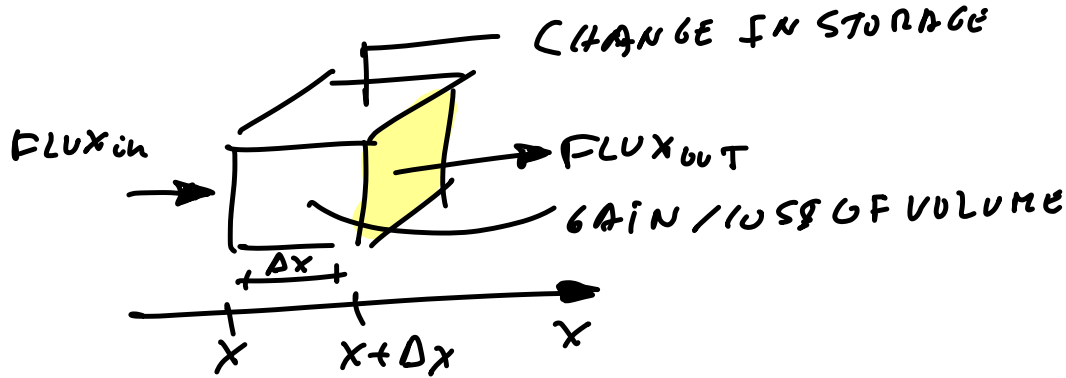


MASS BALANCE 1D



$$V = A \cdot \Delta x$$

$$[t, t + \Delta t]$$

CONSERVATION LAW

$$(\text{FLUX}_{in} - \text{FLUX}_{out}) \cdot A \cdot \Delta t + \underbrace{\text{source} \cdot V \cdot \Delta t}_{\text{sink/sources}} = \text{STORAGE}$$

$$\text{kg/m}^2 \cdot \text{s}$$

$$\text{m}^2 \cdot \text{s}$$

$$\text{kg/m}^3 \cdot \text{s} \cdot \text{m}^3 \cdot \text{s}$$

$$\text{kg}$$

$$[j(x) - j(x + \Delta x)] \cdot A \cdot \Delta t + \sigma \cdot V \cdot \Delta t = \underline{\Phi}(t + \Delta t) - \Phi(t)$$

↑
EXTENSIVE
QUANTITY

Division BY:

$$V = \Delta x \cdot A \quad \Delta t$$

$$\frac{[j(x) - j(x + \Delta x)] A \cdot \Delta t}{\Delta t \cdot \underbrace{\Delta x \cdot A}_V} + \frac{\sigma V \cdot \Delta t}{\Delta t \cdot \underbrace{A \cdot \Delta x}_V} = \frac{\Phi(t + \Delta t) - \Phi(t)}{\underbrace{\Delta x \cdot A \cdot \Delta t}_V}$$

↑
EXTENSIVE
QUANTITY

$$\frac{j(x) - j(x + \Delta x)}{\Delta x} + \sigma = \frac{\Phi(t + \Delta t) - \Phi(t)}{\Delta t}$$

IN THE LIMITS $\Delta t \rightarrow 0$
 $\Delta x \rightarrow 0$

$$\boxed{\frac{-\partial j}{\partial x} + \sigma = \frac{\partial \Phi}{\partial t}}$$

