SCHOOL OF ENGINEERING
MECHANICAL ENGINEERING
LRESE - Laboratory of Renewable Energy Sciences and Engineering

Renewable Energy: Energy Storage solution

In this exercise, you will learn about energy storage solutions.

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1. Application of Flywheels in Cars

   (a) Kinetic Energy: $E_{kin} = \frac{1}{2} M \cdot \nu^2 \approx 320$ kJ ≈ 0.089 kWh

   (b) Losses due to air drag: $P_{air} = F_{air} \cdot \nu = \frac{1}{2} \rho_{air} \cdot c_d \cdot A_{front} \cdot \nu^3 \approx 4.5$ kW

   (c) Necessary $E_{flywheel} = \frac{1}{\eta}(E_{kin} + P_{air} \cdot \frac{d_{range}}{\nu}) \approx 35.1$ MJ ≈ 9.8 kWh

   (d) In a car, there is only space for wheels with a radius R of up to 70 cm. Therefore R is set to 70 cm.

   According to the equation in the slides lecture, $\sigma_{\max} = \rho * R^2 * \omega^2$, the maximum angular frequency can be derived directly: $\omega = \sqrt{\sigma_{\max}/(\rho * R^2)} = 1649.6$ rad/s. This results in $f = \frac{\omega}{2\pi} \approx 263$ Hz (rotations per second).
   Comment: This is a rather high value, which probably causes additional losses due to aerodynamic and bearing drag.
   The energy density of a flywheel can be calculated as follows: $\frac{E}{M} = k \cdot \frac{\sigma_{max}}{\rho}$. With k being the given shape factor and the stored energy of a single flywheel $E_{flywheel} =$ 17.55 MJ, the mass of a single flywheel is $m \approx 21.7$ kg, so 43.4 kg for both.
   The thickness of one flywheel is accordingly: $D = \frac{M}{\pi \cdot R^2 \rho} \approx 9.4$ mm.

   (e) The pair of flywheels should store the kinetic energy of a car moving at a speed of 120 km/h:
   $2E_{flywheel} = E_{kin} = \frac{1}{2} M \cdot \nu^2 \approx 720$ kJ ≈ 0.20 kWh
   Losses due to air resistance are neglected here. There is less space for a supplementary device. As a consequence, the radius of the flywheels R is set to 30 cm.
   The maximal angular frequency is $\omega = \sqrt{\sigma_{\max}/(\rho * R^2)} = 3849$ rad/s, using the formula from the slides.
   Analogously to (d) the mass of a single flywheel is then: $M \approx 44.7$ kg.
   Correspondingly, the thickness is around 1.1 mm.

2. Pumped air storage:

   (a) Uncompressed air: $p_0 \approx 1$ bar ≈ 100 kPa, $T_0 \approx 25\ ^oC$
   Compressed air (gas tank): $p_1 \approx 300$ bar ≈ 30 MPa, $T_1 = T_0 \approx 25\ ^oC$

Released air: $p_2 = p_0 \approx 1$ bar ≈ 100 kPa, $T_2 < T_0$

Isothermal process: $p \cdot V = n \cdot R \cdot T =$ const. or $V(p) = \dfrac{n \cdot R \cdot T}{p}$

Adiabatic process: $p \cdot V^{\kappa} =$ const. or $V(p) = V_1 \cdot (\dfrac{p_1}{p})^{1/\kappa} = \dfrac{n \cdot R \cdot T_1}{p_1} \cdot (\dfrac{p_1}{p})^{1/\kappa}$

