Astrophysics IV : Stellar and galactic dynamics

Exercises

Problem 1 :

Show that the following relations hold :

$$A(R) \equiv \frac{1}{2}\left(\frac{v_c}{R} - \frac{\mathrm{d}v_c}{\mathrm{d}R}\right) = -\frac{1}{2}R\frac{\mathrm{d}\Omega}{\mathrm{d}R}$$

$$B(R) \equiv -\frac{1}{2}\left(\frac{v_c}{R} + \frac{\mathrm{d}v_c}{\mathrm{d}R}\right) = -\left(\Omega + \frac{1}{2}R\frac{\mathrm{d}\Omega}{\mathrm{d}R}\right)$$

$$\Omega = A - B$$

$$\kappa^2 \equiv \left(R\frac{\mathrm{d}(\Omega^2)}{\mathrm{d}R} + 4\Omega^2\right) = -4B(A-B) = -4B\Omega$$

Problem 2 :

Derive the relation between R and $\dot{R}$ in the $z = 0$ plane (approximation of the third integral), if we assume that the total angular momentum is conserved in an axisymmetric potential.

Problem 3 :

Derive the Hamilton equations of motion of a particle in a potential Φ inside a uniformly rotating reference frame $\vec{\Omega}$.

Setup for Numerical Exercises :

To run the following exercise, you must install the `pNbody` and `Ptools` packages. It is highly recommended to use a custom Python virtual environment. If you cannot install these on your local machine, you may use `noto.epfl.ch`, though it provides a less interactive experience.

1. Install `pNbody` from GitLab :
   `pip install git+https://gitlab.com/revaz/pNbody.git`
2. Install `Ptools` from GitLab :
   `pip install git+https://gitlab.com/revaz/Ptools.git`

Detailed installation instructions and documentation can be found at :
`https://www.astro.unige.ch/~revazy/pNbody/rst/Installation.html`

Problem 4 :

Using the scripts MAPPING.PY and MAPPING-RZ.PY, explore the surface of section discussed during the lectures.