BALANCE EQUATION 3D

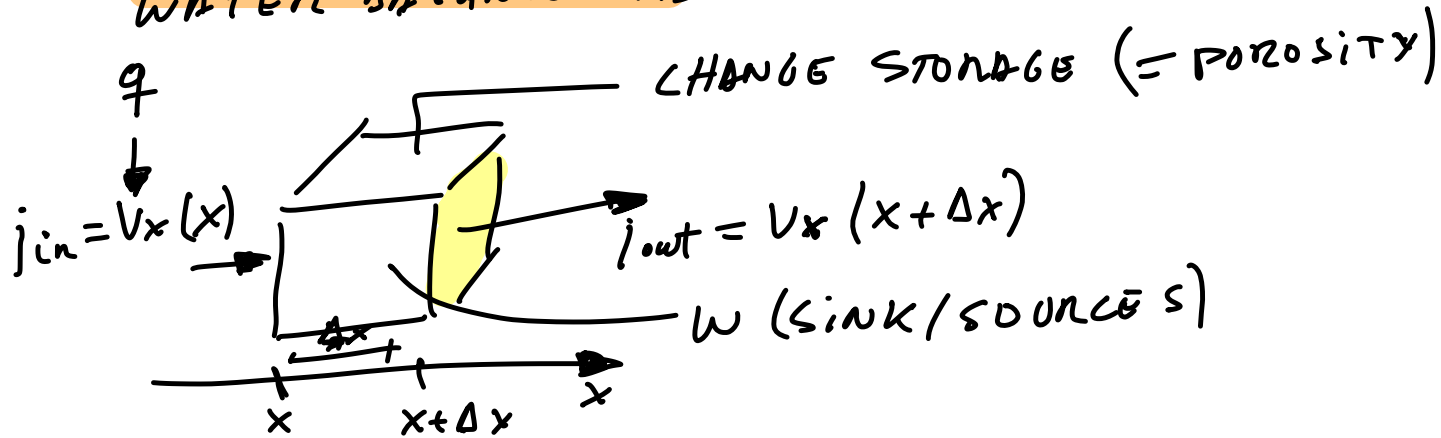
$$-\frac{\partial j_x}{\partial x} - \frac{\partial j_y}{\partial y} - \frac{\partial j_z}{\partial z} + \sigma = \frac{\partial \epsilon}{\partial t}$$

$$-\underset{\uparrow}{\nabla} \cdot \mathbf{j} + \sigma = \frac{\partial \epsilon}{\partial t}$$

DIVERGENCE

OF FLUX VECTOR

WATER BALANCE 1D



CONSERVATION EQUATION

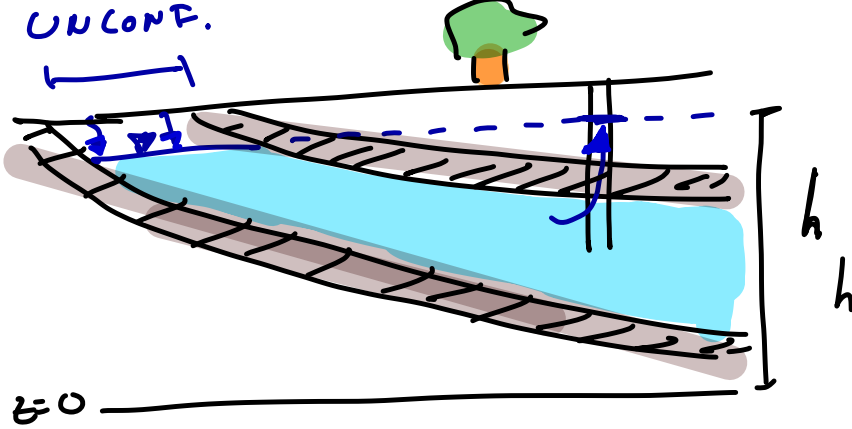
$$[V_x(x) - V_x(x + \Delta x)] A \Delta t + w V \Delta t = V_{water}(t + \Delta t) - V_{water}(t)$$

DIVISION BY Δt $V = \Delta x \cdot A$

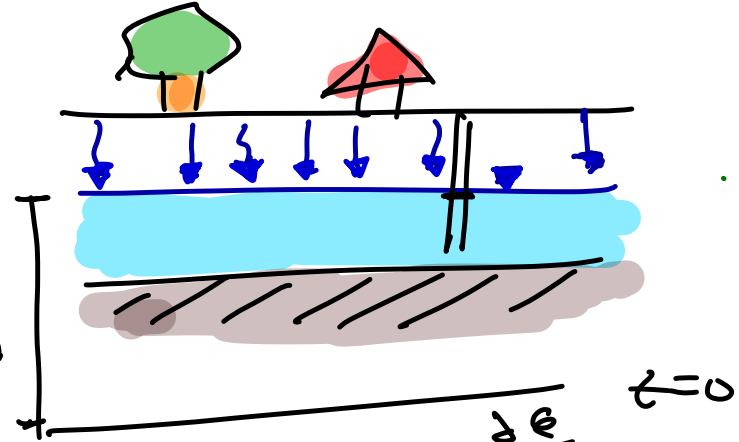
$$-\frac{V_x(x + \Delta x) - V_x(x)}{\Delta x} + w = \frac{\frac{\Delta V_{water}}{V}}{\Delta t} = \frac{\Delta n}{\Delta t}$$

