

Tidal and wave power

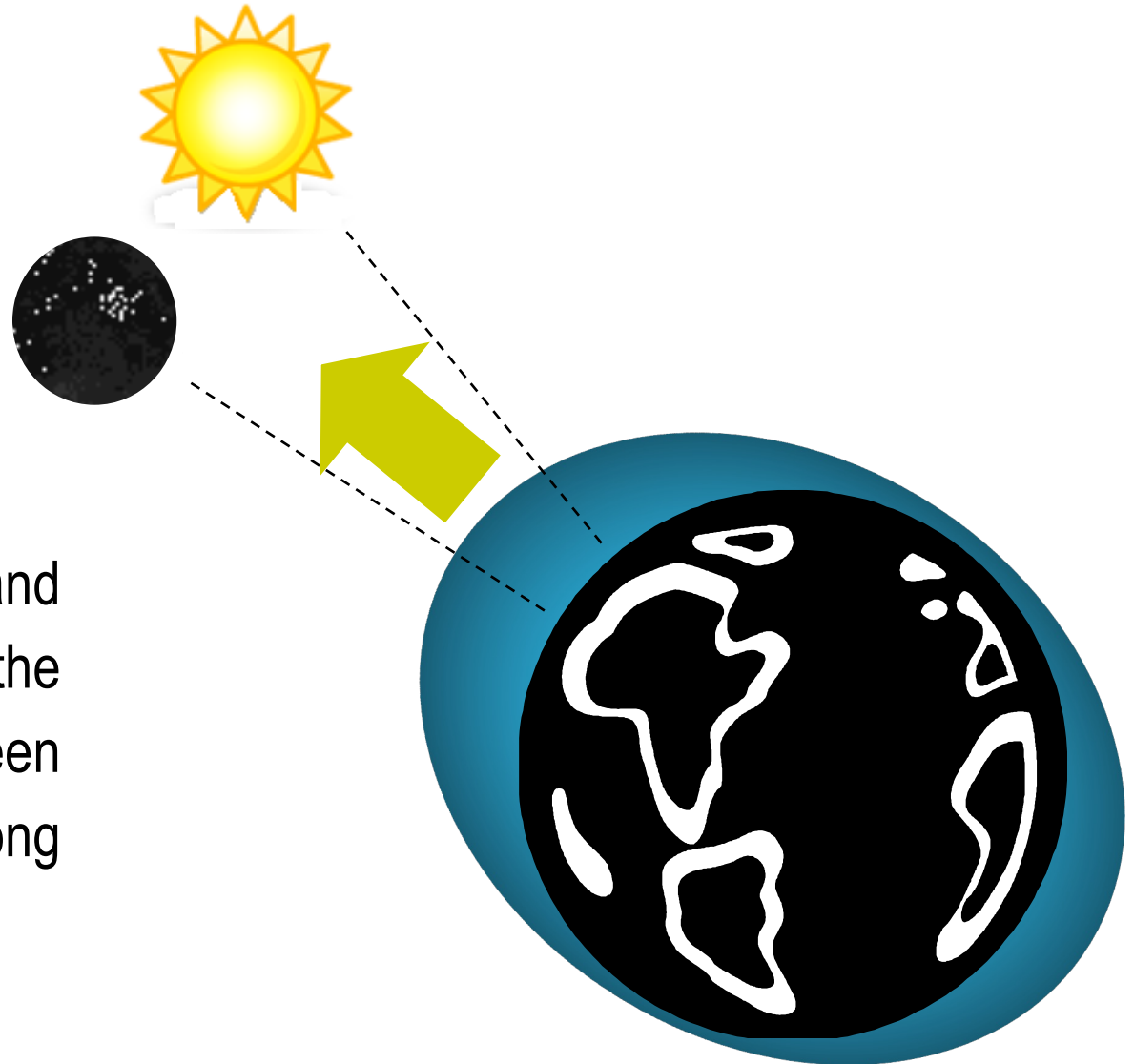
Learning objectives

- Explain the origin of tidal power, how we can exploit it, its advantages and its (real) potential
- Compare tidal turbines with wind turbines
- Explain the origin of wave power, how it is exploited, and its (real) potential

