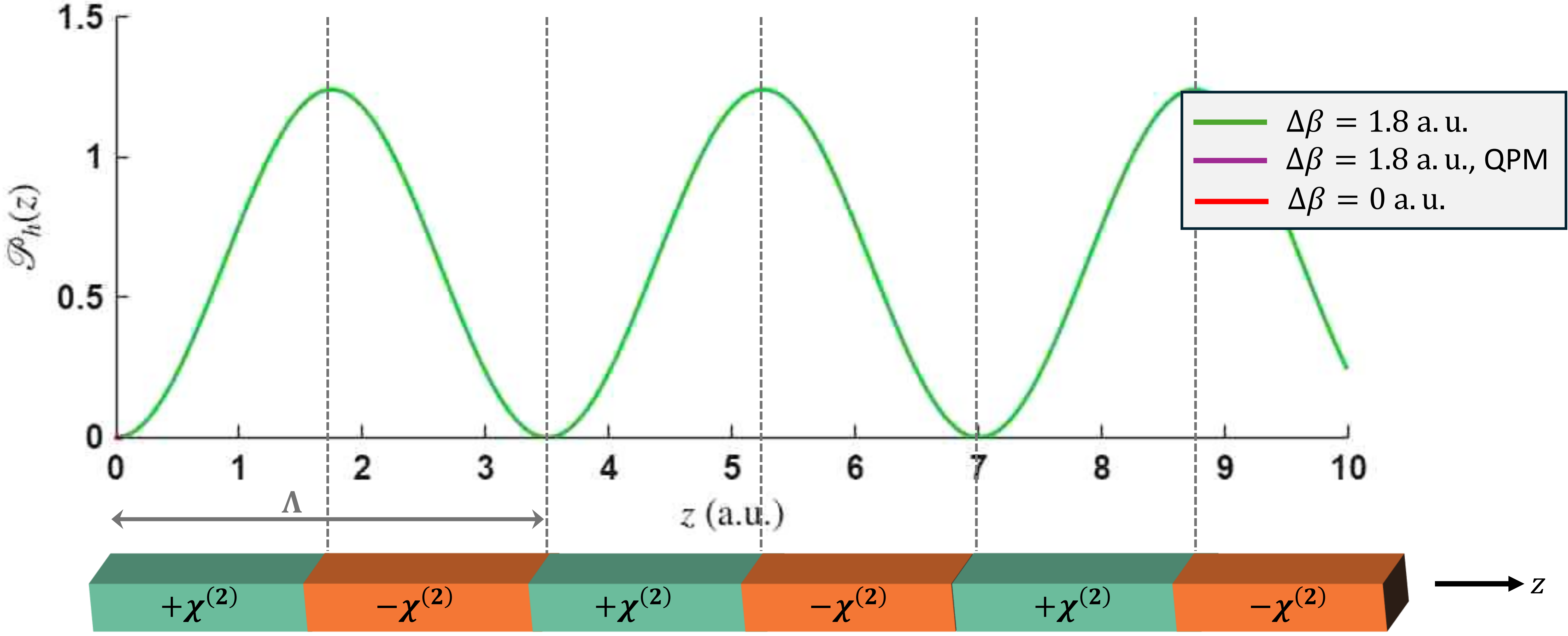
Second harmonic generation



- We can introduce a periodicity (Λ) along the waveguide to compensate the phase-mismatch
  - $\Delta\beta = 2\pi/\Lambda$: Quasi-phase-matching
  - $\chi^{(2)}$ is reversed every-time the SH interference becomes destructive
  - $\mathcal{P}_h(z) \simeq \frac{4}{\pi^2}|\gamma_{sh}|^2\mathcal{P}_p^2 z^2$

Second harmonic generation

- $\Delta\beta = 2\pi/\Lambda \rightarrow$ the SH and pump waves are in phase during propagation, so the SH can grow, $\mathcal{P}_h(z) \simeq (4/\pi^2)|\gamma_{sh}|^2\mathcal{P}_p^2 z^2$



Second harmonic generation

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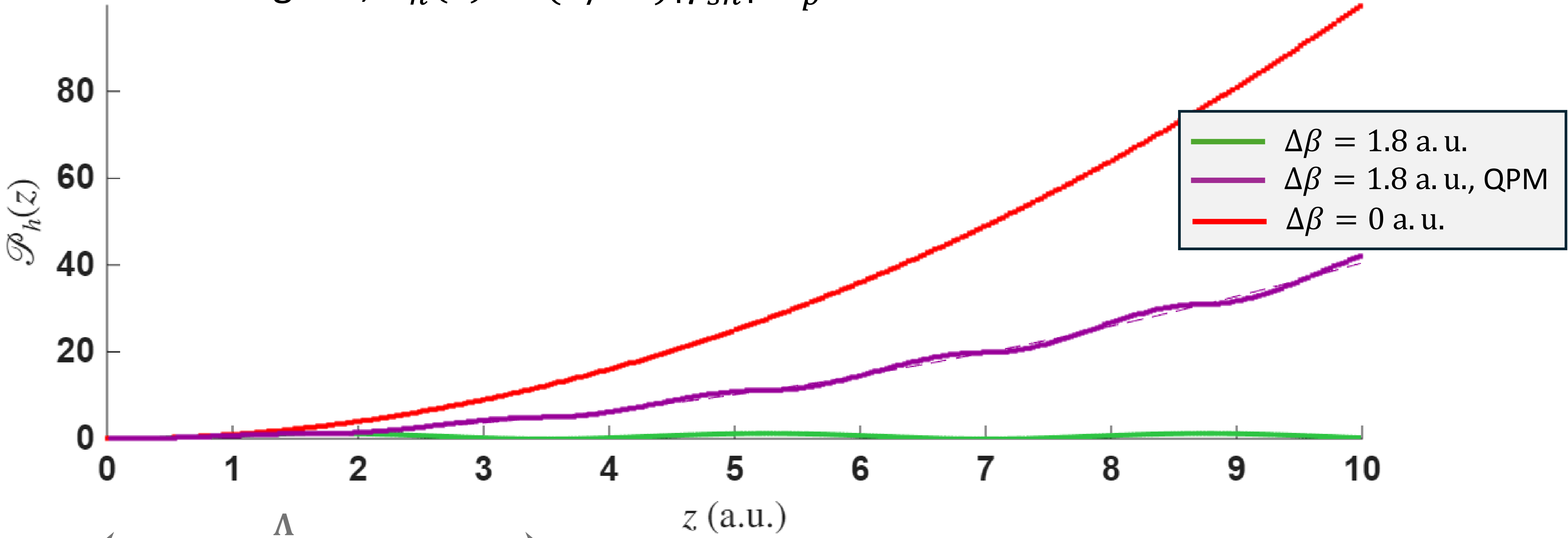
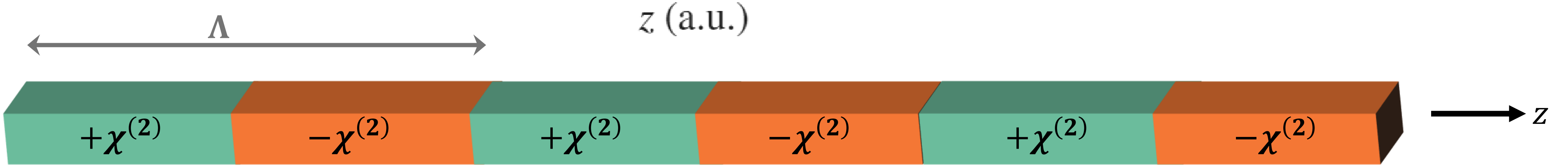
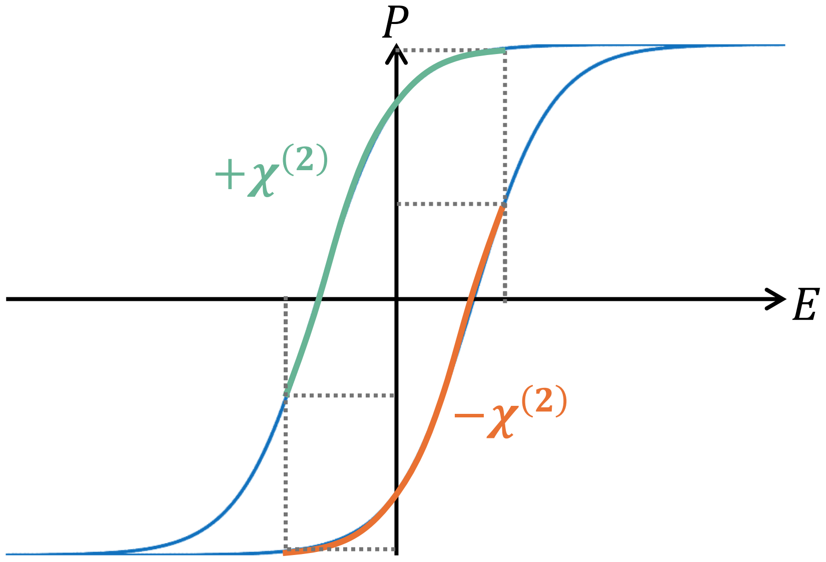




Frequency conversion

Second harmonic generation

- How to reverse $\chi^{(2)}$ on a given fabricated waveguide?
  - Ferroelectric materials!



$$P \simeq \varepsilon_0\left(\chi^{(1)}E + \chi^{(2)}E^2\right)$$

Opposite curvature → opposite $\chi^{(2)}$

Second harmonic generation

- How to reverse $\chi^{(2)}$ on a given fabricated waveguide?
  - Ferroelectric materials!

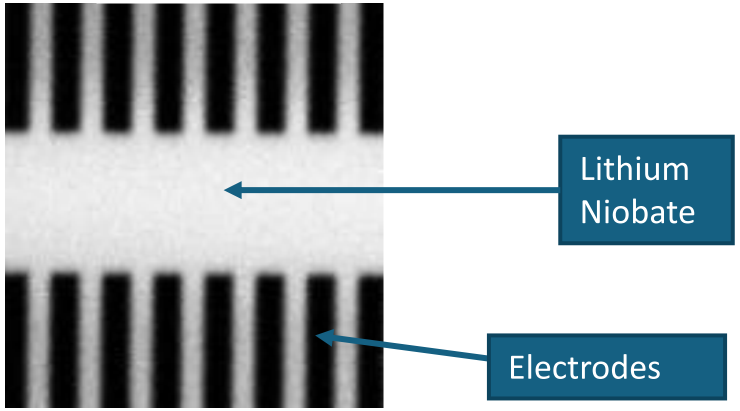
- Example: periodically poled lithium niobate (PPLN) waveguides
  - Electrodes are deposited periodically on the sides of the waveguide
  - High voltage pulses are applied to reverse ferroelectric domains

