

## Astrophysics IV : Stellar and galactic dynamics

Exercises**Problem 1 :**

Demonstrate that the small radial motions around the guiding center in a weakly bared potential of pattern speed  $\Omega_b$  is given by :

$$R_1(\theta_0(t)) = C_1 \cos\left(\frac{\chi_0 \theta_0}{\Omega_0 - \Omega_b} + \alpha\right) - \left[\frac{\partial \phi_b}{\partial R} + \frac{2\Omega_0 \phi_b}{R(\Omega_0 - \Omega_b)}\right]_{R_0} \frac{\cos(m\theta_0)}{\chi_0^2 - m^2(\Omega_0 - \Omega_b)^2}, \quad (1)$$

where :

$$\theta_0(t) = (\Omega_0 - \Omega_b)t, \quad (2)$$

$$\kappa_0^2 = \left(\frac{\partial^2 \phi}{\partial R^2} + 3\Omega^2\right)\Bigg|_{R_0}, \quad (3)$$

is the radial epicycle frequency and  $\Omega_0$  is the circular frequency without the bar. Show that the small azimuthal motion is given by :

$$\dot{\theta}_1(t) = -2\Omega_0 \frac{R_1}{R_0} - \frac{\phi_b(R_0)}{R_0^2(\Omega_0 - \Omega_b)} \cos(m(\Omega_0 - \Omega_b)t) + const, \quad (4)$$

Hints : (1) Assume the rotating potential to be of the form :

$$\phi(R, \theta) = \phi_0(R) + \phi_1(R, \theta) = \phi_0(R) + \phi_b(R) \cos(m\theta), \quad (5)$$

with  $\phi_1 \ll \phi_0$ .

(2) decompose the motion of the stars into two parts :

$$\begin{cases} R(t) = R_0 + R_1(t) \\ \theta(t) = \theta_0(t) + \theta_1(t) \end{cases} \quad (6)$$

with  $R_0$  the radius of the guiding centre (circular orbit).

(3) Develop the equations of motion to the first order.

**Problem 2 :**

Show from a simple geometrical argument that the density of the phase space is conserved for a 1-D harmonic oscillator.

Hint : demonstrate that particles move along ellipses in the phase space.