Bell's Theorem and Quantum Nonlocality

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Introduction

Quantum mechanics challenged foundational assumptions of classical physics, particularly the principle of locality—the idea that objects are influenced only by their immediate surroundings, which was crucial to theories like Newtonian mechanics and Maxwell's electromagnetism. Phenomena such as quantum entanglement suggested that particles could exhibit instantaneous correlations over vast distances—a concept Einstein dismissed as "spooky action at a distance." To preserve locality within the quantum framework, Einstein and his colleagues proposed thought experiments questioning the completeness of quantum mechanics and used the notion of local hidden variables to explain these correlations without violating locality. This sparked profound debates about the nature of reality and the limits of scientific knowledge. In 1964, physicist John Bell built upon these discussions by formulating Bell's theorem, deriving inequalities that any local hidden variable theory must satisfy, thereby transforming the issue into one that could be experimentally tested. The results were decisive: experiments showed that Bell's inequalities are violated, compelling us to abandon the idea of local hidden variables and fundamentally rethink the notion of locality in physics.

By introducing the concept of local causality, Bell formalized the idea that events in one region of spacetime should not influence those in another. This notion is encapsulated in the factorizability condition, which quantum mechanics famously violates through entanglement. However, The distinction between the condition of local causality and the condition of no signalling faster than light reveals that quantum mechanics, while nonlocal in Bell's sense, remains consistent with the relativistic prohibition of faster than light signalling. Finally, we discuss the factorizability condition under the prism of the Evrett Theory. We point out that there might still be room for locality, even when the factorizability condition is not verified, in the special case of the Evrett setting. At the end, we finish our discussion on the difficult problem of Probability in the MWI and how to make sense of it.

1 From EPR to Bell's Theorem

Why does quantum mechanics challenge the classical notion of locality, and how did this lead to the development and disproof of local hidden variable theories ?

- **EPR and the Search for Local Hidden Variables:** The Copenhagen interpretation, while highly successful in predicting measurement results on a microscopic scale, deeply unsettled many scientists due to its philosophical implications. Foundational principles of classical physics, such as locality, causality, and determinism, seemed incompatible with the probabilistic and observer-dependent nature of quantum mechanics. Einstein, determined to defend those pillars of science, built numerous thought experiments (Gedankenexperiments) to highlight what he saw as the absurdity of quantum theory. Among these, "Einstein's Boxes," presented in 1927 and refined in 1931, challenged the principle of locality. By examining Einstein's arguments and Bohr's refutations (Bohr, 1949), we will be able to investigate the conceptual tensions at the heart of this debate. Unable to prove the inconsistency of quantum mechanics, Einstein shifted focus to its incompleteness. This effort culminated in the EPR paradox of 1935, developed with Podolsky and Rosen, which advanced his critique and introduced the hypothesis of local hidden variables (Einstein, 1935). We will explore how this paradox built on his earlier ideas and laid the foundation for the modern debate on locality in quantum mechanics.

- **Bell and the Disproof of Local Hidden Variables:** For decades, non-locality was dismissed as a philosophical issue with no bearing on experimental results. But in 1964, John Bell changed the game. He devised a quantitative framework to test local hidden variable theories, deriving the now-famous Bell inequalities—conditions that any local realistic theory must satisfy (Bell, 1964). His work transformed the debate from a matter of taste to testable science. In the 1980s, Alain Aspect and others conducted experiments that violated Bell inequalities, confirming quantum mechanics and disproving local hidden variable theories (Aspect, 1982). These findings revealed that any hidden variable theory consistent with quantum mechanics must be inherently non-local—ironically, the opposite of what Bell initially hoped to prove. We will explore how Bell's work demonstrates the impossibility of reconciling local hidden variables with quantum mechanics. Additionally, we will discuss the philosophical insights and mathematical tools that enabled Bell to formulate his theorem.

2 Bell's Interpretations of Locality

How locality can be describe ? How strong is Bell's definition of locality ? Quantum mechanics violates local causality, but is it incompatible with relativity ?

- **The factorizability condition:** In this section we will present the key assumptions that Bell says to define locality (local causality in Bell's words). A system is local-causal if the probability of a local beables associated to a distinct space-time region is unchanged by the specification of a local beable associated to an other space-time region. Also, in local causal system, if what happens in the backward light cone of one of the beables is completely specified, we can deduce entirely the probability of this beable. (Bell, 2004) Then, we will see an necessary condition (in probabilistic view) of local causality, the factorizability condition (outcomes and parameters independence), seen as a consequence of locality (Goldstein, 2011). In EPR experiment, we saw that the two outcomes of measurements in the two separated regions are correlated, but how two outcomes intrinsically random, associated with irreducible probability can be correlated ? So, in this sense, we will see the weakness of the factorizability condition (local causality), by discussing the possibility of superluminal signalling, the real independence of parameters and the non-separability. (Esfeld, 2014)

- **Non-locality and relativity:** The distinction between the condition of local causality and the condition of no signalling faster than light is crucial for understanding why quantum mechanics and relativity are consistent despite the apparent violation of Bell's local causality. (Brown, 2015) In relativity, the principle of locality ensures that no information or physical influence can travel faster than the speed of light. However, Bell's notion of local causality, grounded in factorizability, is stronger: it assumes that the outcomes of measurements at one location are entirely independent, the same applies for parameters (Goldstein, 2011) Quantum mechanics, through entanglement, violates this stricter notion. The measurement outcomes of entangled particles are correlated in a way that cannot be explained by any locally causal hidden-variable theory. However, these correlations are purely statistical—they do not enable faster-than-light signaling (c.f. no-siganlling principle), because the outcomes are intrinsically random. (Brown, 2011)

3 Locality under the Evrett Theory

How can locality hold in the frame of the Evrett Theory ? Does the Evrett Theory satisfies completely causal locality ?

- **Evrett Theory and Locality:** Evrett Theory satisfies locality (Brown, 2015). In the EPR experiment, specifically in the case of perfect correlation when a measurements occurs the state of the two photons and the measurements device become entangled. The universe than splits in two branches corresponding to the two possible outcomes of the experiment. Those two branches then evolve independently from each others. They can be seen as worlds. The non uniqueness of the measurement allows us to observe the correlation between the two photon without having to assume interaction at a distance. It is just the case that one branch will contain the "Up Up" state and the other the "Down Down" state. So, in that sense there is no "spooky" interaction at a distance.

- **The factorizability condition in the Evrett Setting:** In its paper of 2015 Brown H. R., Timpson reckons that the Evrett Theory "can be local in the sense of satisfying no-action-at-a-distance, whilst failing to be locally causal" (Brown 2015). There is no immediate action at a distance but the two particle remain correlated and this regardless of how far apart they are from each others. Therefore the factorizability condition does not hold in Evrett Theory. It follows that the Evrett Theory shows that we can have a theory where locality holds while not having the factorizability condition satisfied.

- **Probability and the MWI:** The topic of probability in the MWI is a difficult subject. How to make sense of probability in a multiverse where every outcome exists in its own paralell branch ? Why do we have the subjective experience of probability if all possible outcomes predicted by quantum physics are realised in separate branches ? In this section we will discuss some potential answers that have been provided. For example the idea of "indexicalism" proposed by Wilson in 2013, 2020 (Wilson, Alastair 2013). However we reckon that those propositions might not give a completely satisfying answer to the question of probability. This is the main challenge in the MWI and an attempt to solve this problem in this essay would be completely out of scope.

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