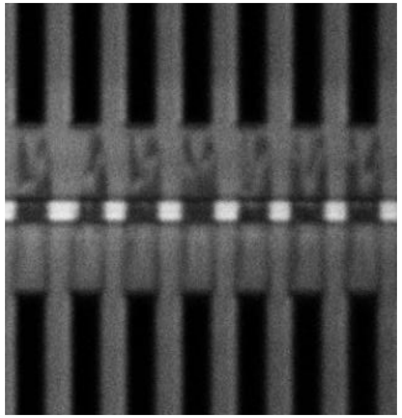
Second harmonic generation

- Example: periodically poled lithium niobate (PPLN) waveguides

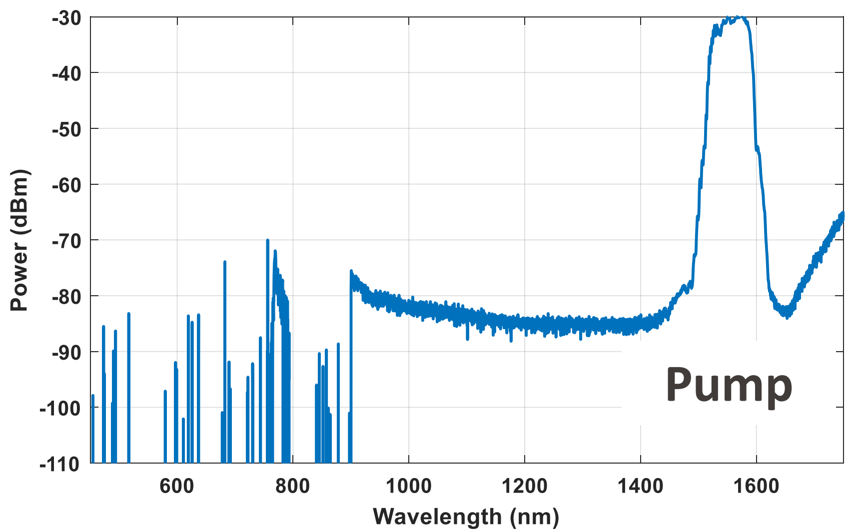
Original | **Poled**

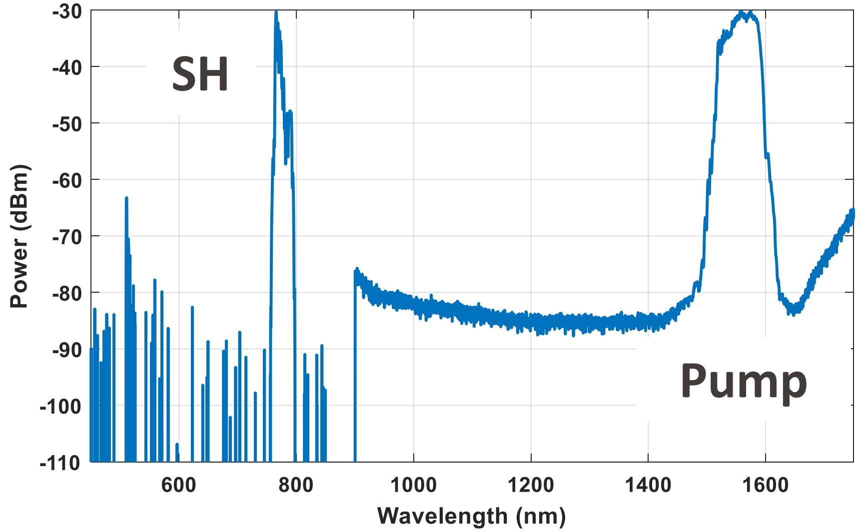
$\chi^{(2)}$ **mapping**





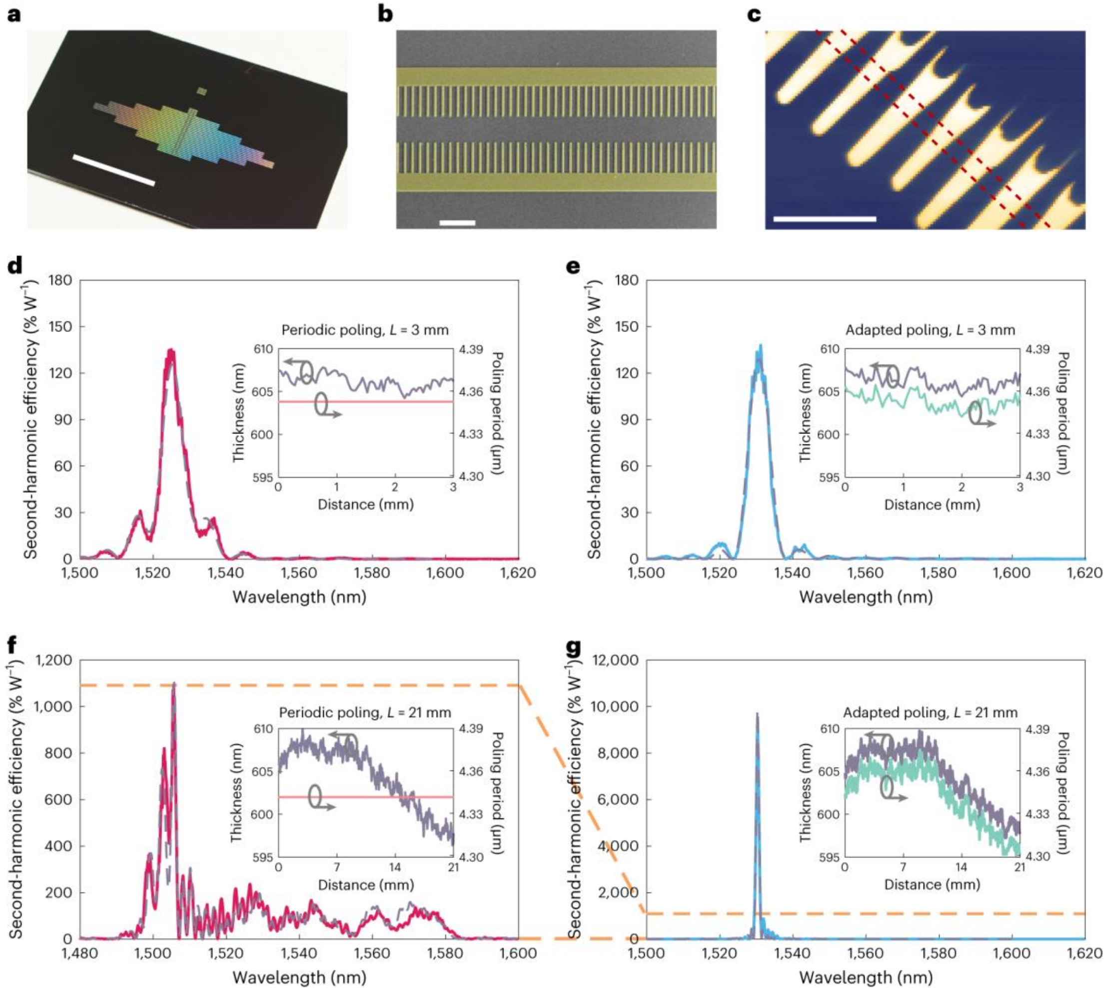
Optical spectrum





Second harmonic generation

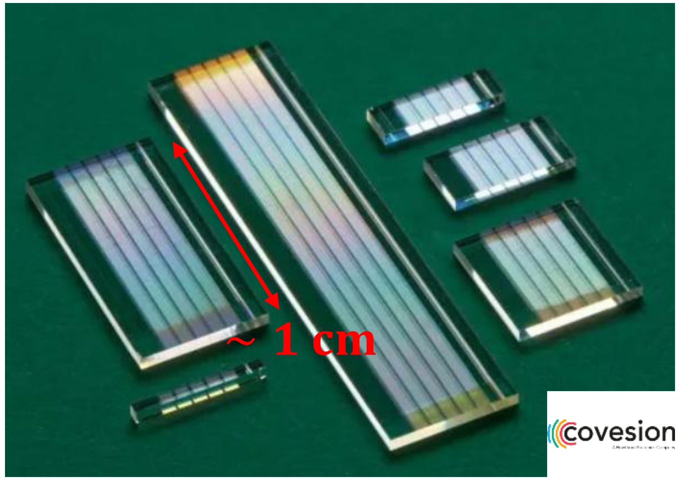
- Example: periodically poled lithium niobate (PPLN) waveguides



Chen, PK., *et al.* Adapted poling to break the nonlinear efficiency limit in nanophotonic lithium niobate waveguides. *Nat. Nanotechnol.* **19**, 44–50 (2024). https://doi.org/10.1038/s41565-023-01525-w

Conversion efficiency: $\sim 1 \cdot 10^4 \ \% \cdot \mathrm{W}^{-1}$

Commercial bulk PPLN



Conversion efficiency: $\sim 1 \ \% \cdot \mathrm{W}^{-1}$

Second harmonic generation

- How to boost efficiency?

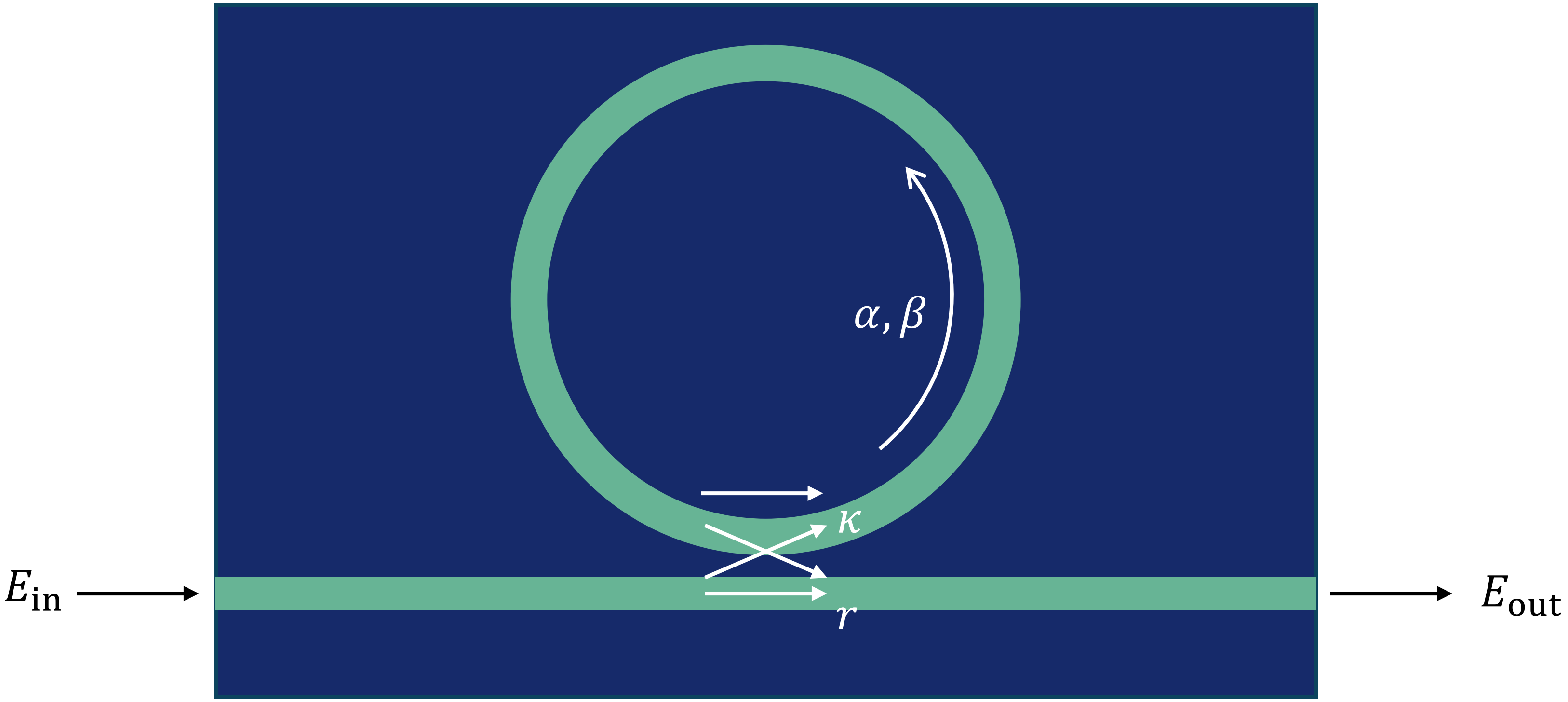
$$\mathcal{P}_{sh}(z) \propto |\gamma_{sh}|^2 \mathcal{P}_p^2 z^2$$

  - Increase modal overlap: limited design parameter space
  - Increase length: limited footprint
  - Increase power: limited laser sources

  - Length and power can be increased using resonators!

Second harmonic generation

- Ring resonators:
  - Light circulates in loops: longer interaction distance + higher intensity (accumulation of optical energy)

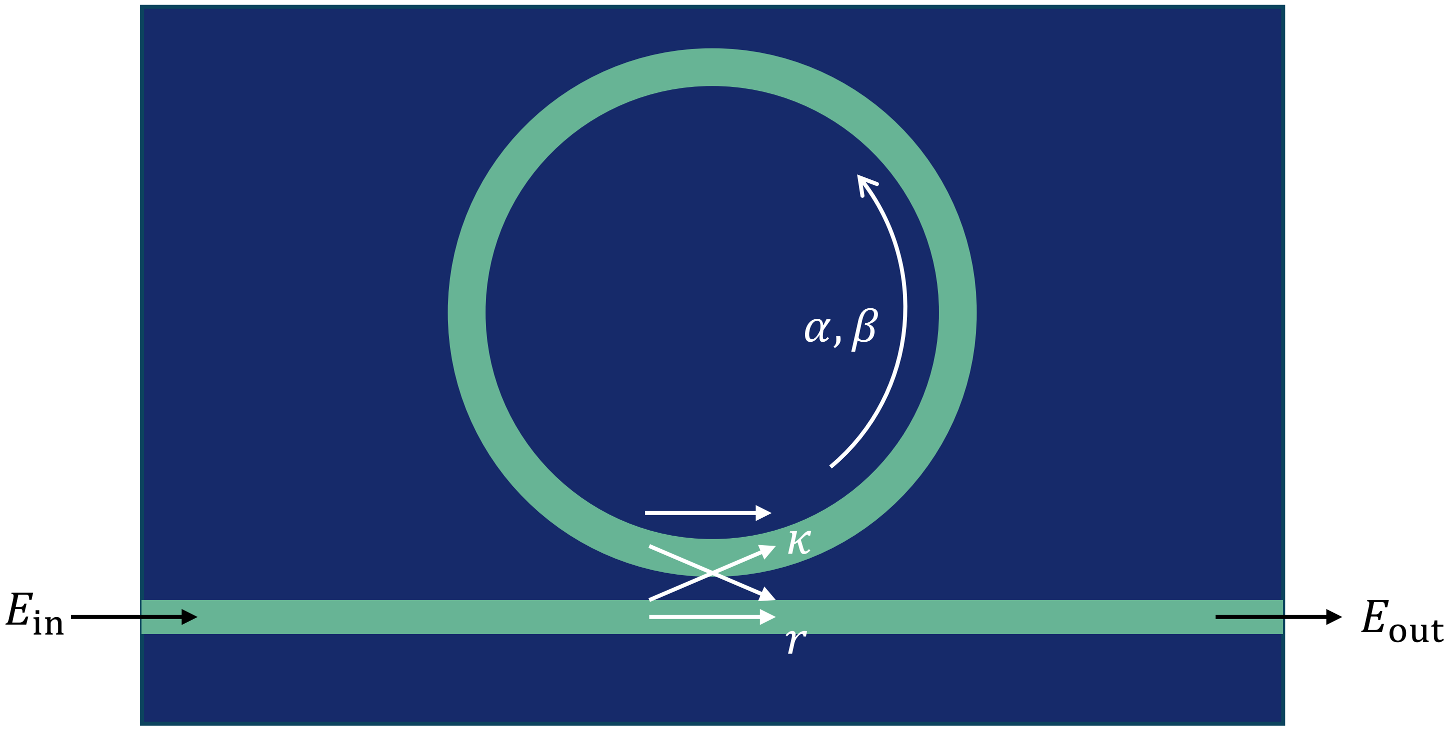


Second harmonic generation

$$\frac{E_{\text{out}}}{E_{\text{in}}} = \frac{r - ae^{j\beta L}}{1 - rae^{j\beta L}}$$

$$a = e^{-\frac{\alpha L}{2}} \ ; \ L = 2\pi R \ ; \ |r|^2 + |\kappa|^2 = 1$$

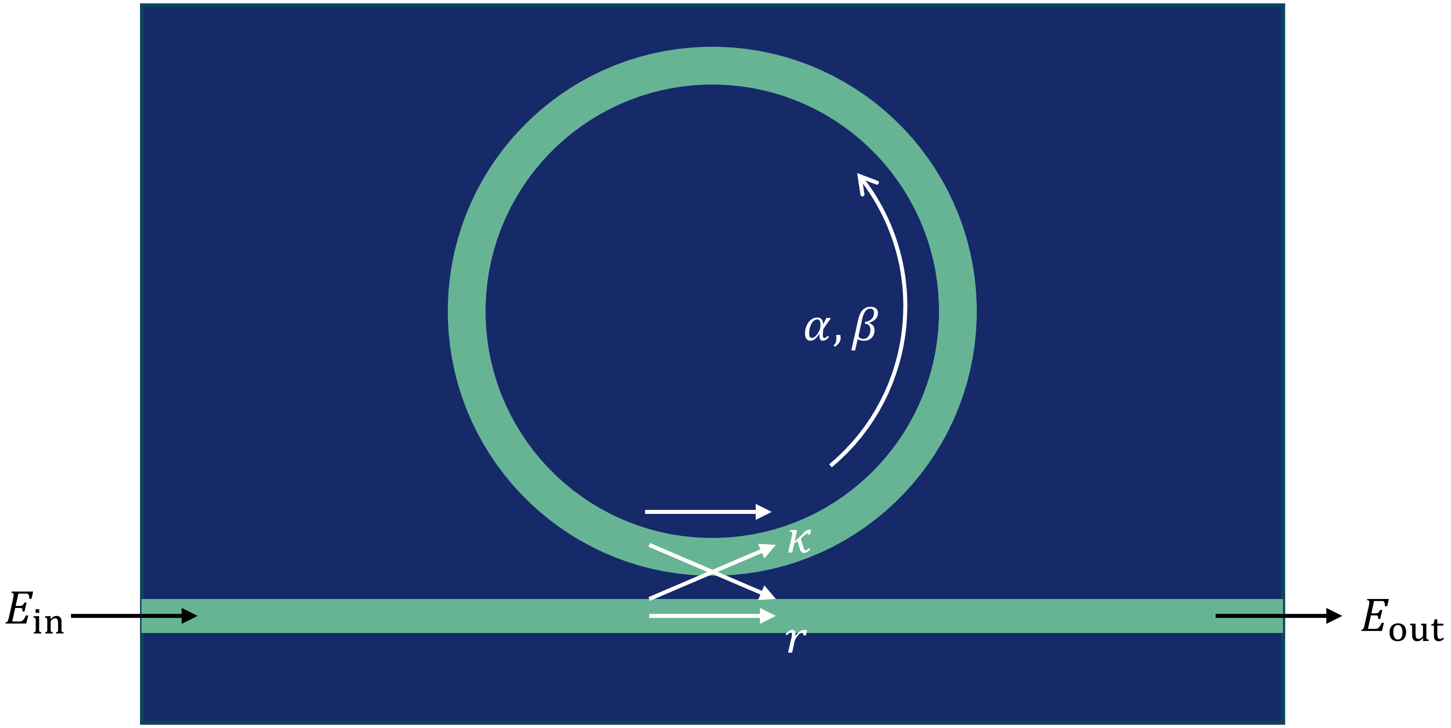
$$\Rightarrow \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{r^2 - 2ra\cos(\beta L) + a^2}{1 - 2ra\cos(\beta L) + (ra)^2}$$



