Astrophysics IV: Stellar and galactic dynamics

Exercises

Problem 1:

Show that in a spherical potential the vertical epicycle and circular frequencies are equal.

Problem 2:

Using the epicycle approximation, prove that the azimuthal angle $\Delta\phi$ between successive pericenters lies in the range $\pi \leq \Delta\phi \leq 2\pi$ in the gravitational field arising from any spherical mass distribution in which the density decreases outwards.

Problem 3:

Prove that circular orbits in a given potential are unstable if the angular momentum per unit mass on a circular orbit decreases outward.

Problem 4:

Prove that the mean-square velocity on a bound orbit in a spherical potential $\Phi(r)$ is:

$$\langle v^2 \rangle = \left\langle r \frac{\mathrm{d}\Phi(r)}{\mathrm{d}r} \right\rangle , \tag{1}$$

where, $\langle . \rangle$ denotes a time average.

Problem 5:

Let $\Phi(R, z)$ be the Galactic potential. At the solar location, $(R, z) = (R_0, 0)$, prove that:

$$\left. \frac{\partial^2 \Phi(R,z)}{\partial z^2} \right|_{z=0} = 4\pi G \rho_0 + 2\left(A^2 - B^2\right), \tag{2}$$

where ρ_0 is the density in the solar neighborhood and A and B are the Oort constants.