Tidal power

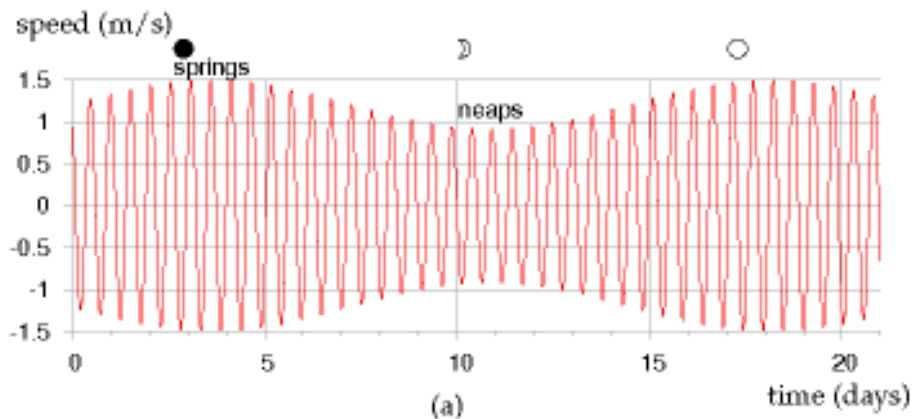
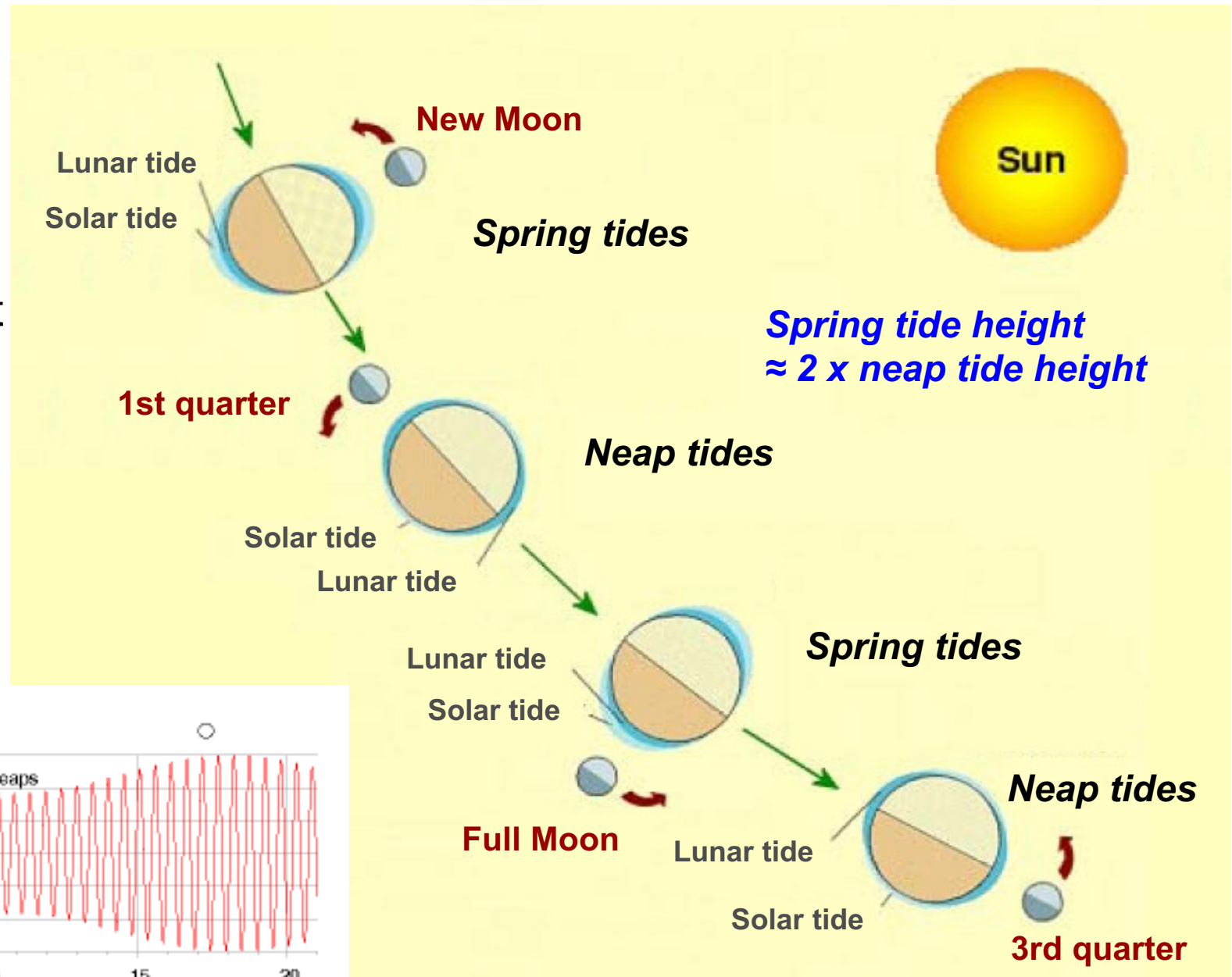
Tides result from the **sum** of gravitational pulls of the **Moon** and **Sun** on the surface of the spinning Earth.