Second harmonic generation

Critical coupling : $r = a$

$$\Rightarrow \frac{P_{\text{out}}}{P_{\text{in}}} = 2a^2 \frac{1 - \cos(\beta L)}{1 - 2a^2\cos(\beta L) + a^4}$$



Second harmonic generation

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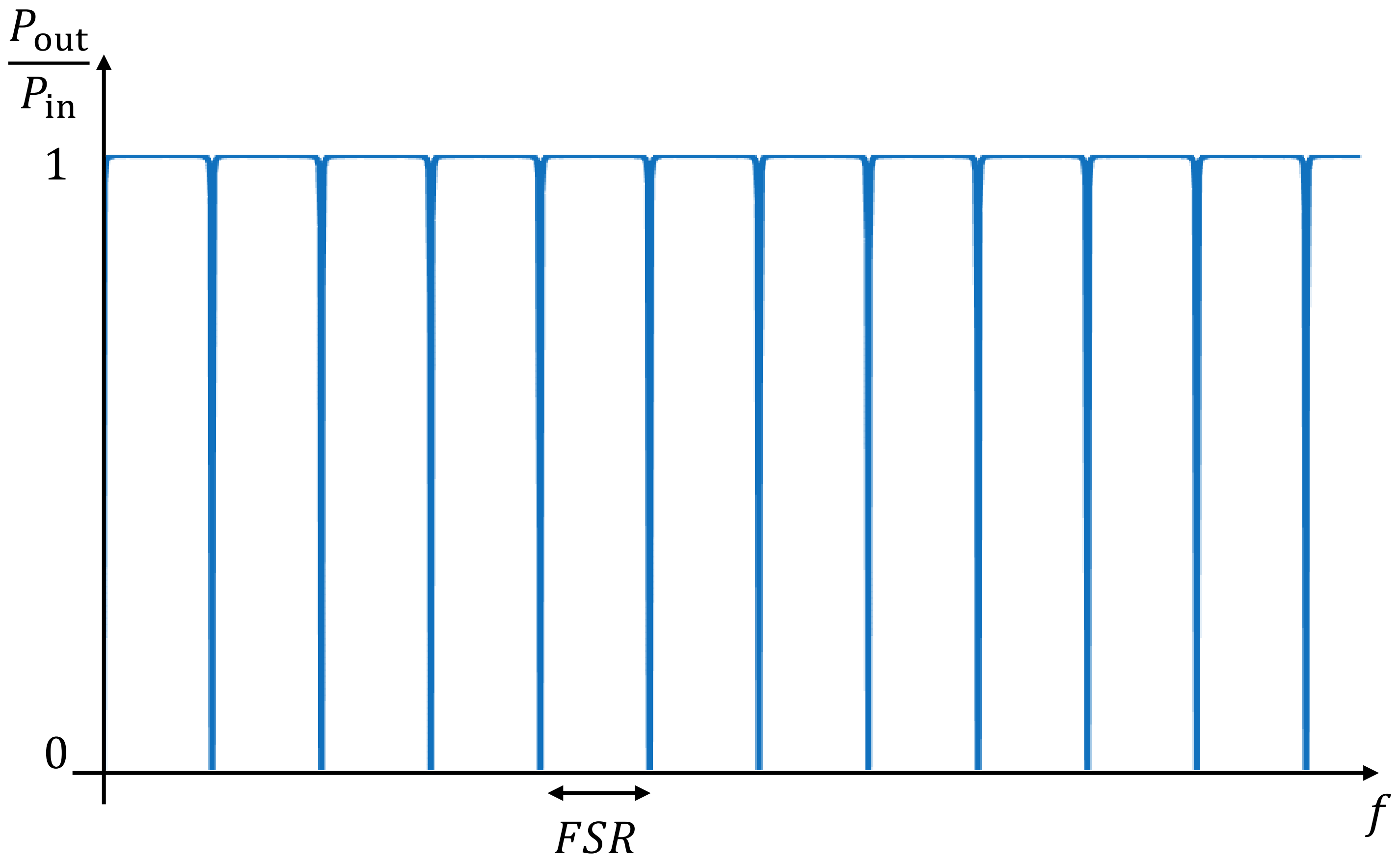


$$FSR = \frac{1}{\beta_1 L} = \frac{c}{n_g L}$$

Second harmonic generation

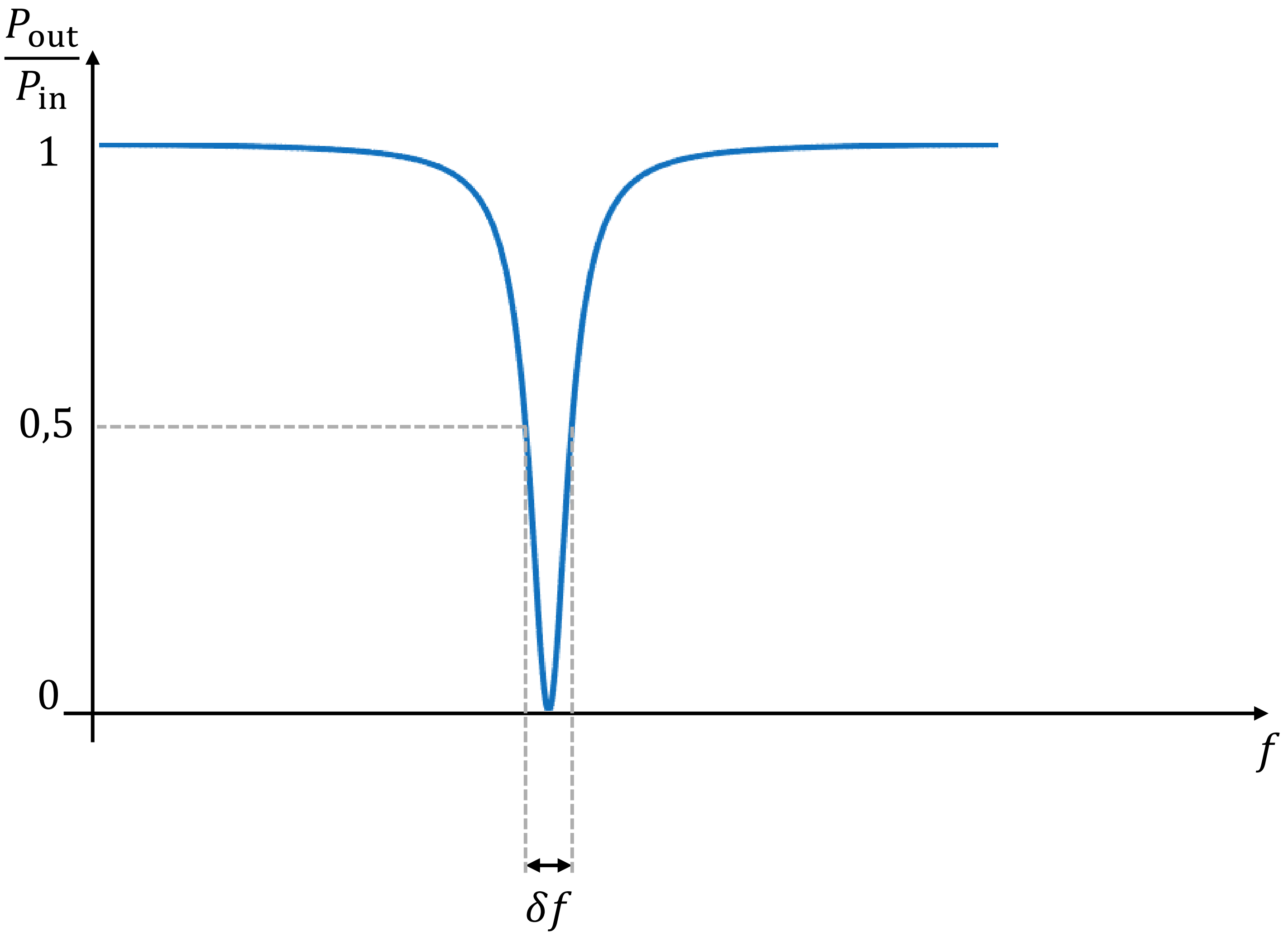
Critical coupling : $r = a$

$$\Rightarrow \frac{P_{\text{out}}}{P_{\text{in}}} = 2a^2 \frac{1 - \cos(\beta L)}{1 - 2a^2 \cos(\beta L) + a^4}$$

Quality factor: $Q = \frac{f_0}{\delta f}$

Depends on loss: lower loss, higher Q

Finnesse: $\mathcal{F} = \frac{FSR}{\delta f}$

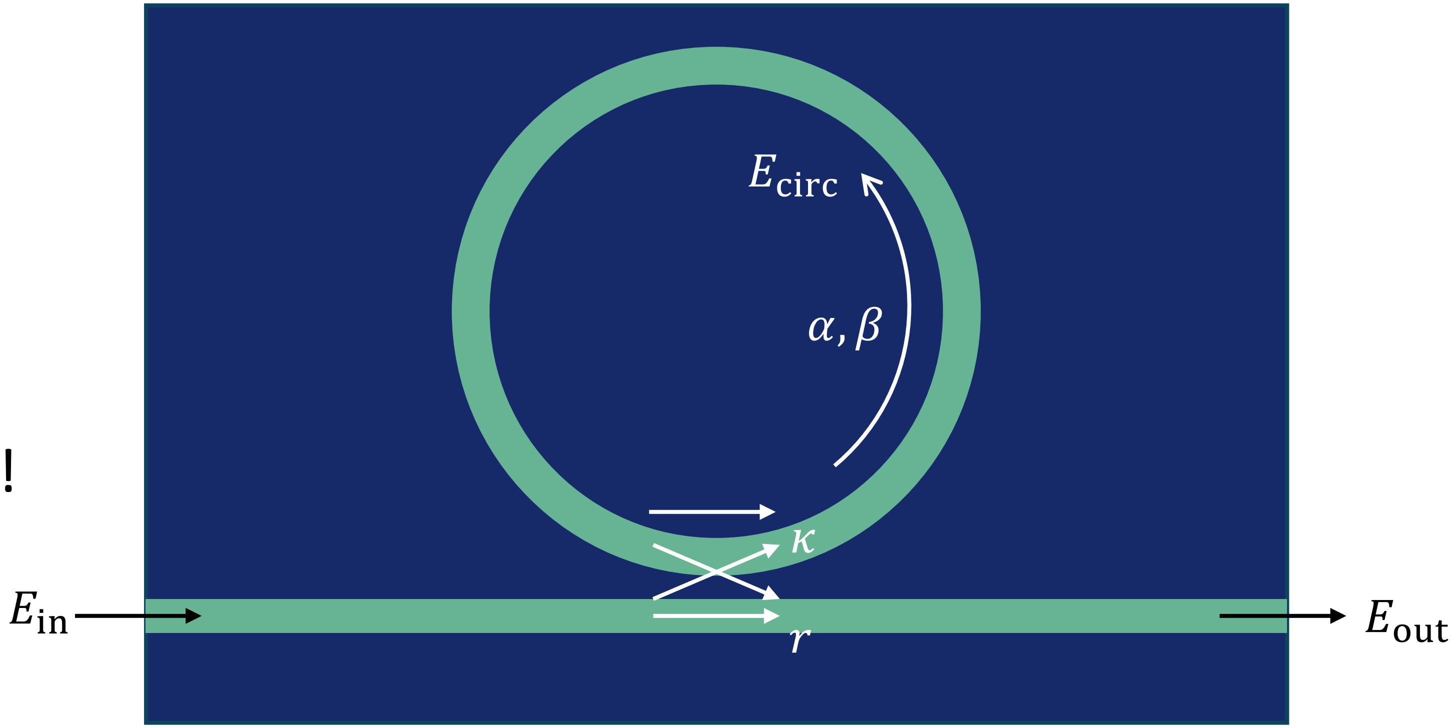


Second harmonic generation

What about the field inside the resonator?

$$\frac{P_{\text{circ}}}{P_{\text{in}}} = \frac{\mathcal{F}}{\pi}$$

→ The power inside the ring is enhanced!



