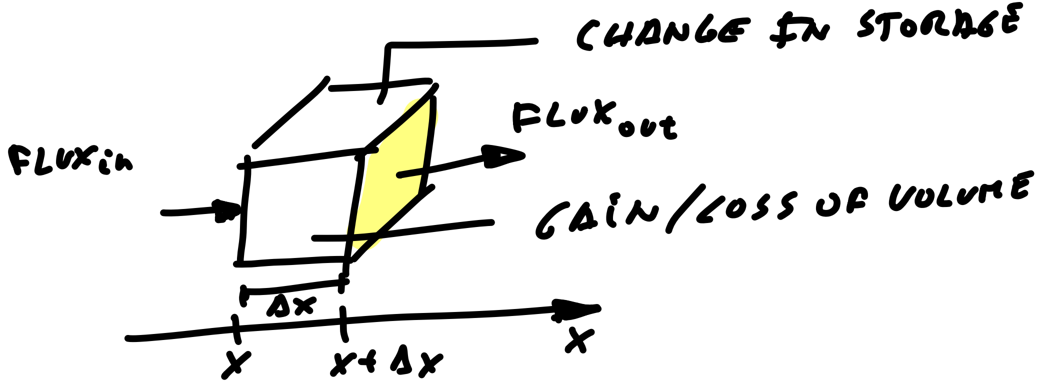
MASS BALANCE 1D



$V = A \cdot \Delta x$

$[t, t+\Delta t]$

CONSERVATION LAW

$$\left(FLUX_{in} - FLUX_{out}\right) A \cdot \Delta t + \underbrace{SOURCE \cdot V \cdot \Delta t}_{SINK/SOURCES} = STORAGE$$

$kg/m^2 \cdot s$ $\quad m^2 \quad s \quad kg/m^3 \cdot s \quad m^3 \quad s \quad kg$

$$[j(x)-j(x+\Delta x)]\cdot A\cdot\Delta t+\sigma\cdot V\cdot\Delta t=\underset{\substack{\uparrow\\ \text{EXTENSIVE QUANTITY}}}{\Phi}(t+\Delta t)-\Phi(t)$$

DIVISION BY:

$V=\Delta x\cdot A \qquad \Delta t$

$$\frac{[j(x)-j(x+\Delta x)]\,A\cdot\Delta t}{\Delta t\cdot\underbrace{\Delta x\cdot A}_{V}}+\frac{\sigma V\cdot\Delta t}{\Delta t\cdot\underbrace{\Delta x\cdot A}_{V}}=\frac{\Phi(t+\Delta t)-\Phi(t)}{\underbrace{\Delta x\cdot A}_{V}\cdot\Delta t}$$

INTENSIVE QUANTITY

$$\frac{j(x)-j(x+\Delta x)}{\Delta x}+\sigma=\frac{\varepsilon(t+\Delta t)-\varepsilon(t)}{\Delta t}$$

IN THE LIMITS $\quad \Delta t\to 0$, $\quad \Delta x\to 0$

$$\boxed{-\frac{\partial j}{\partial x}+\sigma=\frac{\partial\varepsilon}{\partial t}}$$

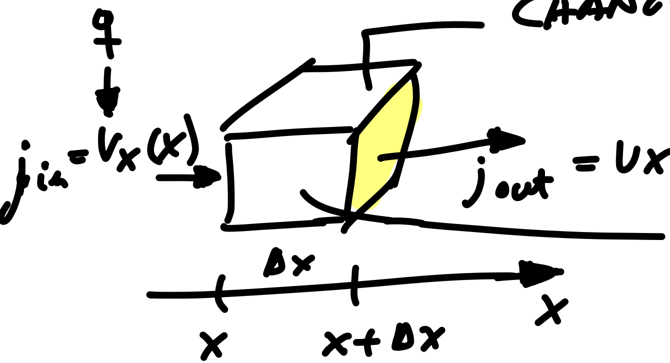
BALANCE EQUATION 3D

$$-\frac{\partial j_x}{\partial x} - \frac{\partial j_y}{\partial y} - \frac{\partial j_z}{\partial z} + \sigma = \frac{\partial \epsilon}{\partial t}$$