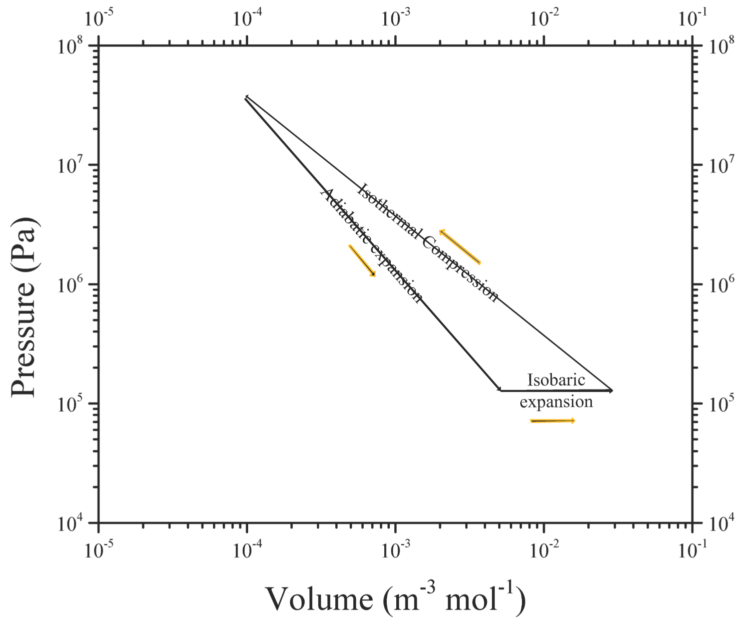

Figure 1: P-V diagram

(b) Isothermal compression work:

$$W_{comp} = -\int_0^1 p \cdot dV = -\int_{p_0}^{p_1} p \frac{dV}{dp}|_{isothermal} dp$$

$$= nRT_0 \int_{p_0}^{p_1} \frac{dp}{p} = nRT_0 \cdot ln(\frac{p_1}{p_0}) \approx 14.1 \text{ kJ/mol}$$

Adiabatic expansion work:

$$W_{exp1} = -\int_1^2 p \cdot dV = -\int_{p_1}^{p_0} p \frac{dV}{dp}|_{adiabatic} dp$$

$$= \frac{p_1^{1/\kappa} \cdot V_1}{\kappa} \int_{p_1}^{p_0} p^{-1/\kappa} \cdot dp = \frac{p_1^{1/\kappa} \cdot V_1}{\kappa} \frac{\kappa}{\kappa - 1} (p_0^{\frac{\kappa-1}{\kappa}} - p_1^{\frac{\kappa-1}{\kappa}}) = nRT_1 \frac{p_1^{\frac{1-\kappa}{\kappa}}}{\kappa - 1} (p_0^{\frac{\kappa-1}{\kappa}} - p_1^{\frac{\kappa-1}{\kappa}}) = \frac{nRT_1}{\kappa - 1}((\frac{p_0}{p_1})^{\frac{\kappa-1}{\kappa}} - 1) \approx \text{-5.0 kJ/mol}$$

Isobaric expansion work:

$$W_{exp2} = -\int_2^0 p \cdot dV = -p_0(V_0 - V_2) = nRT_0((\frac{p_0}{p_1})^{\frac{\kappa-1}{\kappa}} - 1) \approx \text{-2.0 kJ/mol}$$

Losses: $W_{losses} = W_{comp} - |W_{exp1}| - |W_{exp2}| \approx 7$ kJ/mol
Efficiency: $\eta = \dfrac{W_{exp1} + W_{exp2}}{W_{comp}} \approx 50\%$

(c) From Problem 1c:
Energy needed for 120 km: $E_{drvie} = P_{air} \cdot \dfrac{d_{range}}{\nu} \approx 24.2$ MJ
Released work from pumped air storage: $W_{released} = |W_{exp1} + W_{exp2}| \approx 7.0$ kJ/mol
$\rightarrow$ Minimal amount of air : $n = \dfrac{E_{drive}}{W_{released}} \approx 3478$ mol$, V_{air} = \dfrac{R \cdot T_1}{p_1} \dfrac{E_{drive}}{W_{released}} \approx 0.287$ m^3
There should be enough space in a car for a 300 litre tank.

3. Pumped water storage:

(a) Potential energy of 1 m^3 water: $E_{pot} = m \cdot g \cdot \Delta h = 1000 \cdot 9.81 \cdot 1000 \approx 9.81$ MJ
Annual production of 100 MW$_p$ PV plant:
$E_{prod} = \eta \cdot P_p \cdot t = 0.15 \cdot 10^8 \cdot 365 \cdot 24 \cdot 3600 \approx 4.7 \cdot 10^{14}$ J
Amount of water: $V_{water} = \eta_{pump} \cdot \dfrac{E_{prod}}{E_{pot}} = 0.85 \cdot \dfrac{4.7 \cdot 10^{14}}{9.8 \cdot 10^6}$ m$^3 \approx 4.1 \cdot 10^7$ m^3

