

Astrophysics III : Stellar and galactic dynamics

Exercises**Problem 1 :**

The surface density of our Galaxy's disk is $\sim 50 M_{\odot}/\text{pc}^2$ and its thickness is $\sim 500 \text{ pc}$. Given that its mass is $\sim 1/45$ of the total mass of our Galaxy ($M_{\text{tot}} = 2 \times 10^{12} M_{\odot}$), estimate its radius and its mean density. Given that the Sun is at $R_{\text{sun}} = 8 \text{ kpc}$ from the Galaxy center and that its rotation period is 220 Myr, estimate the mass inside a sphere of R_{sun} .

Problem 2 :

For a galaxy cluster and for a galaxy, estimate the ratio between the volume of the N components and the total volume of the system. Consider a mean radius of $R_{\star} = 10^6 \text{ km}$ for the stars.

Problem 3 :

For a galaxy cluster and for a galaxy, estimate the ratio between the volume of the tube travelled by one of the component during $t = 10^{10}$ years and the total volume of the system. Consider a mean radius of $R_{\star} = 10^6 \text{ km}$ for the stars.

Problem 4 :

Estimate the gravitational influence radius R_G for a galaxy moving within a galaxy cluster and for a star moving within a galaxy.

Problem 5 :

Assuming that the disk of galaxies are (uniformly) randomly oriented, what fraction will be seen face-on (say under 10°) between their axis of symmetry and line of sight? What fraction are seen edge-on, 10° between their equatorial plane and the line of sight?

Problem 6 :

Estimate the relaxation time of the following systems, assuming that all stars are solar type ones :

1. An open cluster (typical radius : ~ 2 parsecs, typical velocity $\sim 0.5 \text{ km s}^{-1}$, mass $\sim 300 M_{\odot}$).
2. A globular cluster (typical radius : ~ 3 parsecs, typical velocity $\sim 6 \text{ km s}^{-1}$, mass $\sim 2 \times 10^5 M_{\odot}$).
3. A dwarf spheroidal galaxy (typical radius : ~ 500 parsecs, typical velocity $\sim 10 \text{ km s}^{-1}$, mass $\sim 10^7 M_{\odot}$).

Problem 7 :

Discuss briefly, why the relaxation time of a system will increase with the number of members assuming the size is held constant. (This will also help with the next problem.)

Problem 8 :

In a first part, estimate the relaxation time of an ultra-faint dwarf galaxy (UFD) containing only about 1'000 stars and being as compact as 50 pc. In a second part, assume that the typical velocity of the stars is about 4 km/s and thus, that the stellar component must be embedded in a massive dark matter halo. How is the relaxation time changed from the first to the second case?

Hint : Assume that the mass of the stars is about one solar mass.