porosity $n = \phi$

CONFINED AQUIFER



UNCONFINED AQUIFER



$$q = v_x = -k \frac{\partial h}{\partial x}$$

HYDRAULIC
CONDUCTIVITY

DARCY
LAW

HYDRAULIC
GRADIENT

$$\frac{\partial v_x}{\partial x} + w = \frac{\partial h}{\partial t} \frac{\partial h}{\partial x}$$

S_0 = STORAGE
SPECIFIC STORAGE

$$\frac{\partial}{\partial x} \left(k \frac{\partial h}{\partial x} \right) + w = S_0 \frac{\partial h}{\partial t} \quad 1D$$

GENERALIZATION 3D

$$-\frac{\partial v_x}{\partial x} - \frac{\partial v_y}{\partial y} - \frac{\partial v_z}{\partial z} + w = S_0 \frac{\partial h}{\partial t}$$

$$\underline{V} = -\underline{K} \cdot \underline{\nabla h}$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = \underline{\nabla \cdot V}$$

DIVERGENCE
IN Darcy
FLUX

WATER BALANCE EQUATION FN 2D

2D CONFINED

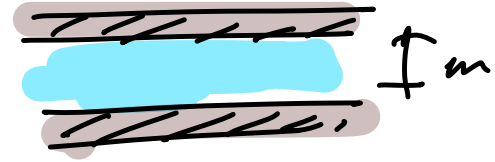
$$\nabla \left(\underset{\downarrow}{T} \nabla h \right) + w = \underset{\uparrow}{S} \frac{\partial h}{\partial t}$$

TRANSMISSIVITY

$$T = K \cdot m$$

STORAGE COEFFICIENT

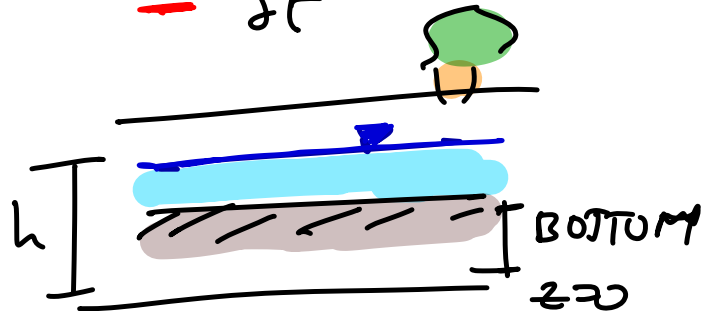
$$S = S_0 \cdot m$$



2D UNCONFINED

$$\nabla \left(K \underbrace{(h - \text{bottom})}_{\substack{\text{SATURATED} \\ \text{THICKNESS}}} \nabla h \right) + w = \underset{\downarrow}{S} \frac{\partial h}{\partial t}$$