Second harmonic generation

- Ring resonators:
  - Light circulates in loops: longer interaction distance + higher intensity (accumulation of optical energy)

  - Can be **resonant for the pump wave**.
    Conversion efficiency enhancement:

$$\frac{\mathcal{F}_p^2}{\pi^2}$$

Second harmonic generation

- Ring resonators:
  - Light circulates in loops: longer interaction distance + higher intensity (accumulation of optical energy)

  - Can be **resonant for the SH wave**.
    Conversion efficiency enhancement:

$$\frac{\mathcal{F}_{sh}}{\pi}$$

Second harmonic generation

- Ring resonators:
  - Light circulates in loops: longer interaction distance + higher intensity (accumulation of optical energy)

  - Can be **resonant for the pump and SH waves**.
    Conversion efficiency enhancement:

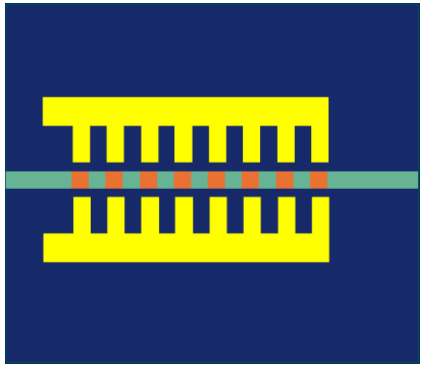
$$\frac{\mathcal{F}_p^2 \mathcal{F}_s}{\pi^3}$$

Second harmonic generation

- Example: PPLN in a racetrack resonator

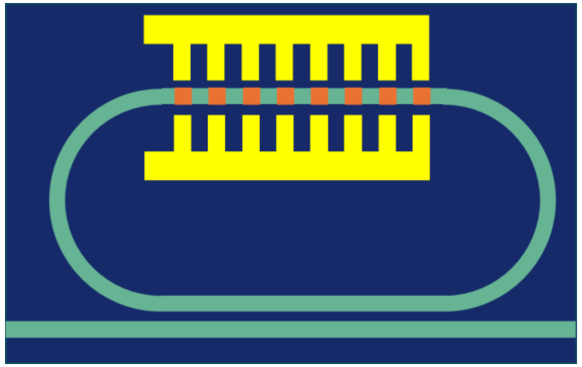
Length $L = 780$ nm

Straight PPLN waveguide



$FSR \sim 50$ GHz
$Q \sim 5 \cdot 10^5$
$F \sim 125$

PPLN waveguide in a resonator (pump resonant)



Second harmonic generation

- Example: PPLN in a racetrack resonator

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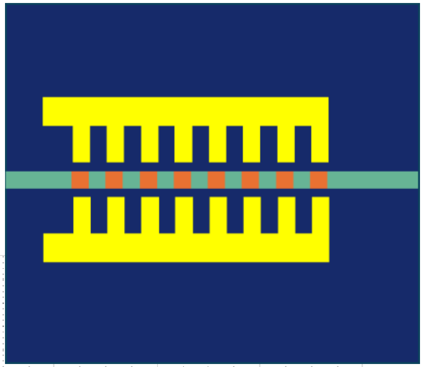

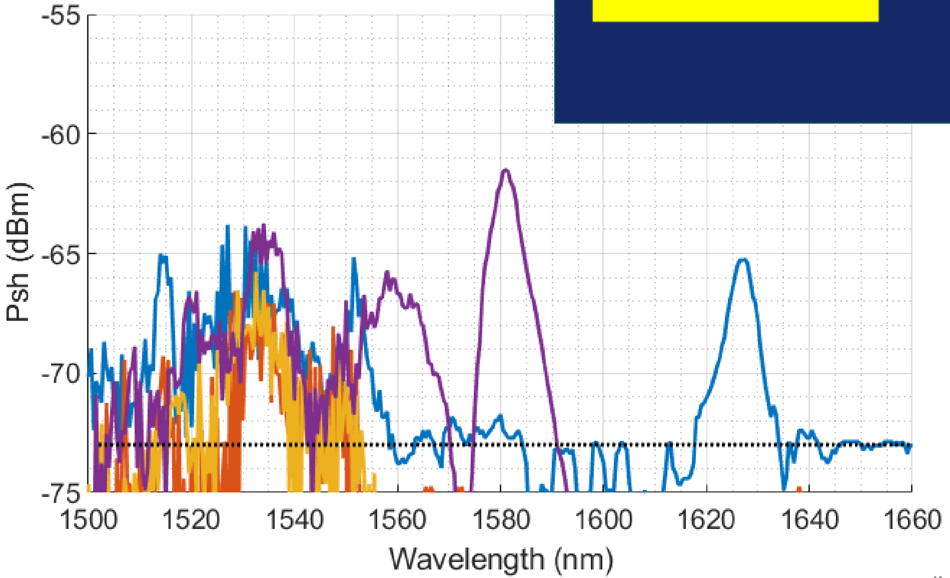

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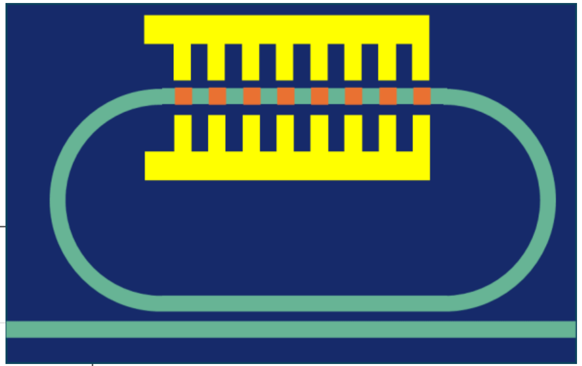
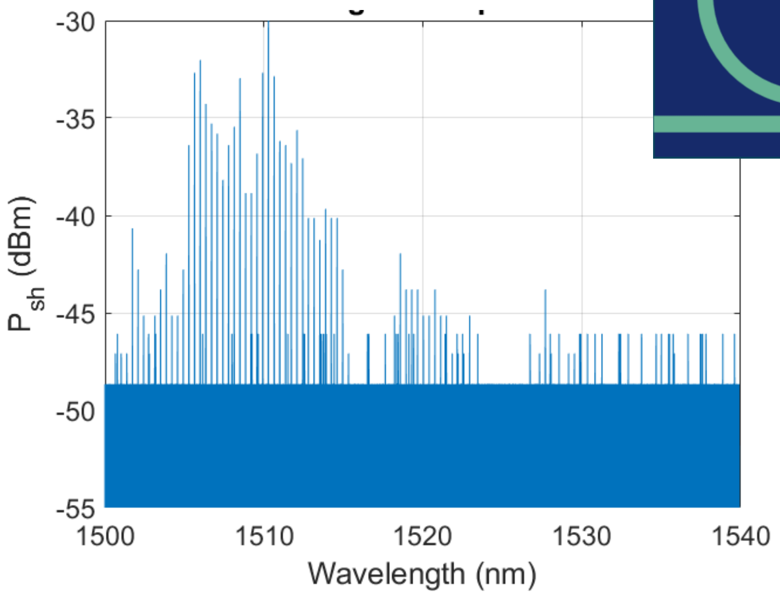
PPLN waveguide in a resonator (pump resonant)

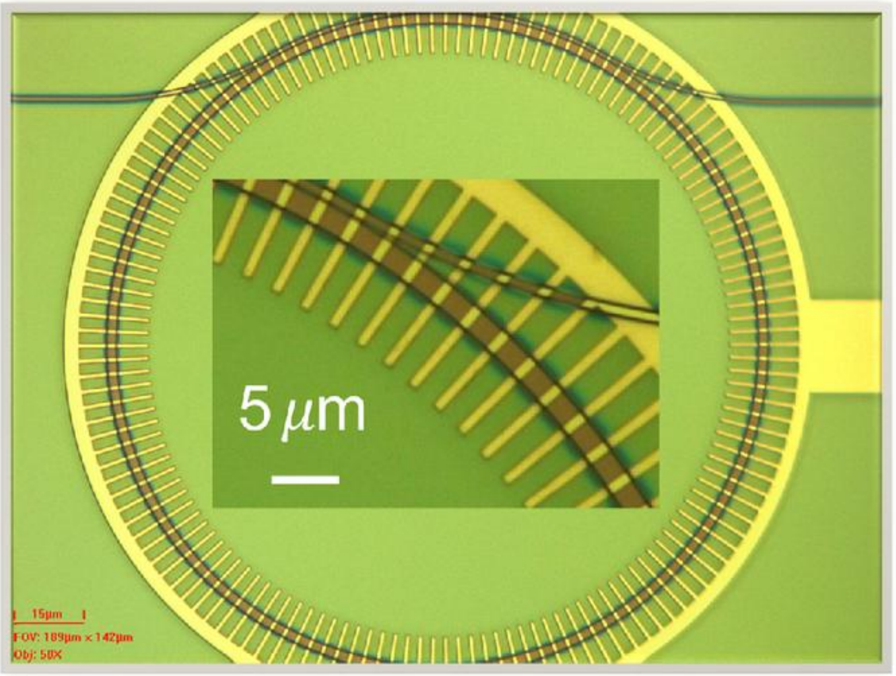




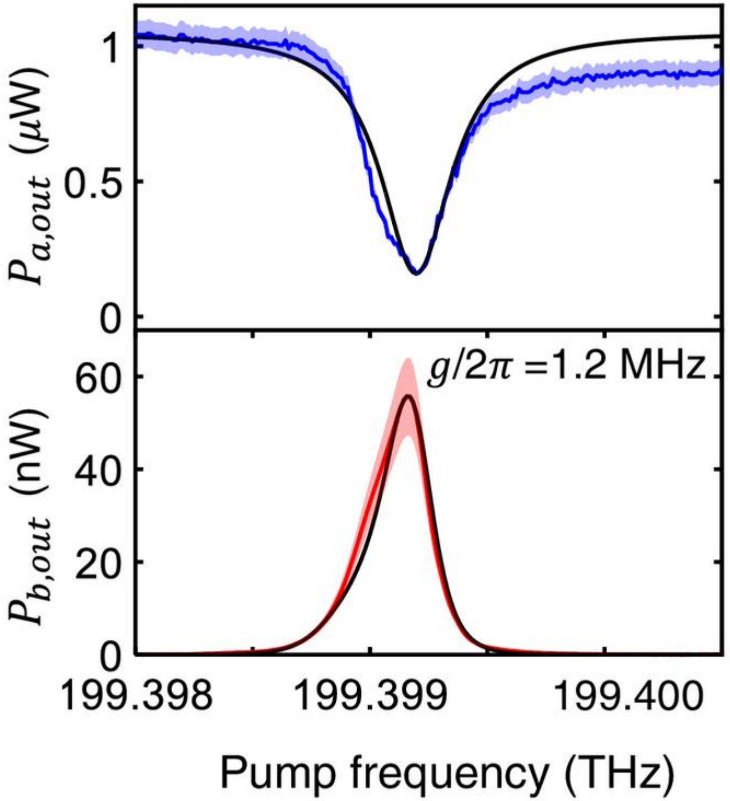
>30 dB enhancement!

Second harmonic generation

- Example: PPLN in a ring resonator



Juanjuan Lu, *et al.*, "Toward 1% single-photon anharmonicity with periodically poled lithium niobate microring resonators," Optica 7, 1654-1659 (2020)



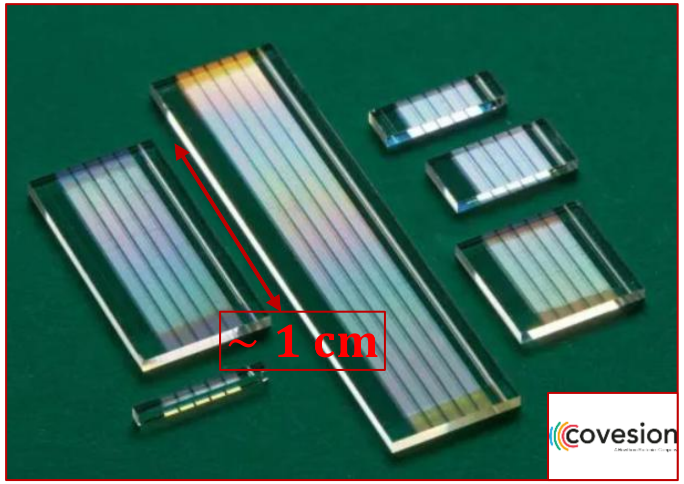
$$Q_p \simeq Q_h \simeq 5 \cdot 10^5$$

Conversion efficiency: $\sim (5 \pm 1) \cdot 10^6 \,\% \cdot \mathrm{W}^{-1}$

Second harmonic generation

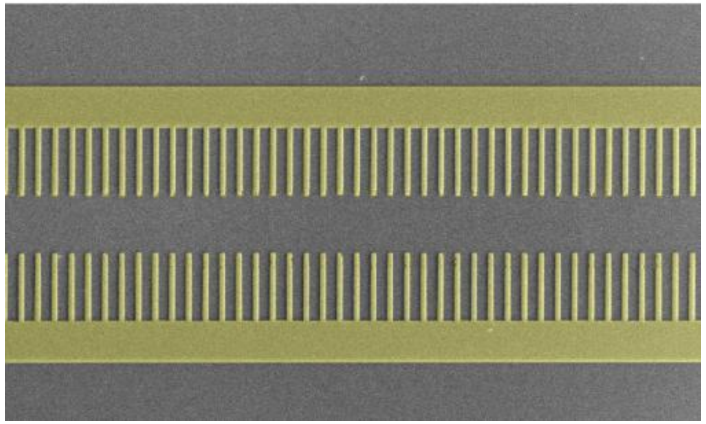
- Example: PPLN in a ring resonator

Commercial bulk PPLN



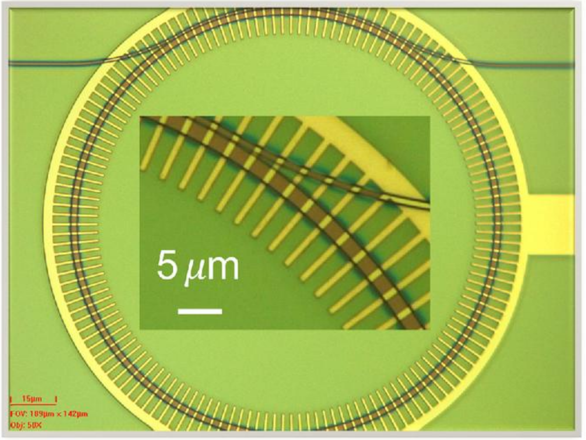
Conversion efficiency: $\sim 1\ \% \cdot \mathrm{W}^{-1}$