The shape of the shore and adjacent seafloor affects the **tidal range** (difference between high and low tides) along specific coast-lines.



The origin of tides

Earth orbits around its axis in 24h, while the Moon orbits around Earth in 28 days. Someone standing at the coast at not very high latitudes therefore sees 2 high tides and 2 low tides during one day (once every $6\frac{1}{4}$ h).



Explanations

- when Sun and Moon **align** to 'pull' **together** on the Oceans' masses, tides are **highest** and called '**spring**' (New Moon, Full Moon)
- when Sun and Moon **misalign** 'pulling' on the Oceans' masses at 90° angle (1st & 3rd quarter), tides are **lowest** and called '**neap**'.
- Earth orbits around its axis in 24 h, while the Moon orbits around Earth in 28 days; i.e. for 1 terrestrial day, the Moon is almost 'stationary' (it moves on 1/28th of its orbit, or by 360/28 = 13°)

⇒ a person standing at the coast at not very high latitudes sees **2 high tides and 2 low tides during one day** (once every ≈6¼ h).

(it is ≈6¼ h and not 6 h because the Moon is not 'stationary': for the person to find the Moon in the same position he has to execute 1 Earth day rotation (2π rad) + x rad where

$$t[day] = \frac{x[rad]}{\omega_{Moon}[rad/day]} = \frac{(2\pi + x[rad])}{\omega_{Earth}[rad/day]} \Rightarrow t = \frac{28}{27} day = 24.89h$$

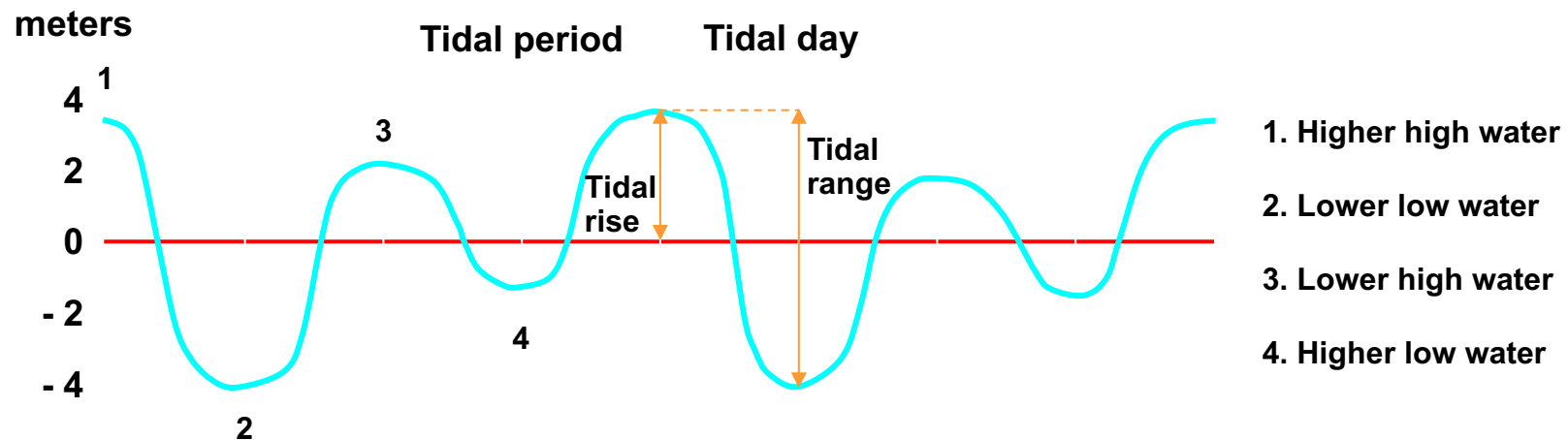
$$\Rightarrow 1 \text{ tide} = \frac{24.89}{4} h = 6.22h = 6h13m20s$$

Tide mechanism



The Earth-Moon system in actual proportions

‘Tides’ = twice-daily rises and falls of water level relative to land



The **Coriolis** force makes the tidal range differ at West banks and East banks (i.e. different at the European Atlantic Coast and the American Atlantic Coast).