$$S = n$$



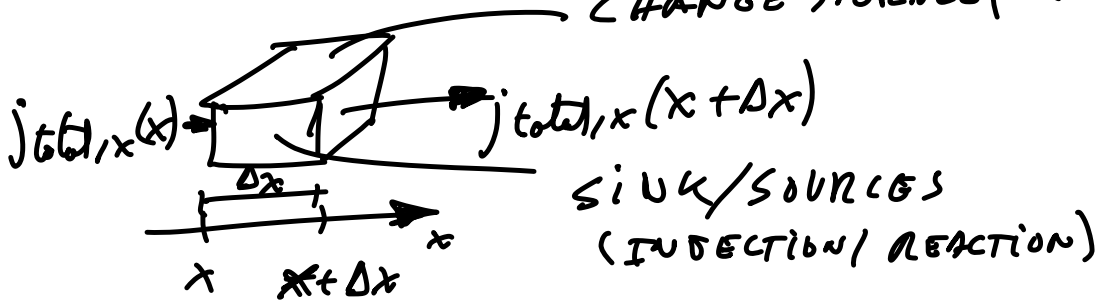
TRANSPORT OF A SOLUTE

- ADVECTIVE FLUX $j_{adv} = v \cdot c$
- DIFFUSIVE FLUX $j_{diff} = -n_e \underset{\substack{\uparrow \\ \text{POROSITY}}}{P_m} \cdot \nabla \underset{\substack{\uparrow \\ \text{CONCENTRATION}}}{c}$ MOLECULAR DIFFUSION
- DISPERSIVE FLUX $j_{disp} = -n_e \underset{\substack{\uparrow \\ \text{DISPERSION COEFFICIENT}}}{D} \cdot \nabla c$

$$j_{total} = j_{adv} + j_{diff} + j_{disp}$$

MASS BALANCE IN

CHANGE STORAGE (DISSOLVED SOLUTE)



CONSERVATION OF MASS

$$[j_{total,x}(x) - j_{total,x}(x+\Delta x)] A \cdot \Delta t - \overbrace{\lambda c \cdot n V \cdot \Delta t}^{\text{SINK (DEGRADATION)}} + \overbrace{W \cdot c_{in} V \cdot \Delta t}^{\text{SOURCE (INFL.)}}$$

$$= m(t + \Delta t) - m(t)$$

DIVISION BY Δt $V = A \cdot \Delta x$

$$\frac{-j_{total,x}(x+\Delta x) + j_{total,x}(x)}{\Delta x} - \lambda n_e \cdot c + W c_{in} = \frac{V_w \cdot c(t+\Delta t) - V_w \cdot c(t)}{V \Delta t}$$

$$-\frac{\partial j_{total,x}}{\partial x} - \lambda \cdot n_e \cdot c + W \cdot c_{in} = \frac{\partial (n_e \cdot c)}{\partial t}$$

$$\frac{V_w}{V} = n$$

$$V = A \cdot \Delta x$$

$$V_w \cdot c = m$$

$$-\frac{\partial (v_x \cdot c)}{\partial x} + \frac{\partial}{\partial x} \left(n_e (D_m + D_L) \frac{\partial c}{\partial x} \right) - \lambda n_e \cdot c + W \cdot c_{in} = \frac{\partial n_e \cdot c}{\partial t}$$

$$\boxed{-\frac{\partial (u_x \cdot c)}{\partial x} + \frac{\partial}{\partial x} \left((D_m + D_2) \frac{\partial c}{\partial x} \right) - \lambda \cdot c + \frac{w \cdot c_{in}}{n_e} = \frac{\partial c}{\partial t}}$$

1D ADVECTION-DISPERSION EQ.

$n_e = \tau \cdot e$

GENERALIZATION TO 3D

$$\frac{\partial (u_x \cdot c)}{\partial x} + \frac{\partial (u_y \cdot c)}{\partial y} + \frac{\partial (u_z \cdot c)}{\partial z} = \nabla \cdot (u \cdot c)$$

$$j_{disp} + j_{diff} = n_e (D_m + \mathbf{D}) \underbrace{\nabla c}_{\frac{\Delta c}{\Delta x}}$$

$$\mathbf{D} = \begin{bmatrix} D_{xx} & \dots \\ \vdots & \ddots \end{bmatrix}$$

$$\boxed{-\nabla \cdot (u \cdot c) + \nabla \cdot \left[(D_m + \mathbf{D}) \nabla c \right] - \lambda \cdot c + \frac{w \cdot c_{in}}{n_e} = \frac{\partial c}{\partial t}}$$

$$\mathbf{D} = \begin{bmatrix} P_L & 0 & 0 \\ 0 & P_T & 0 \\ 0 & 0 & P_T \end{bmatrix}$$

3D