Integrated PPLN



$\sim 1 \cdot 10^{4}\ \% \cdot \mathrm{W}^{-1}$

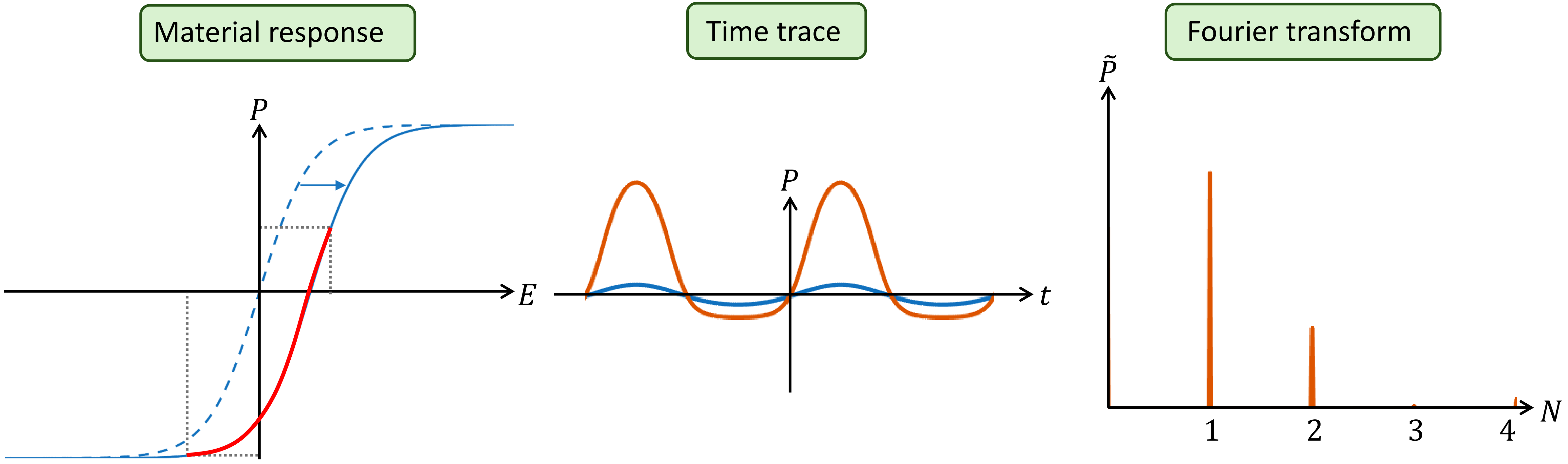
Integrated resonant PPLN



$\sim (5 \pm 1) \cdot 10^{6}\ \% \cdot \mathrm{W}^{-1}$

Second harmonic generation

- Other way: electric field induced SHG



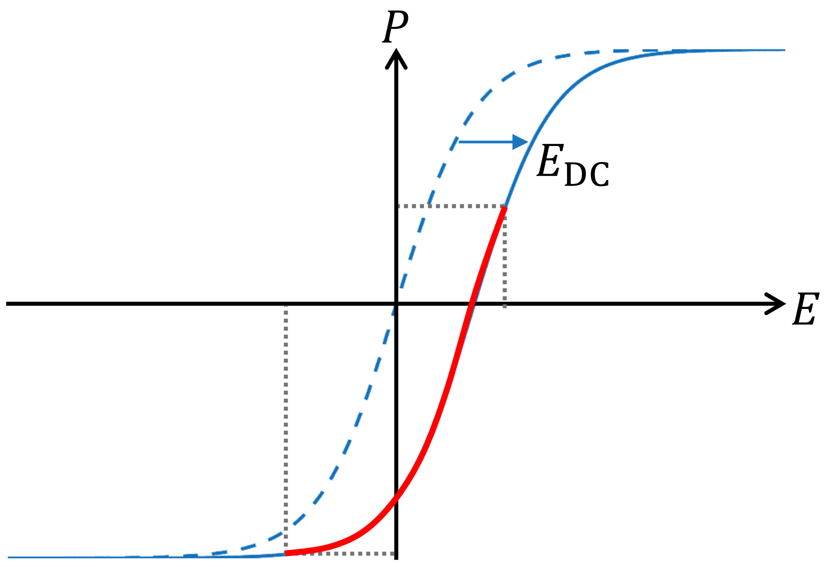
$P \simeq \varepsilon_0\left(\chi^{(1)}E + \chi^{(2)}E^2 + \chi^{(3)}E^3\right)$

Distorted sine wave,
But not symmetric!

Fundamental +2nd (+3rd) harmonic

- Other way: electric field induced SHG
  - $\chi^{(2)}_{\text{eff}} \propto \chi^{(3)} E_{\text{DC}}$

External field



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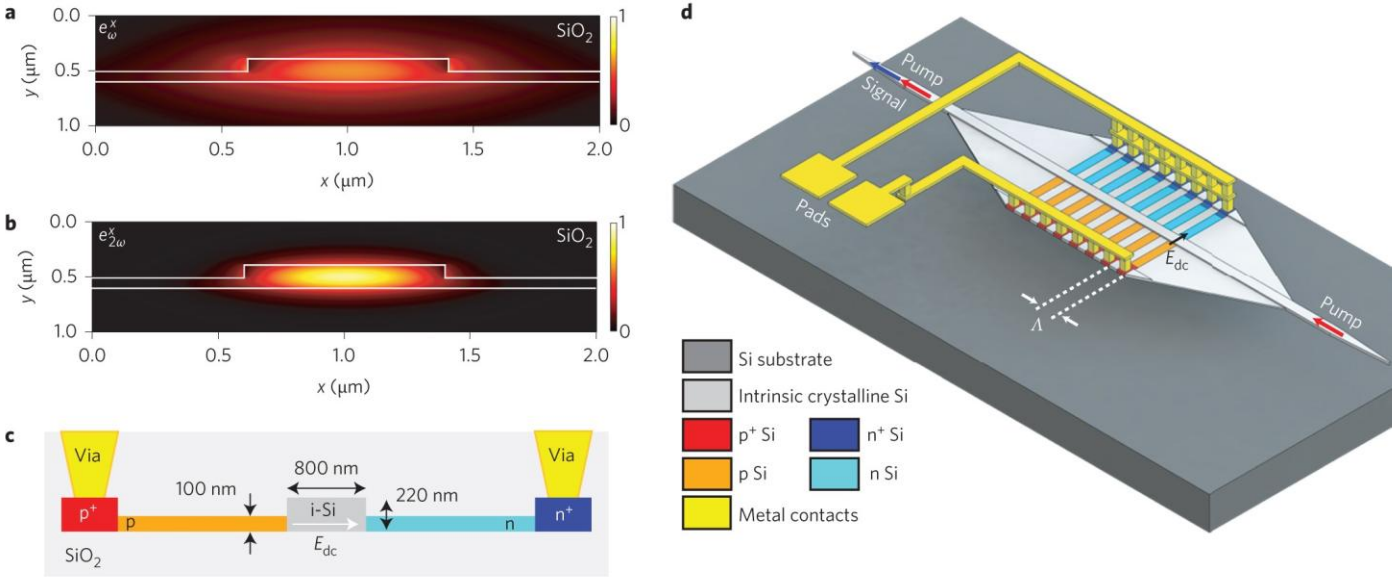

Timurdogan, E., Poulton, C., Byrd, M. *et al.* Electric field-induced second-order nonlinear optical effects in silicon waveguides. *Nature Photon* **11**, 200–206 (2017). https://doi.org/10.1038/nphoton.2017.14

Second harmonic generation

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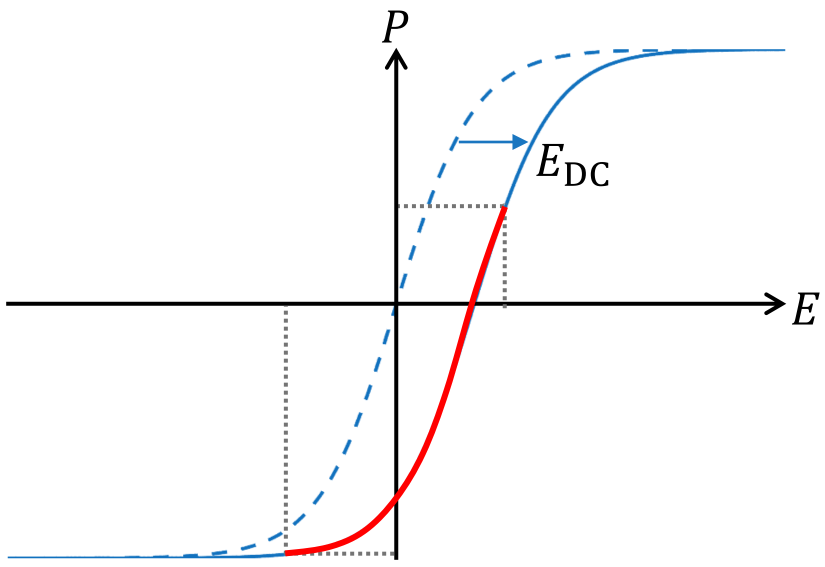

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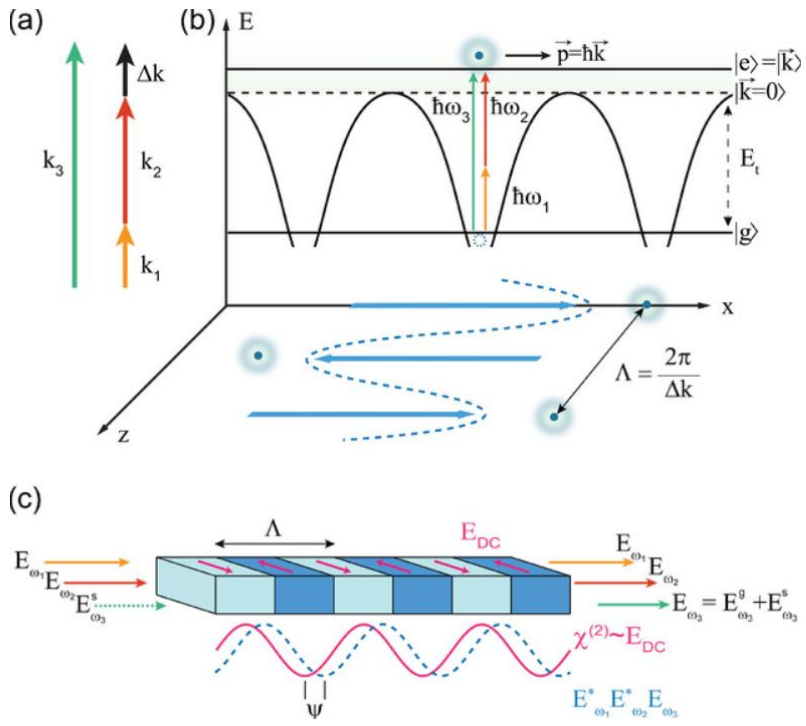
Optically induced field



O. Yakar, E. Nitiss, J. Hu, C.-S. Brès, Generalized Coherent Photogalvanic Effect in Coherently Seeded Waveguides. *Laser Photonics Rev* 2022, 16, 2200294. https://doi.org/10.1002/lpor.202200294

Second harmonic generation

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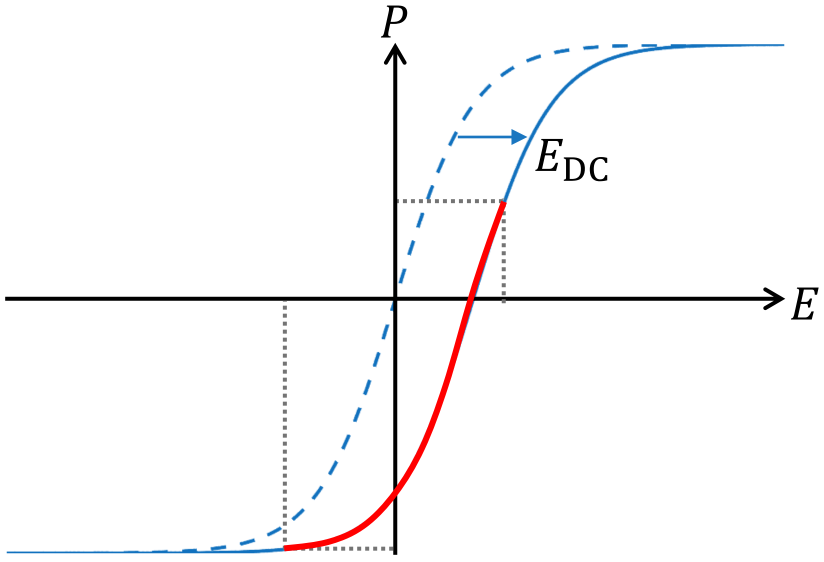

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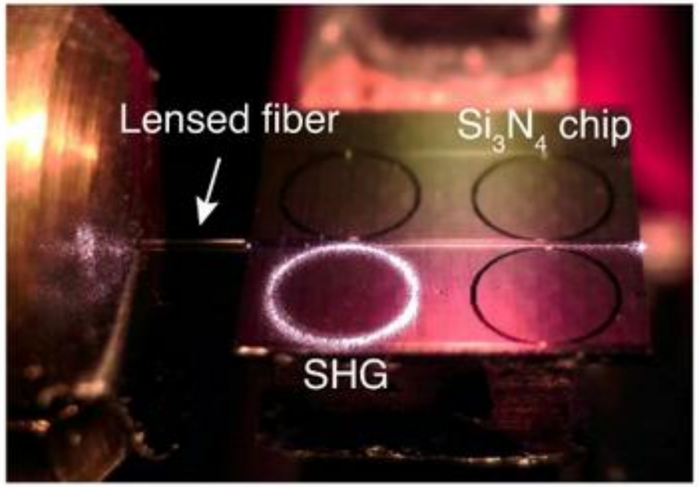
Optically induced field

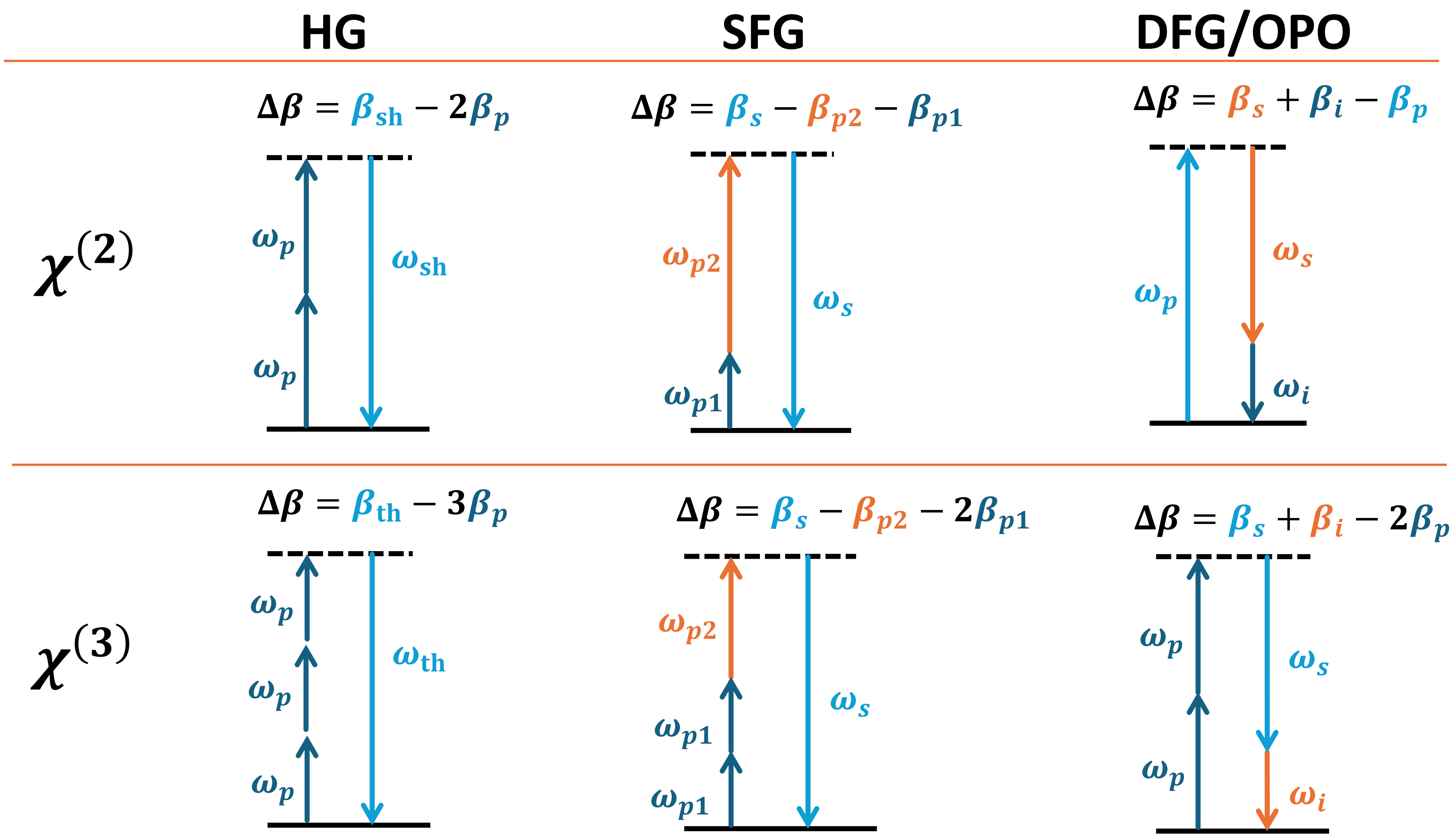


Conversion efficiency: $\sim 250\ \% \cdot \mathrm{W}^{-1}$

Clementi, M., Nitiss, E., Liu, J. *et al.* A chip-scale second-harmonic source via self-injection-locked all-optical poling. *Light Sci Appl* **12**, 296 (2023). https://doi.org/10.1038/s41377-023-01329-6