Gravitational forces of Moon and Sun

Newton's Law of gravitation : $F = G \frac{M_1 M_2}{d^2}$

G = $6.674 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$, universal gravitational constant

M_{Sun} = $1.989 \times 10^{30} \text{ kg}$ (333'000 x Earth's mass)

M_{Earth} = $5.9736 \times 10^{24} \text{ kg}$

M_{Moon} = $7.3477 \times 10^{22} \text{ kg}$ (1.2% of Earth's mass)

$d_{\text{Earth-Sun}}$ = $1.496 \times 10^{11} \text{ m}$ (150 million km)

$d_{\text{Earth-Moon}}$ = $3.843 \times 10^8 \text{ m}$ (0.384 million km)

→ $F_{\text{Sun-Earth}}$ = $3.54 \times 10^{22} \text{ N}$

→ $F_{\text{Moon-Earth}}$ = $1.98 \times 10^{20} \text{ N}$

Ratio of the forces of gravity : 'Sun-Earth' = 179 * 'Moon-Earth'

It's the difference of gravity between 'near' and 'far' side of the Earth that counts

Earth diameter = 12.756×10^6 m

Near side of Earth is 12756 km closer to both the Sun and the Moon than its **far** side.

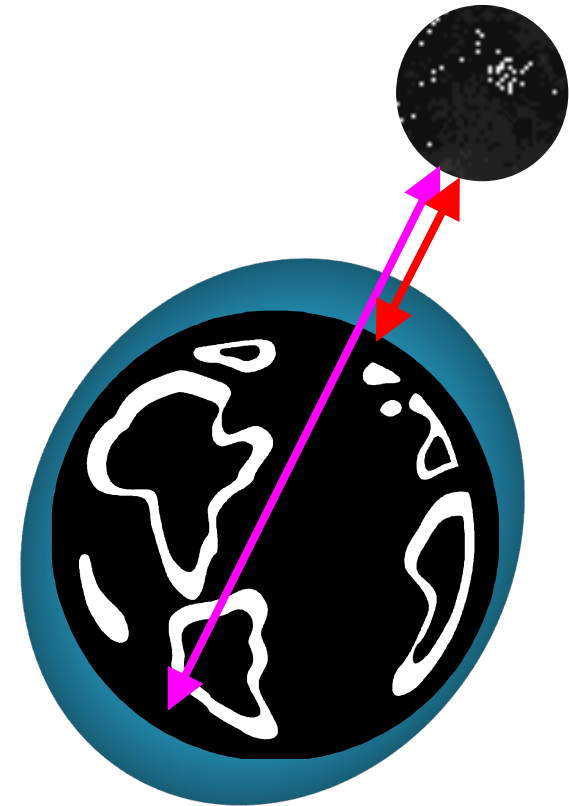
$d_{\text{Earth-Sun}} = 1.496 \times 10^{11}$ m

$\rightarrow \Delta d_{\text{near-far}} = 0.00856\% \rightarrow \Delta F_{\text{near-far}} = 0.0173\%$

(square of distance dependence)

$d_{\text{Earth-Moon}} = 3.843 \times 10^8$ m

$\rightarrow \Delta d_{\text{near-far}} = 3.3\% \rightarrow \Delta F_{\text{near-far}} = 7\%$




The acceleration force due to gravity of the Sun is 179 times that of the Moon. The **Sun's tidal effect** (the difference in force between far and near side of the Earth) is then $179 * 0.0173\% = 3.1\%$, compared to the **Moon's** (7%).

Hence total tidal force is for $2/3^{\text{rd}}$ due to the Moon, and for $1/3^{\text{rd}}$ due to the Sun.

