
Exercise Set 8
Quantum Computation

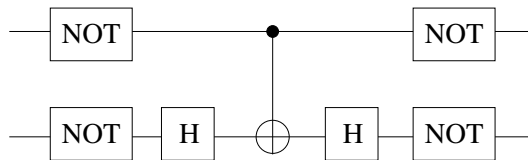
Exercise 1 *Grover's algorithm for $N = 4$*

Let $x \in \{x_0, x_1, x_2, x_3\}$ and $f(x) = 1$ if and only if $x = x_0$. Otherwise $f(x) = 0$. We search x_0 thanks to an "oracle" which returns the value of f when queried with an entry.

- (a) What is the theoretical prediction for the number of queries of the oracle in the quantum setting when we use Grover's algorithm ?
- (b) Show that the following

$$U = \mathbb{I} - 2 \underbrace{|00\dots 0\rangle\langle 00\dots 0|}_{n \text{ times}}$$

is unitary and show also that for $n = 2$ it can be implemented by the following circuit:



- (c) Take Grover's circuit and for $N = 4$ compute the quantum state at each step of the algorithm. Draw a geometrical representation in an appropriate two dimensional space (like in class). Confirm that the measurement of the final state indeed gives x_0 and that only one query of the oracle was needed.

