

Astrophysics IV: Stellar and galactic dynamics

Exercises**Problem 1:**

Derive the Jeans equations in spherical coordinates, starting from the collisionless Boltzmann equation. Assume a spherically symmetric system.

Hints:

- For the zeroth moment, you can express the zeroth moment of the Boltzmann equation

$$\frac{\partial \nu}{\partial t} + \sum_i \frac{\partial}{\partial x_i} (\nu \bar{v}_i) = 0, \quad \nu \equiv \int d^3\mathbf{v} f$$

first in vectorial notation, then in spherical coordinates.

- For the first moment, use the collisionless Boltzmann equation in spherical coordinates, and compute the first moment by multiplying with the radial velocity v_r and integrating over all velocity space.
- Finally, find the solution assuming the system is stationary, isotropic, and express the equation using velocity dispersion σ .
- Here are a few properties to keep in mind:

$$\begin{aligned} 1) f \rightarrow 0 \text{ when } |v_i| \rightarrow \infty & \quad 2) m \int f d^3\mathbf{v} = \rho & \quad 3) m \int v_i f d^3\mathbf{v} = \rho \bar{v}_i \\ 4) m \int v_i v_j f d^3\mathbf{v} = \rho \overline{v_i v_j} & \quad 5) \overline{v_i v_j} + \sigma_{ij}^2 = \bar{v}_i \bar{v}_j \end{aligned}$$

Problem 2:

Using the Jeans equations in spherical coordinates, determine the velocity dispersion analytically (assuming it is isotropic) for a Plummer sphere in equilibrium.

Problem 3:

Derive the Jeans equations in cylindrical coordinates, starting from the collisionless Boltzmann equation. Assume an axi-symmetric system.