Exploitation of tidal power

1. Based on **height difference**:

- dams built in estuaries or coastal bays with important **tidal range**
- so-called 'lagoons' built out in the sea

$$P = \rho_{H_2O} \cdot \dot{V} g \Delta h$$


2. Based on the moving **water stream** :


- ◆ different tidal stream turbine designs
- ◆ = like 'underwater' **wind turbines**

$$P = \frac{1}{2} \rho_{H_2O} \cdot C_P \cdot A v_{H_2O}^3$$

20 to 50%, depending on design



typically 1-2 m/s (ocean current)



Height difference: dams in bays

To be a practical source for electricity generation, the **tidal range** in a coastal area must typically be **at least 4 m**.

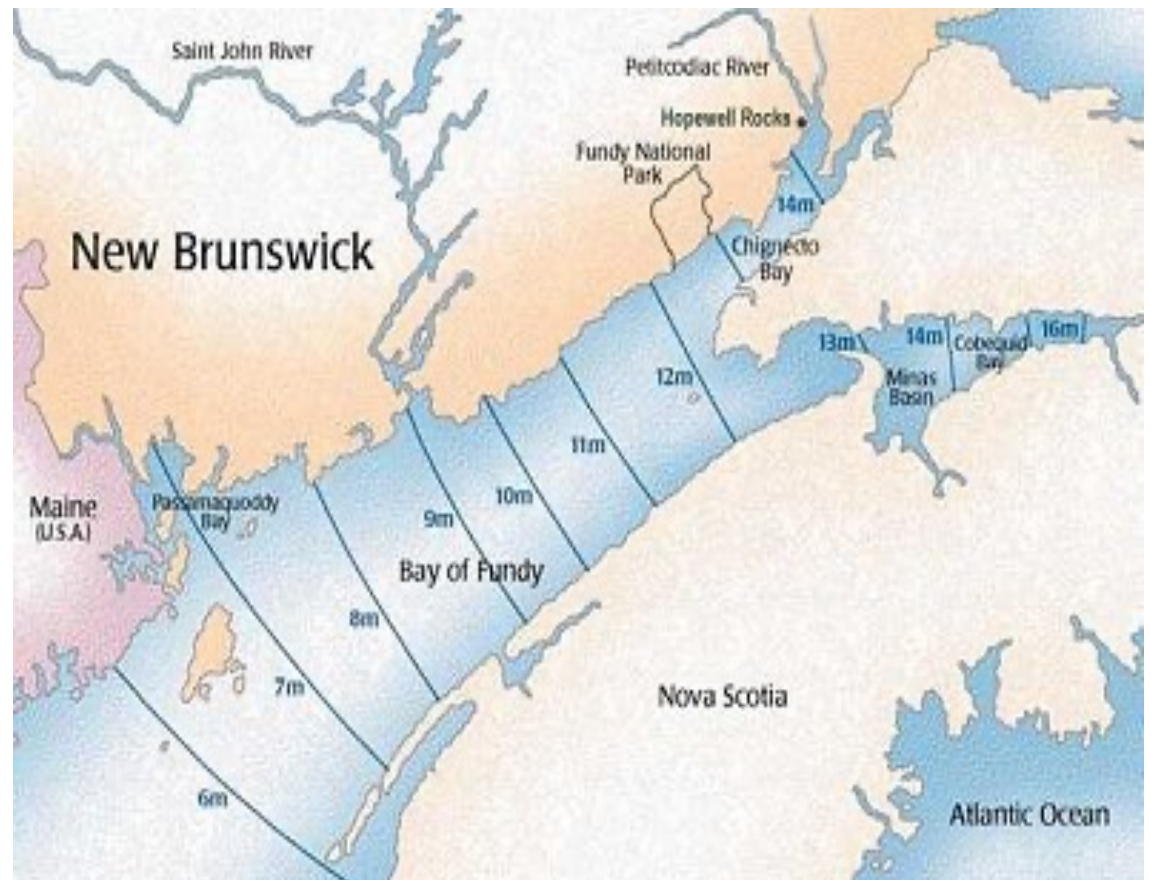


Only a limited number of suitable tidal power station locations have been identified worldwide.