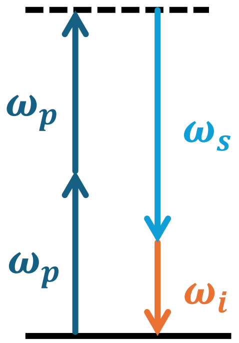
Other frequency conversion processes

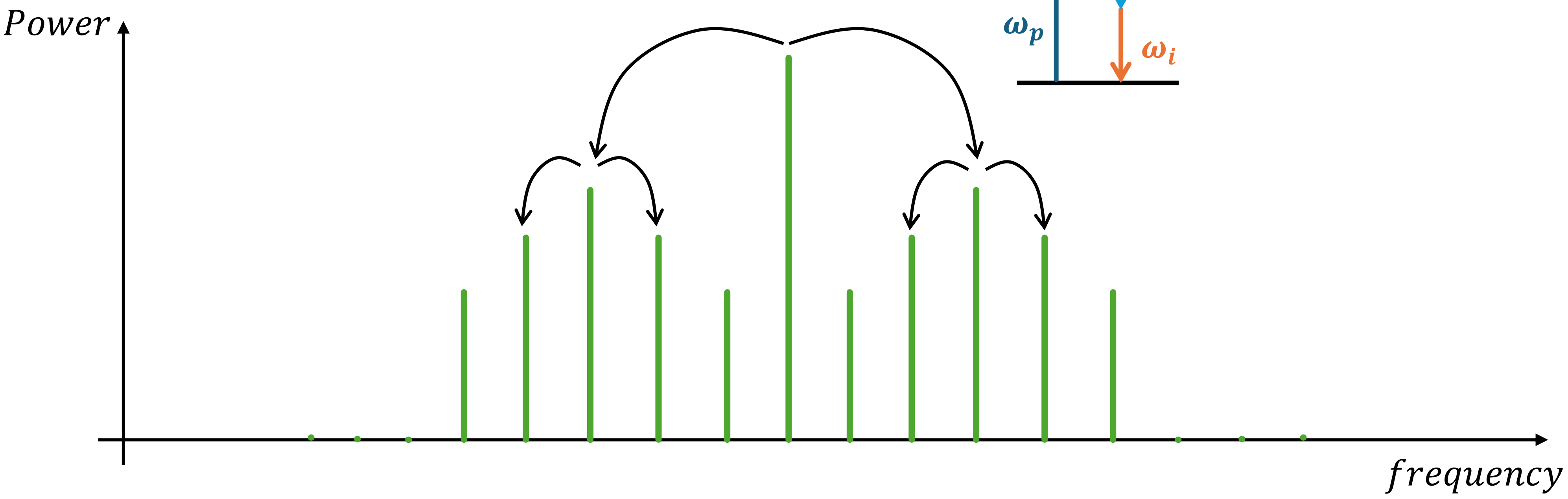
- The process must conserve energy
- The efficiency depends on dispersion
- Resonant processes increase efficiency



Frequency comb generation

- Nonlinear processes can cascade





Frequency comb generation

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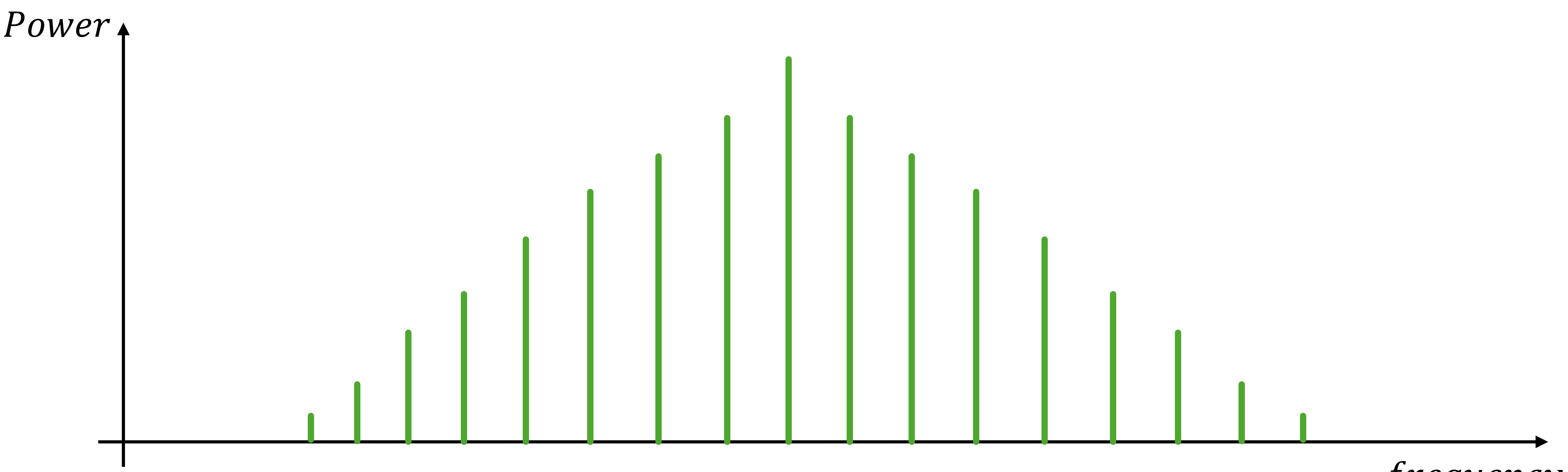


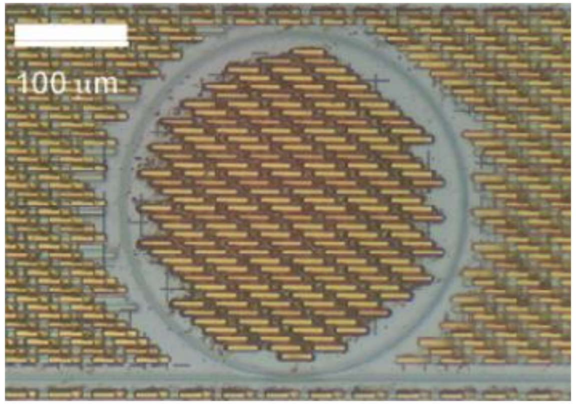
Frequency comb generation

- Nonlinear processes can cascade
- Multiple waves can be generated
- With enough power, a frequency comb can be generated

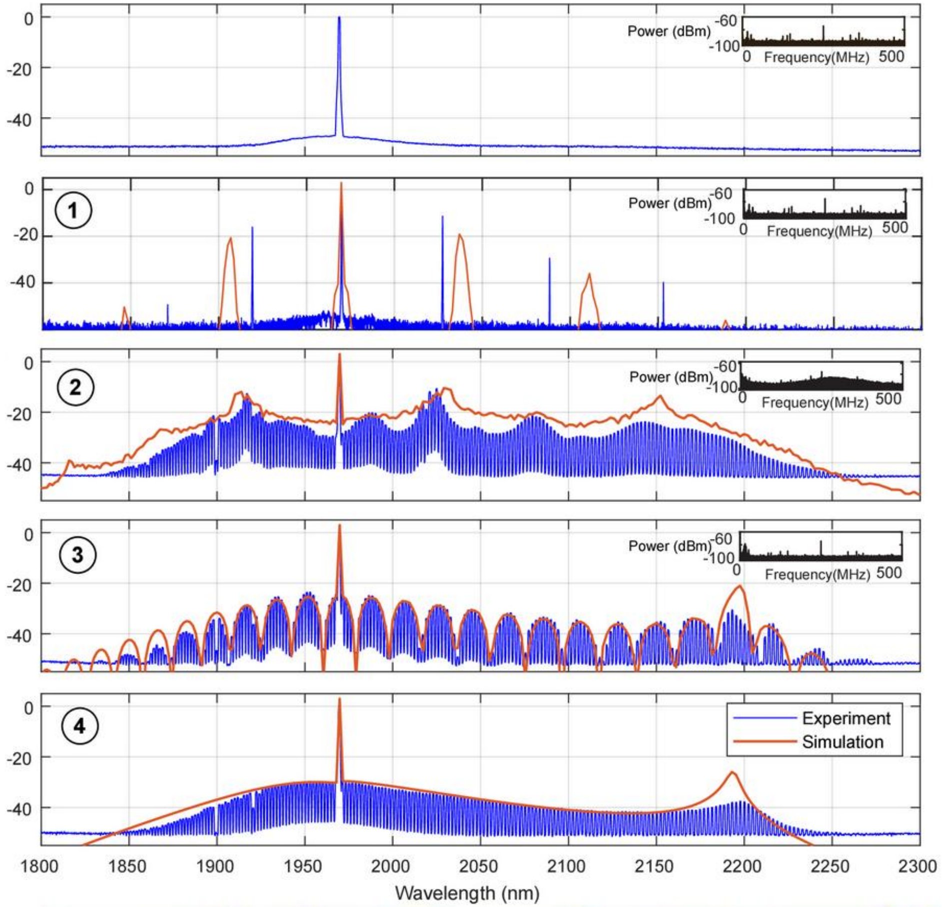
- Useful for:
  - Frequency metrology (e.g. atomic clocks)
  - Communication technologies
  - Spectroscopy

Frequency comb generation

- Example: silicon nitride ring resonator



Anamika Nair Karunakaran, *et al.*, "Dissipative Kerr soliton generation at 2µm in a silicon nitride microresonator," Opt. Express **32**, 14929-14939 (2024)



Summary

- Nonlinear optics permits to generate new optical frequencies
  - Access unconventional wavelength bands
  - Generate frequency combs

- Integration improves nonlinear performances
  - Higher optical intensity
  - Smaller footprint
  - Many degrees of freedom for dispersion engineering/material engineering

- Advanced design further increases conversion efficiency
  - Resonant structures

Electro-optic modulation

Electro-optic phase-shifter

- In presence of a perturbation on the dielectric polarization, the propagation equation is modified:

$$\nabla^2 \underline{\mathrm{E}}(r,t) - \frac{n(x,y)^2}{c^2}\frac{\partial^2 \underline{\mathrm{E}}}{\partial t^2}(r,t) = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2 \underline{\mathrm{P}}^{\mathrm{NL}}}{\partial t^2}(r,t)$$

$$\underline{\mathrm{P}}^{\mathrm{NL}} = \varepsilon_0 \chi^{(2)} \underline{\mathrm{E}}^2$$

$$\underline{\mathrm{E}} = \underline{\mathrm{E}}_{\mathrm{opt}} + \underline{\mathrm{E}}_{\mathrm{DC}}$$

Electro-optic phase-shifter

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Electro-optic phase-shifter

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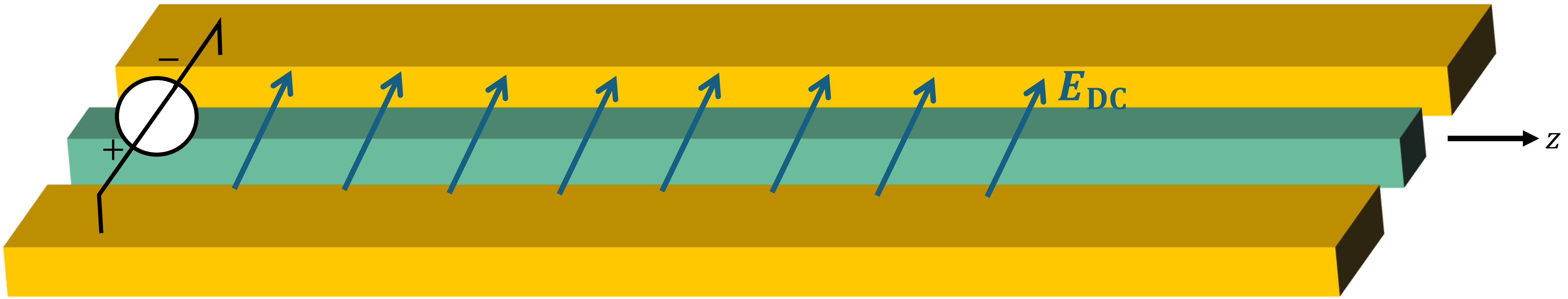
- Projecting the equation on terms oscillating at the optical angular frequency ω, we obtain:

$$\nabla^2 \underline{\mathrm{E}}_{\mathrm{opt}}(r,t) - \frac{n(x,y)^2 + 2\chi^{(2)}\underline{\mathrm{E}}_{\mathrm{DC}}}{c^2}\frac{\partial^2 \underline{\mathrm{E}}_{\mathrm{opt}}}{\partial t^2}(r,t) = 0$$

Electro-optic phase-shifter

$$\nabla^2 \underline{\mathrm{E}}_{\mathrm{opt}}(r,t) - \frac{n(x,y)^2 + 2\chi^{(2)}\underline{\mathrm{E}}_{\mathrm{DC}}}{c^2} \frac{\partial^2 \underline{\mathrm{E}}_{\mathrm{opt}}}{\partial t^2}(r,t) = 0$$

- We get a refractive index that depends on the DC field
- We can tune the refractive index by changing an externally applied DC field



Electro-optic phase-shifter

$$\nabla^2 \underline{\mathrm{E}}_{\mathrm{opt}}(r,t) - \frac{n(x,y)^2 + 2\chi^{(2)}\underline{\mathrm{E}}_{\mathrm{DC}}}{c^2}\frac{\partial^2 \underline{\mathrm{E}}_{\mathrm{opt}}}{\partial t^2}(r,t) = 0$$