$$-\nabla \cdot j + \sigma = \frac{\partial \epsilon}{\partial t}$$

DIVERGENCE
OF FLUX VECTOR

WATER BALANCE 1D



CONSERVATION EQUATION

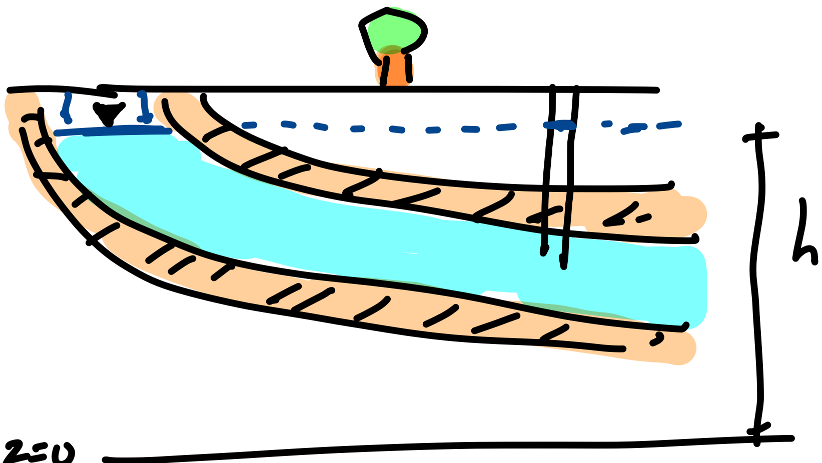
$$[v_x(x) - v_x(x+\Delta x)] \cdot A \cdot \Delta t + w \cdot V \cdot \Delta t = V_{water}(t+\Delta t) - V_{water}(t)$$

DIVISION BY Δt ; $V = \Delta x \cdot A$

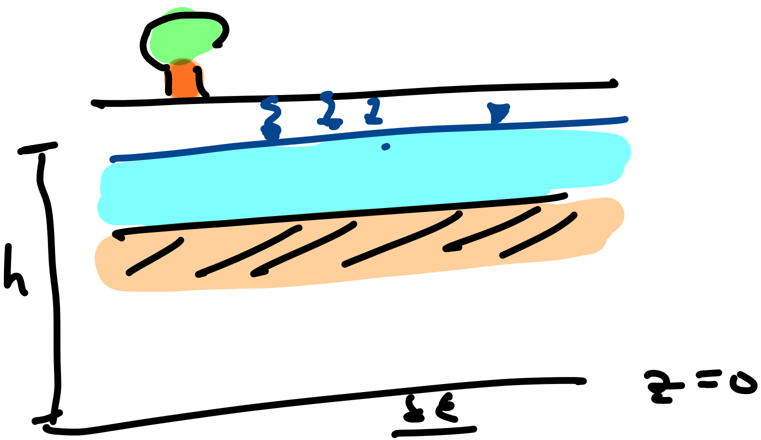
$$-\frac{v_x(x+\Delta x) - v_x(x)}{\Delta x} + w = \frac{\frac{\Delta V_{water}}{V}}{\Delta t} = \frac{\Delta n}{\partial t}$$

POROSITY $n = \phi$

CONFINED AQUIFER



UNCONFINED AQUIFER



$$q = \boxed{v_x = -K \frac{\partial h}{\partial x}}$$

DARCY LAW

K: HYDRAULIC CONDUCTIVITY

$\frac{\partial h}{\partial x}$: HYDRAULIC GRADIENT

$$-\frac{\partial v_x}{\partial x} + w = \overbrace{\boxed{\frac{\partial \varepsilon}{\partial h}} \frac{\partial h}{\partial t}}^{\frac{\partial \varepsilon}{\partial t}}$$

S_0 = STORATIVITY

SPECIFIC STORAGE

$$\boxed{\frac{\partial}{\partial x}\left(K \frac{\partial h}{\partial x}\right) + w = S_0 \frac{\partial h}{\partial t}}$$ 1D

GENERALIZATION 3D

$$-\frac{\partial v_x}{\partial x} - \frac{\partial v_y}{\partial y} - \frac{\partial v_z}{\partial z} + w = S_0 \frac{\partial h}{\partial t}$$

$$V = -K \underbrace{\nabla h}_{\frac{\partial h}{\partial x}}$$

$$\underbrace{\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = \nabla \cdot V}_{}$$

DIVERGENCE IN DARCY FLUX

$$V = q = m/d$$

DARCY VELOCITY

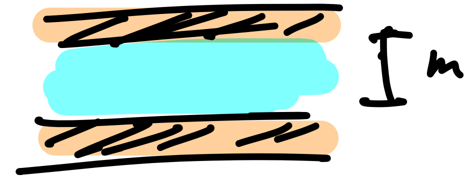
DARCY VELOCITY

$$\boxed{u = \frac{V}{n = \phi}}$$

VELOCITY

WATER BALANCE EQUATION IN 2D

2D CONFINED



$$\nabla(T\nabla h) + w = S\frac{\partial h}{\partial t}$$

TRANSMISSIVITY

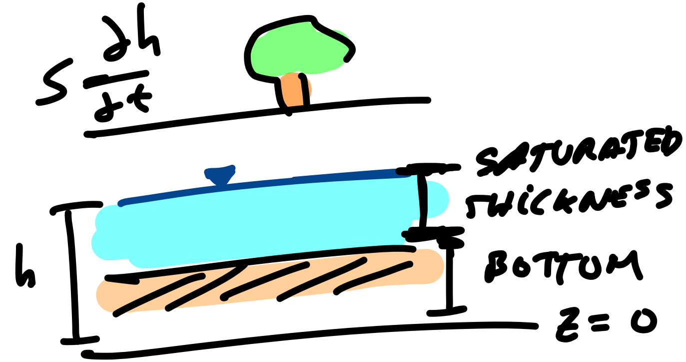
$T = K \cdot m$

STORAGE COEFFICIENT

$S = S_0 \cdot m$

2D UNCONFINED

$$\nabla\left(K(h-\text{bottom})\nabla h\right) + w = S\frac{\partial h}{\partial t}$$