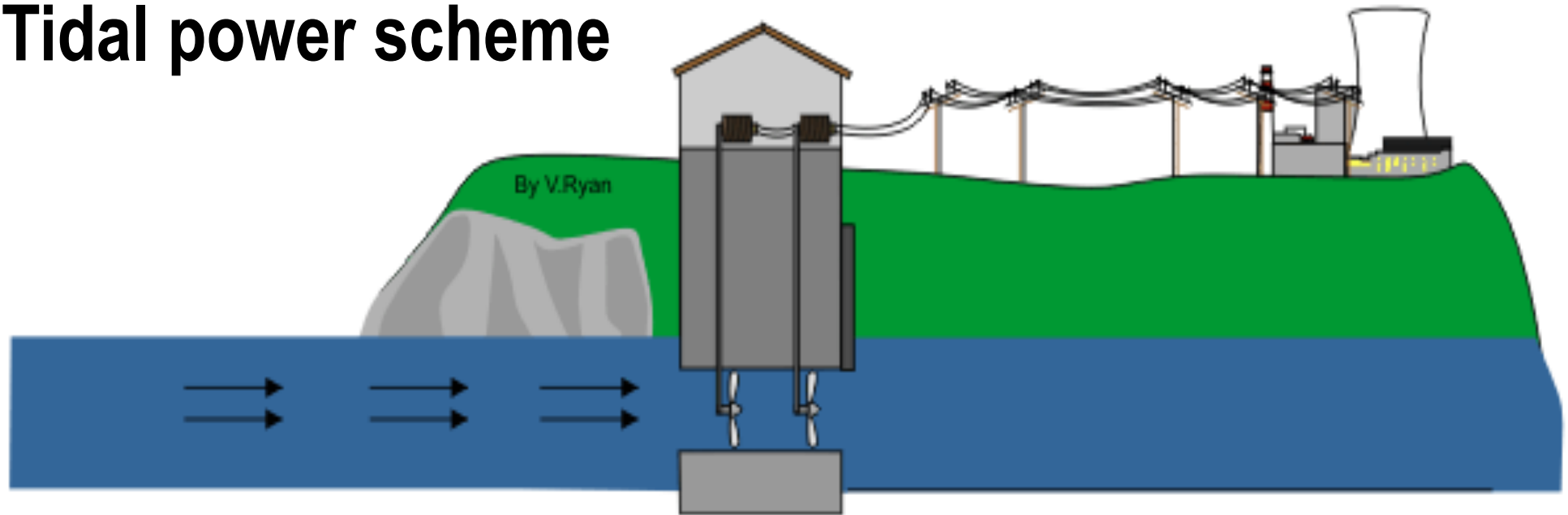
Theoretical world power in tides : 3000 GW