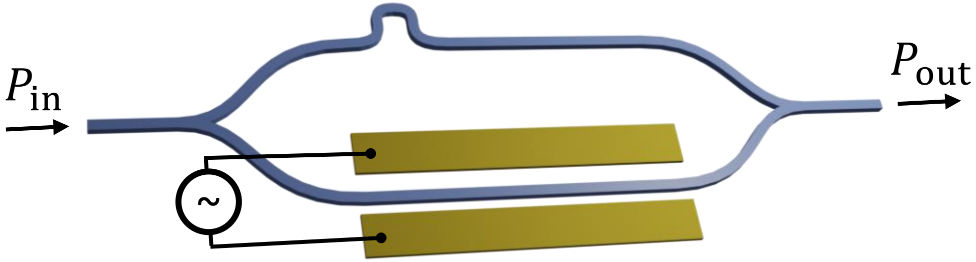
- The effective index of the propagating mode is changed by a DC voltage:

$$\Delta n_{\mathrm{eff}} = \frac{\chi^{(2)}\gamma_{eo}}{n_0} \cdot U$$

$$\gamma_{eo} = \frac{n_0}{n_{\mathrm{eff}}} \frac{\iint_{\sigma_2} F_{\mathrm{DC}}(x,y,1\ \mathrm{V}) \cdot \left|F_{\mathrm{opt}}(x,y)\right|^2 dxdy}{\iint_{\infty} \left|F_{\mathrm{opt}}(x,y)\right|^2 dxdy}$$

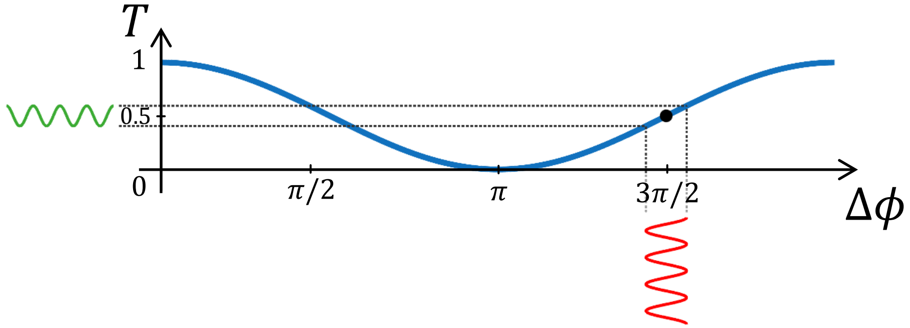
Electro-optic intensity modulator

- We can introduce the phase-shifter inside an interferometric device
  - Example: Mach-Zehnder modulator
  - Useful for communications: data encoding



$$\Delta\phi = \frac{\omega \Delta n_{\mathrm{eff}} L}{c}$$

$$T = \frac{P_{\mathrm{out}}}{P_{\mathrm{in}}} = \frac{1}{2}\left(1 + \cos(\Delta\phi)\right)$$



Electro-optic intensity modulator

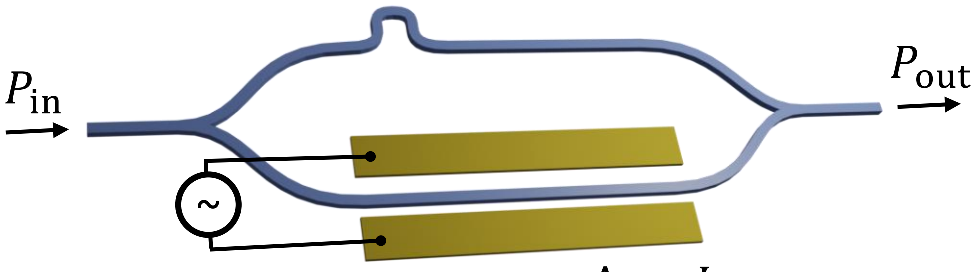
- We can introduce the phase-shifter inside an interferometric device

- Modulation depth depends only on architecture

Maximum efficiency: quadrature point (-3dB optical transmission)

$$\Delta T \simeq \frac{1}{2}\Delta\phi$$

$$T = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{1}{2}(1 + \cos(\Delta\phi))$$



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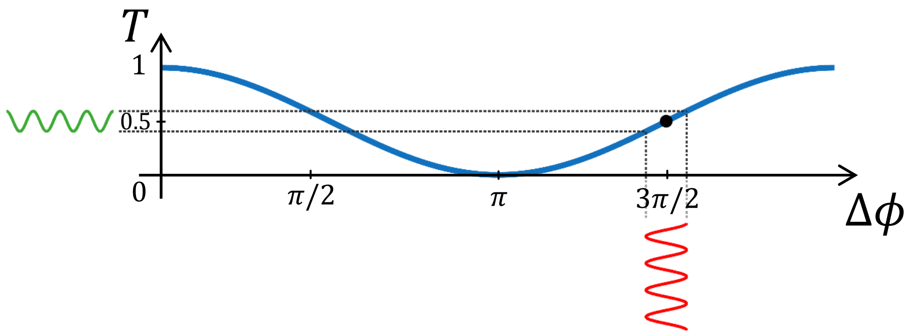
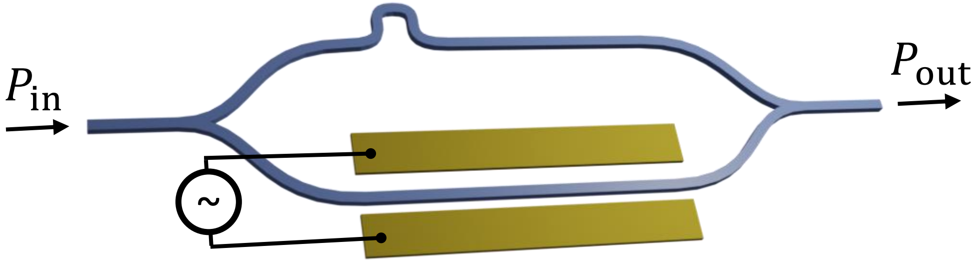


Electro-optic intensity modulator

- We can introduce the phase-shifter inside an interferometric device

- Speed depends on geometry (and architecture)

Maximum speed: given by delay between RF and optical waves



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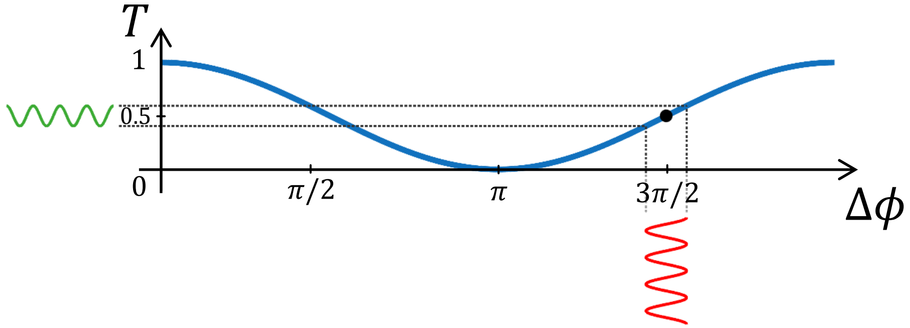
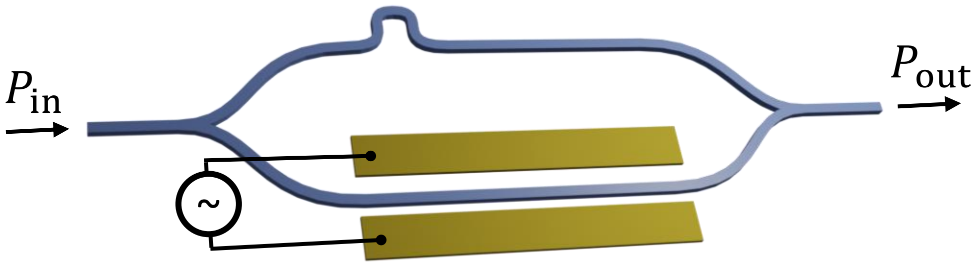


Electro-optic intensity modulator

- Mach-Zehnder modulators:
  - Easy to fabricate
  - Good linearity of $\Delta\phi$ with voltage U
  - Can be insensitive to temperature
  - Very fast (>100 GHz)



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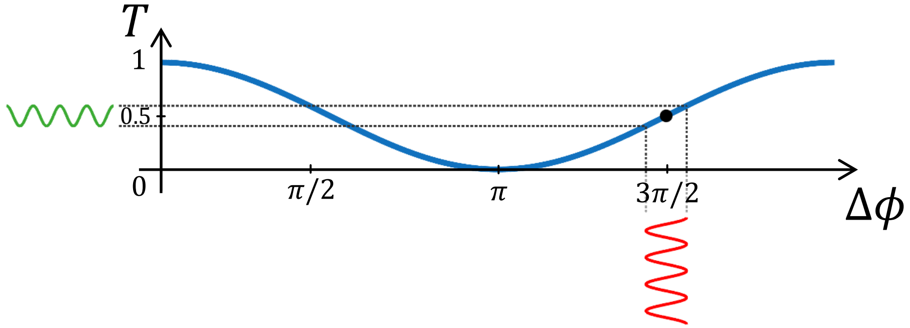

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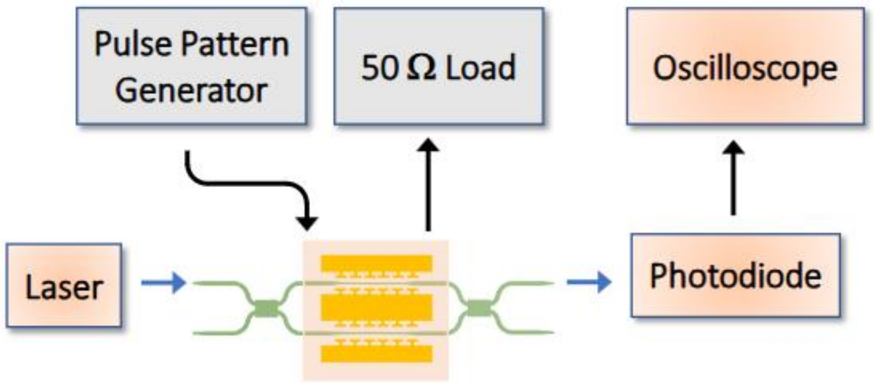
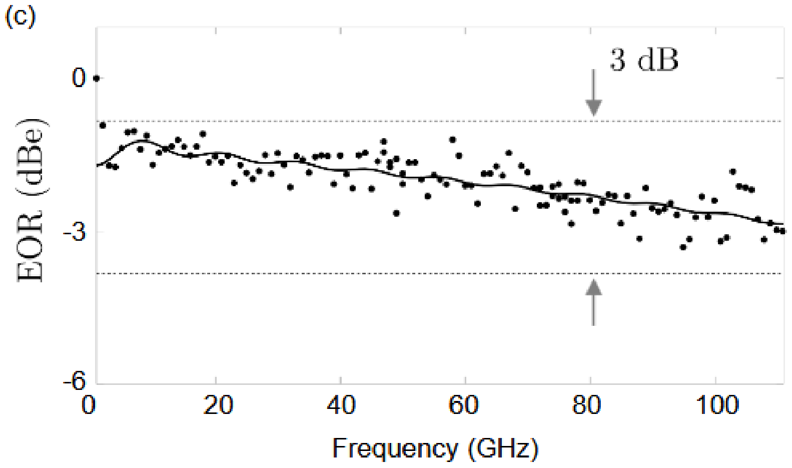
A. Rahman, *et al.* "High-performance Hybrid Lithium Niobate Electro-optic Modulators Integrated with Low-loss Silicon Nitride Waveguides on a Wafer-scale Silicon Photonics Platform", https://doi.org/10.48550/arXiv.2504.00311





Electro-optic modulation

Electro-optic intensity modulator

- Mach-Zehnder modulators:
  - Easy to fabricate
  - Good linearity of $\Delta\phi$ with voltage U
  - Can be insensitive to temperature
  - Very fast (>100 GHz)

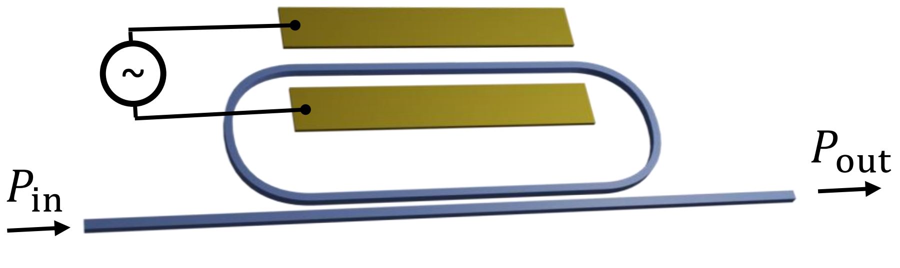
  - Needs RF design (impedance matching) for high speed operation
  - Consumes power (continuously driving current)
  - Large footprint (need long interaction length)

Electro-optic intensity modulator

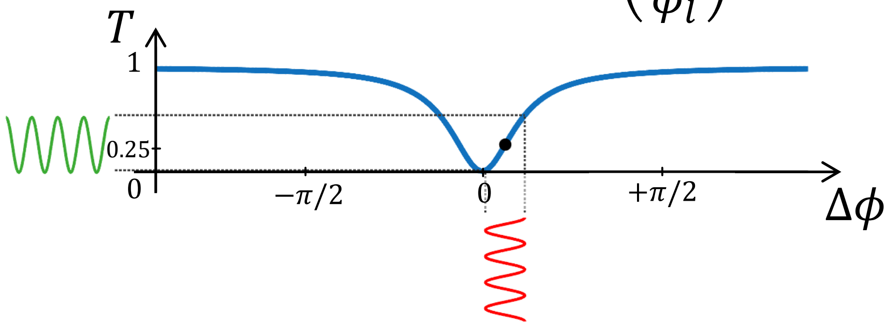
- We can now control the phase of light with an external voltage

- We can introduce the phase-shifter inside an interferometric device
  - Example: micro-ring modulator

$$\Delta\phi = \frac{\omega \Delta n_{\text{eff}} L}{c}$$



$$T = \frac{P_{\text{out}}}{P_{\text{in}}} = 1 - \frac{1}{1 + \left(\frac{\Delta\phi}{\phi_l}\right)^2}$$