$S = n_e$

S_y = SPECIFIC YIELD

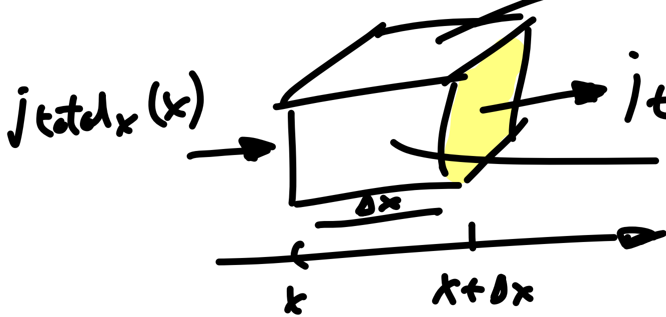
TRANSPORT OF A SOLUTE

- ADVECTIVE FLUX $j_{adv} = v \cdot c$
- DIFFUSIVE FLUX $j_{diff} = -n_e \cdot D_m \cdot \nabla c$ MOLECULAR DIFFUSION
- DISPERSIVE FLUX $j_{disp} = -n_e \mathbf{D} \cdot \nabla c$ DISPERSION COEFFICIENT

$$j_{total} = j_{adv} + j_{diff} + j_{disp}$$

MASS BALANCE 1D

CHANGE STORAGE (DISSOLVED SOLUTE)



SINK/SOURCES (INJECTION; REACTION)

CONSERVATION OF MASS

$$\left[j_{total,x}(x) - j_{total,x}(x+\Delta x)\right] A \cdot \Delta t - \underbrace{\lambda \cdot c \cdot h \cdot V \cdot \Delta t}_{\text{SINK (DEGRADATION)}} + \underbrace{w \cdot c_{in} \cdot V \cdot \Delta t}_{\text{SOURCE (INS.)}}$$

$$= m(t+\Delta t) - m(t)$$

DIVISION BY Δt; $V = A \cdot \Delta x$

$$-\frac{j_{total,x}(x+\Delta x) - j_{total,x}(x)}{\Delta x} - \lambda h_e \cdot c + w\, c_{in} = \frac{\frac{V_w \cdot c(t+\Delta t) - V_w \cdot c(t)}{V}}{\Delta t}$$

$$-\frac{\partial \boxed{j_{total}},x}{\partial x} - \lambda \cdot h_e \cdot c + w \cdot c_{in} = \frac{\partial (h_e \cdot c)}{\partial t}$$

$\frac{V_w}{V} = h$

$V = A \cdot \Delta x$

$V_w \cdot c = m$

$$-\frac{\partial (v_x \cdot c)}{\partial x} + \frac{\partial}{\partial x}\left(h_e \left(D_m + D_L\right) \frac{\partial c}{\partial x}\right) - \lambda \cdot h_e \cdot c + w \cdot c_{in} = \frac{\partial h_e \cdot c}{\partial t}$$

$$\boxed{-\frac{\partial (u_x \cdot c)}{\partial x} + \frac{\partial}{\partial x}\left(\left(D_m + D_L\right)\frac{\partial c}{\partial x}\right) - \lambda \cdot c + \frac{w\, c_{in}}{n_e} = \frac{\partial c}{\partial t}}$$

$n_e = \text{cte}$

1D ADVECTION - DISPERSION EQ.

GENERALIZATION TO 3D

$$\frac{\partial (u_x \cdot c)}{\partial x} + \frac{\partial (u_y \cdot c)}{\partial y} + \frac{\partial (u_z \cdot c)}{\partial z} = \nabla \cdot (\mathbf{u} \cdot c)$$

$$j_{disp} + j_{diff} = n_e \left(D_m \cdot \mathbf{D}\right) \cdot \nabla c \qquad \mathbf{D} = \begin{bmatrix} D_{xx} & \cdots & \cdots \\ \vdots & \ddots & \\ \vdots & & \ddots \end{bmatrix}$$

$$\boxed{-\nabla \left(\mathbf{u} \cdot c\right) + \nabla \left(D_m + \mathbf{D}\right) - \lambda \cdot c + \frac{w\, c_{in}}{n_e} = \frac{\partial c}{\partial t}} \qquad \mathbf{D} = \begin{bmatrix} D_L & 0 & 0 \\ 0 & D_T & 0 \\ 0 & 0 & D_T \end{bmatrix}$$

3D