Actually exploitable : 120 – 400 GW

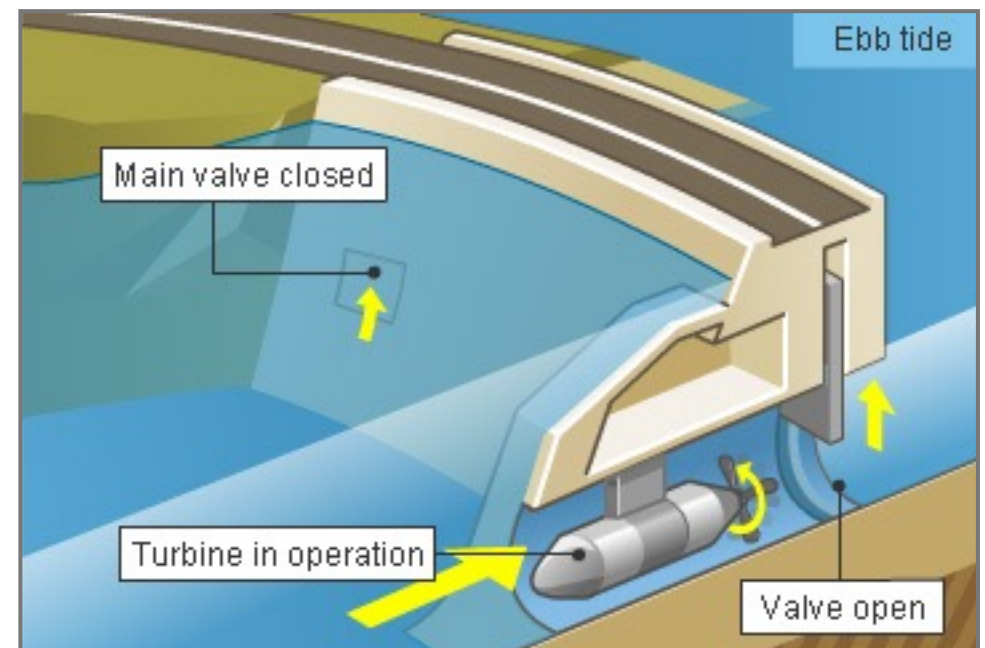
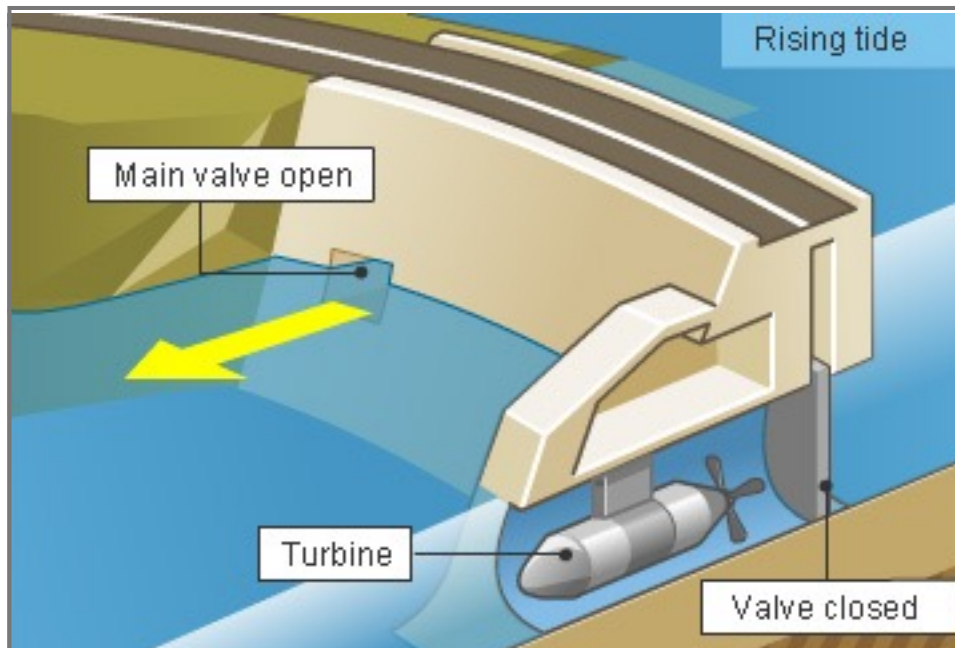
Actually running : 0.5 GW



