Mini-project description

1 The N^2 queens problem

Consider the following problem: on a 3D $N \times N \times N$ chessboard, we would like to place N^2 queens such that no any two queens attack each other. What we mean exactly by this is the following: a queen placed in position (i, j, k) on a 3D $N \times N \times N$ chessboard attacks any position sharing *two* common coordinates, as well as any position along a 2D or 3D diagonal of the chessboard containing position (i, j, k). The aim of the present mini-project is to solve this problem by the use of the Markov Chain Monte-Carlo (MCMC) method.

2 MCMC simulation

Your first tasks in this mini-project are the following:

1. Choose a state space that describes the positioning of the N^2 queens on the 3D chessboard.
2. Choose an energy function that is minimum (more precisely equal to zero) when no two queens attack each other on the 3D chessboard.
3. Design a Monte-Carlo Markov chain on this state space and run it from an initial positioning of the N^2 queens until it reaches a position of minimal energy.

For each of these three tasks, many choices are possible; justify your choices, paying attention to the fact that some choices lead to better performance than others (notably regarding runtime).

3 Further tasks and questions

1. For a fixed value of N, represent the energy of the successive states obtained by your algorithm as a function of time (in order to obtain a nice curve here, you should average the energy over multiple runs of your algorithm).
2. For a fixed value of N, does the use of simulated annealing (with an appropriate cooling schedule) help improve the performance of your algorithm or not? Plot curves (averaged again over multiple runs) to support your claim.
3. Represent the minimal energy reached by your MCMC algorithm as a function of the size N of the 3D chessboard.
   Note: Of course, as N increases, more and more runtime is needed to reach a minimal value.
4. Can you identify values of N where the minimal energy is significantly lower than others?

4 Deadline

Please submit a short report (4-5 pages) addressing the above mentioned points on Moodle, as well as your code, by Sunday, December 14 at 11:59 PM.

. . . and be prepared for the competition that will take place on Tuesday, December 16 at 8:15 AM in room BS 260.