Exercise Set 9

Quantum Computation

Exercise 1 *Let* $C = \{0000, 1111, 0011, 1100\}$.

1. *Compute the minimum distance* $d(C)$.
2. *What is the length* n *and the size* $|C|$*?*
3. *Compute the rate* $R = \frac{k}{n}$, *where* $k = \log_2 |C|$.

Exercise 2 *Show the following about linear codes.*

1. *Determine whether the following set is a linear code:*

$$C = \{000, 011, 101, 110\}.$$

2. *Show that a linear code must contain the zero vector.*
3. *Prove that in a linear code, the minimum distance is equal to the minimum weight of a nonzero codeword.*

Exercise 3 *Let*

$$G = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}.$$

1. *List all codewords of the code generated by* G.
2. *What are the parameters* (n, k) *of this code?*

Exercise 4 *Let* C *be a code with minimum distance* d. *Prove that:*

1. C *can detect up to* $d - 1$ *errors.*
2. C *can correct up to* $\left\lfloor \frac{d-1}{2} \right\rfloor$ *errors.*

Exercise 5 *Let* C *be a linear code with generator matrix* G *and parity-check matrix* H.

1. *Show that the rows of* H *generate a linear code* $C^{\perp}$, *called the* dual code.
2. *Prove that* $C^{\perp} = \{x \in \mathbb{F}_2^n : x \cdot c = 0 \text{ for all } c \in C\}$.
3. *Show that* $\dim(C) + \dim(C^{\perp}) = n$.