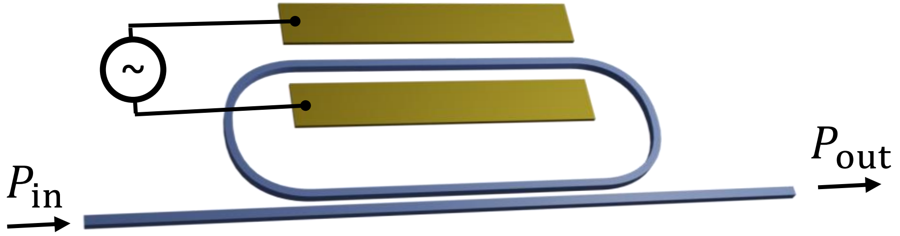
Electro-optic intensity modulator

- We can now control the phase of light with an external voltage
- Modulation depth is proportional to Q

Maximum efficiency: at maximum slope

$$\Delta T \simeq \frac{9}{8\sqrt{3}} \frac{\mathcal{F}}{\pi} \cdot \Delta\phi$$

$$\Delta\phi = \frac{\omega \Delta n_{\text{eff}} L}{c}$$



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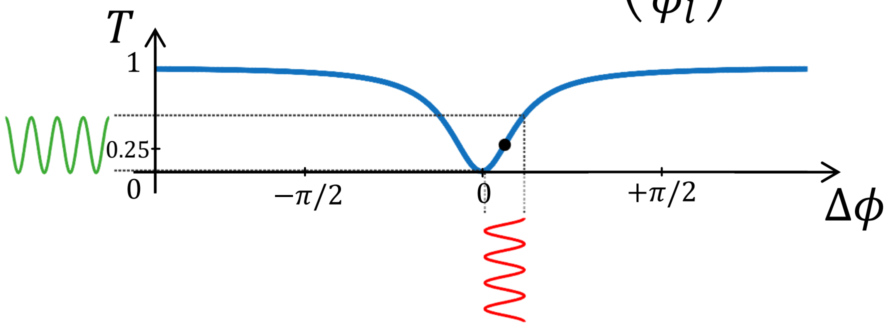


Electro-optic intensity modulator

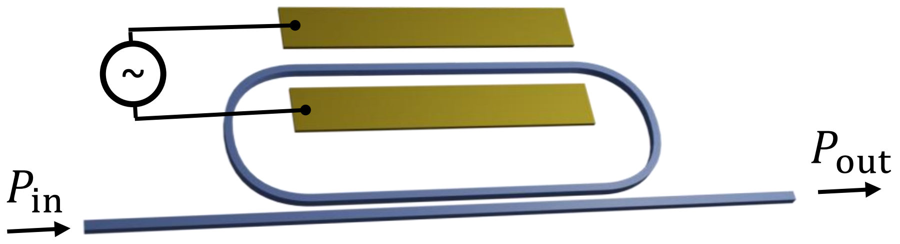
- We can now control the phase of light with an external voltage

- Speed is inversely proportional to Q

Maximum speed: limited by photon lifetime in the cavity

$$f_{\text{cut-off}} \simeq \frac{1+\sqrt{3}}{2\sqrt{3}} \cdot \frac{f_0}{Q}$$

$$\Delta\phi = \frac{\omega \Delta n_{\text{eff}} L}{c}$$



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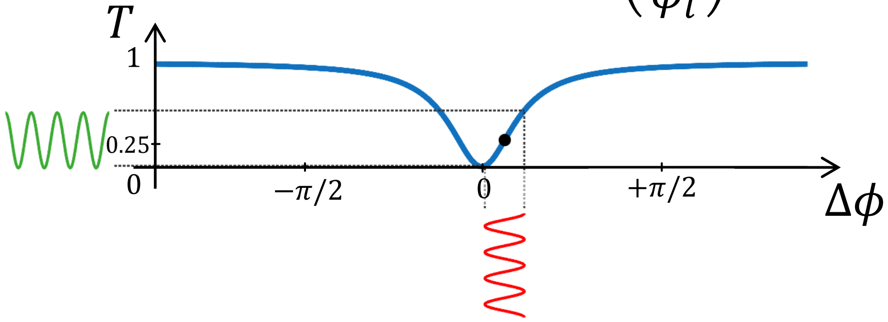
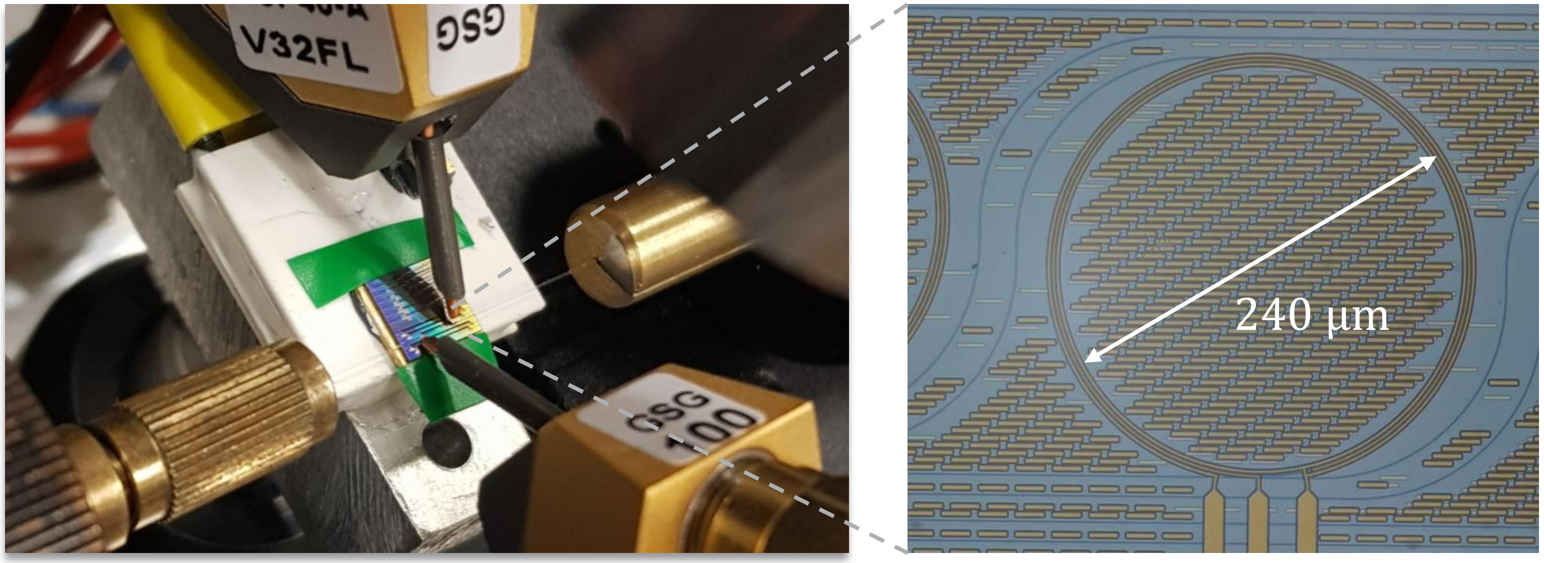
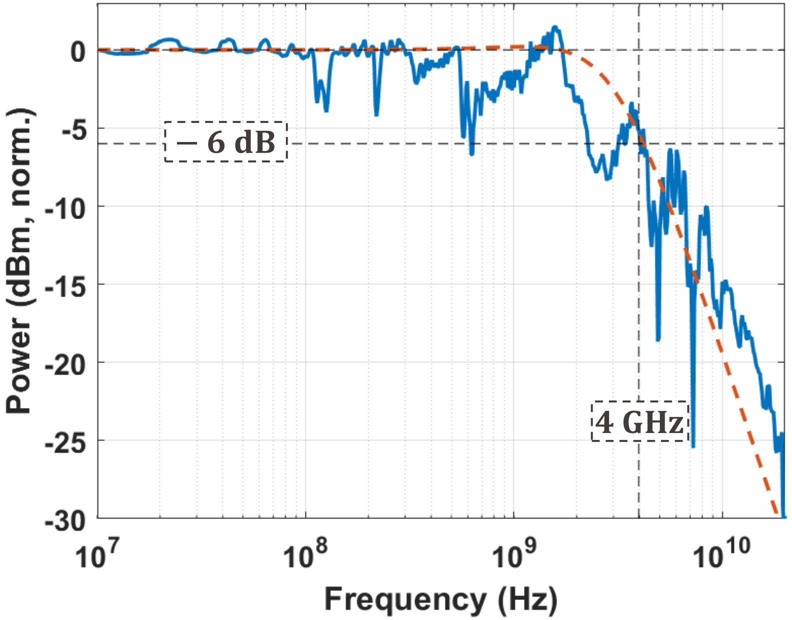


Electro-optic intensity modulator

- Example: silicon nitride micro-ring modulator





Lafforgue, C., Zabelich, B. & Brès, CS. Monolithic silicon nitride electro-optic modulator enabled by optically-assisted poling. *Commun Phys* **8**, 142 (2025). https://doi.org/10.1038/s42005-025-02071-8

Electro-optic intensity modulator

- Micro-ring modulators:
  - High Q → strong modulation (stronger slope)
  - Small footprint
  - No need for RF design (lumped element circuit)
  - (Almost) no power consumption (open circuit)

  - Sensitive to fabrication deviation
  - Sensitive to temperature
  - Trade-off on speed (~ 10 GHz)

Summary

- Nonlinear optics permits to control the refractive index of materials
  - Optical phase-shifting
  - Intensity modulation in interferometric devices

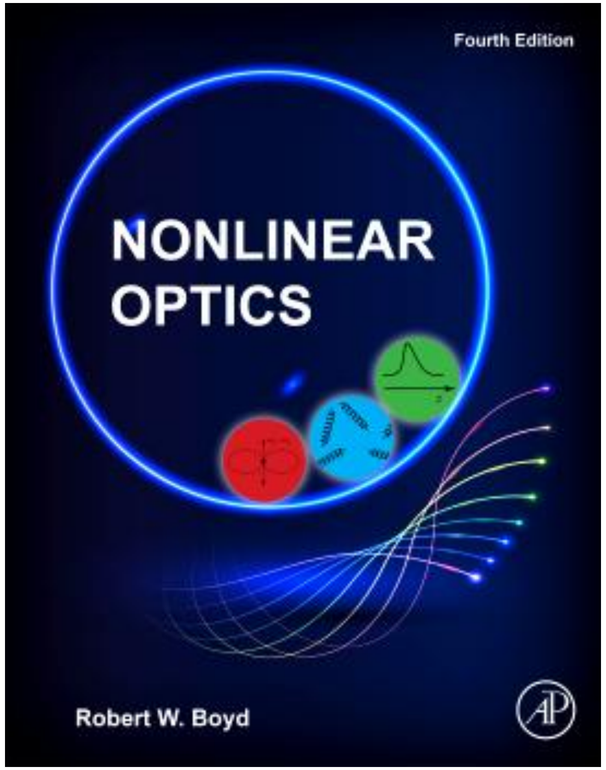
- Integration increases the design space
  - Different structures (Mach-Zehnder, ring, Fabry-Pérot…)
  - Larger parameter space (EO optimization, electrical engineering…)

Further information:
C. Lafforgue and Camille-Sophie Brès, "Electro-optic modulation in amorphous silicon nitride photonic integrated platforms [Invited]," J. Opt. Soc. Am. B **42**, B81-B94 (2025)

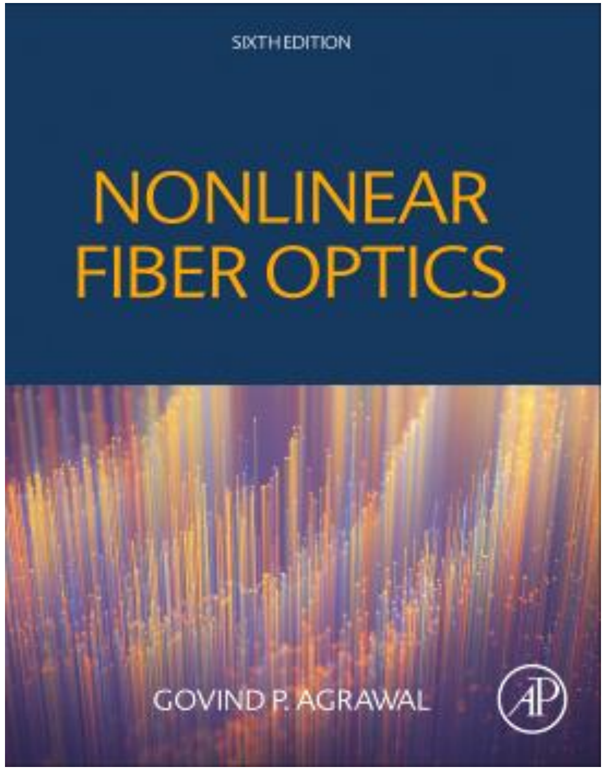
Jorge Parra, Juan Navarro-Arenas, Pablo Sanchis "Silicon thermo-optic phase shifters: a review of configurations and optimization strategies," *Advanced Photonics Nexus* 3(4), 044001 (24 May 2024) https://doi.org/10.1117/1.APN.3.4.044001

Reading advice

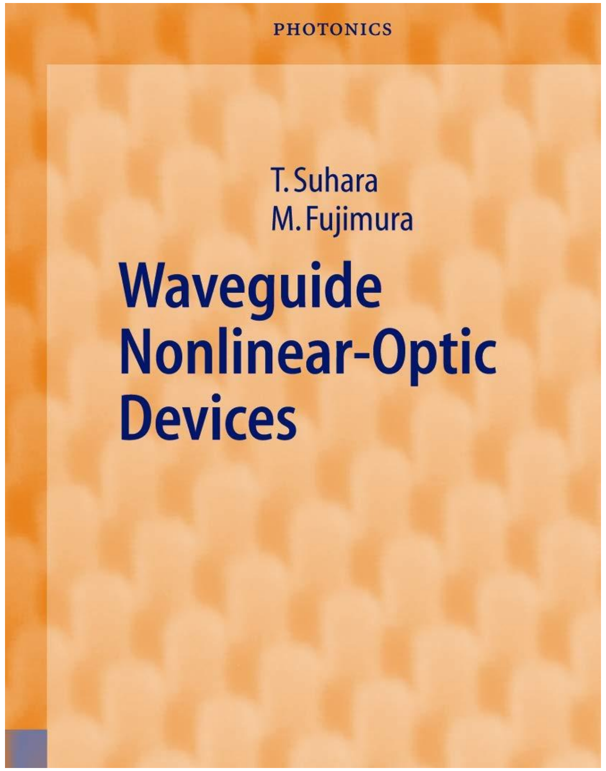
Fundamental nonlinear optics:
R. Boyd, Nonlinear optics



$\chi^{(3)}$ fiber nonlinear optics
G. Agrawal, Nonlinear fiber optics



Integrated nonlinear optics modeling
T. Suhara and M. Fujimura,
Waveguide nonlinear-optic devices



Ring resonators
D. G. Rabus and C. Sada, Integrated ring resonators