(b) Annual production of 100 MW$_{av}$ PV plant:
$E_{prod} = P_{av} \cdot t = 10^8 \cdot 365 \cdot 24 \cdot 3600 \approx 3.2 \cdot 10^{15}$ J
Amount of water: $V_{water} = \eta_{pump} \cdot \dfrac{E_{prod}}{E_{pot}} = 0.85 \cdot \dfrac{3.2 \cdot 10^{15}}{9.8 \cdot 10^6}$ m$^3 \approx 2.7 \cdot 10^8$ m^3

4. Batteries:

(a) for the discharge:
Anode: $Pb + SO_4^{2-} \longrightarrow PbSO_4 + 2e^-$
Cathode: $PbO_2 + 4H^+ + SO_4^{2-} + 2e^- \longrightarrow PbSO_4 + 2H_2O$

(b) Equation for the electrochemical equilibrium: $U^0 = \Delta E^0 = -\dfrac{\Delta G^0}{z \cdot F}$
ΔG^0 for Pb-Acid and F are given, it is possible to see from point a) that z=2.
$\longrightarrow U^0 = 2.047$ V
If a 24 V battery is required, a series of at least 12 Pb-Acid cells is needed $\longrightarrow$ 24.56 V

(c) How many moles of Pb got converted? (= moles of $PbSO_4$ formed on the anode only)
$n_{C_d} = m_{C_d}/M_{C_d} = \dfrac{11.6g}{207.2g/mol} = 0.056$ mol

With the help of the Faraday constant (which defines the mol-specific charge of matter), we can now calculate the overall charge in Anode side we get, when the 56 mmol are converted. Note from the half-cell reaction, that there are 2 electrons involved when 1 Pb is converted.

$F = \frac{Q_0}{z \cdot n}$ $\quad Q_0 = F \cdot z \cdot n = 96485\text{As/mol} \cdot 2 \cdot 0.056 \text{ mol} = 10803 \text{ C}$

To determine the time it will take to recharge the battery, we divide the charge by the given current:

$\frac{10803 \text{ As}}{1.5 \text{ A}} = 7202.2 \text{ s} = 2.0 \text{ h}$

(d) For obtaining the mass specific charge Q in Ah/kg we use the Faraday law again. Note, that all the charge-carrying species (educts, left side of the overall reaction equation) are involved in the calculation by their molar masses:

$Q = \frac{z \cdot F}{\sum_i M_i}$; $\sum_i M_i = 1\cdot$ M(Pb) + 1 $\cdot$ M(PbO_2) +2 $\cdot$ M(H_2SO_4)

From the given molar masses for Pb,O,S,H to be 207.2, 16, 32, 1 g/mol respectively,it is possible to obtain: $\sum_i M_i = 642.4$ g/mol

Having in mind that z is still 2, the specific charge now calculates to $Q = 300.39$ C/g = 83.44 Ah/kg.

The energy density can be obtained from the charge density (= mass specific charge) by multiplying by the reversible cell voltage, since $U[V] \cdot I[A] = P[W]$ and $P[W] \cdot t[h] = E[Wh]$:

$E = Q \cdot U^0$; using U^0 from above = 2.047 V, it follows: E=170.8 Wh/kg.

(e)

i. Equation for the electrochemical equilibrium: $U^0 = \Delta E^0 = -\frac{\Delta G^0}{z \cdot F}$,
$\longrightarrow U^0 = 4.20$ V.

ii. $Q = \frac{z \cdot F}{\sum_i M_i}$; $\sum_i M_i$ = 1.M(LiC_6) + 1.M(CoO_2)=169.8 g/mol; z=1
$\longrightarrow Q_{Li-ion} = 157.84$ Ah/kg $\longrightarrow U^0_{Li-ion} \longrightarrow E_{Li-ion} = 662.54$ Wh/kg
compare:
$\longrightarrow Q_{Pb-Acid} = 83.44$ Ah/kg $\longrightarrow U^0_{Pb-Acid} \longrightarrow E_{Pb-Acid} = 170.8$ Wh/kg

iii. reason 1): reversible cell voltage has doubled
reason 2): less weight of the charged electrode and electrolyte
$\rightarrow$ both parameters bring big advantage in economic feasibility of a battery system