Tidal power scheme



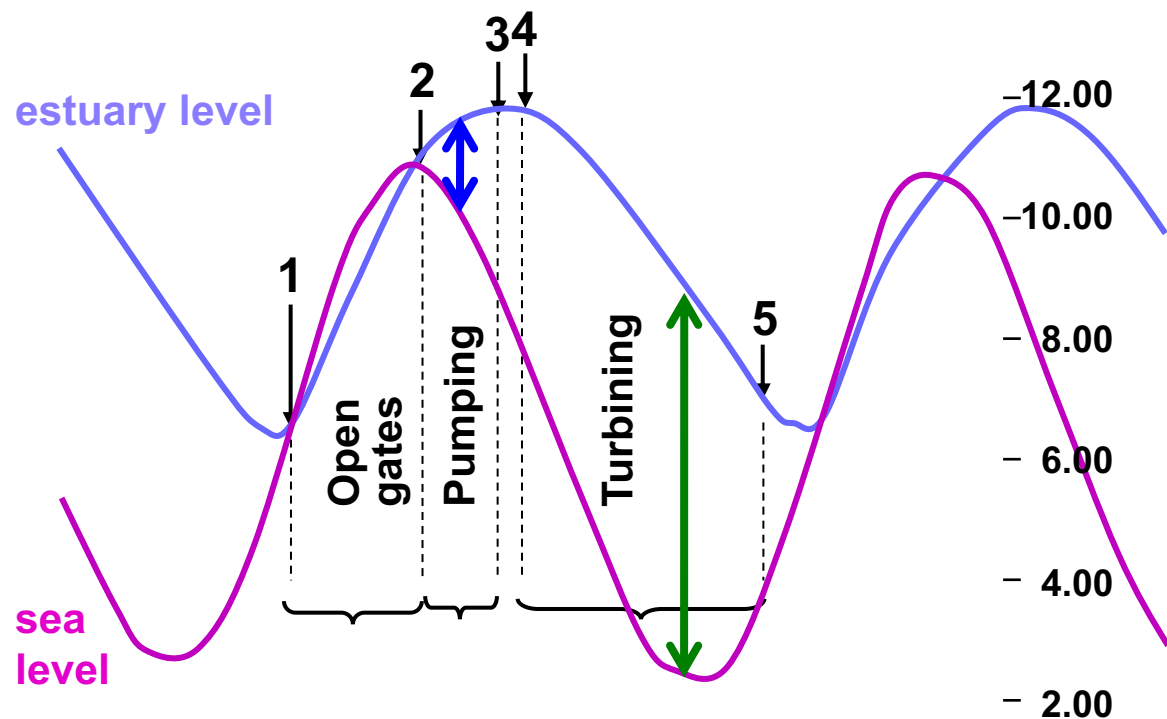
TIDE COMING IN



One-way / two-way exploitation

The variation of the sea level resulting from the tide movements can be exploited to produce electricity in two different ways: **simple effect** and **double effect**

Simple effect: for middle tides or neap tides

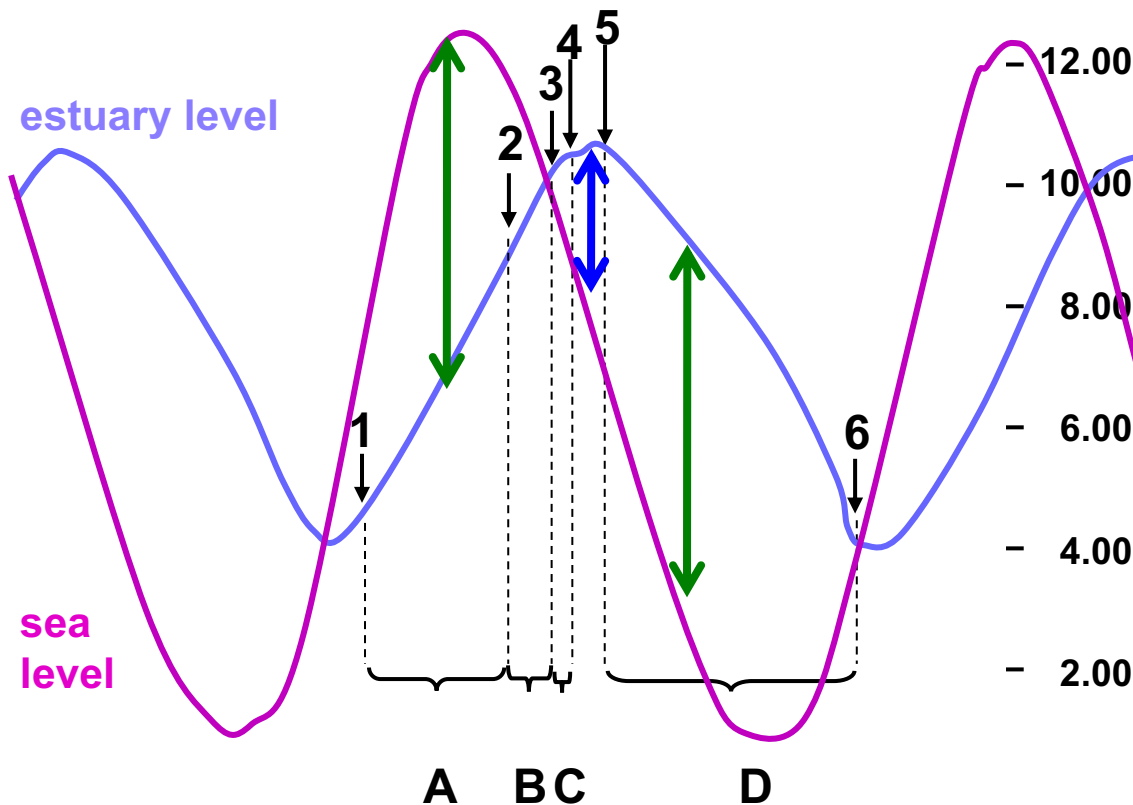


5 successive transitions
per tide cycle

Power invested in **pumping** (small Δh) is only a fraction of the gain in **turbinning** (larger Δh)

Double effect

For important tides, the use of the sea and estuary variation levels is based on a double effect cycle with 6 transitions per cycle.



Double effect:

- A** Inverse **turbinning**
- B** Open gates
- C** **Pumping**
- D** Direct **turbinning**

Example : La Rance (F)



Largest and oldest commercial tidal power plant in operation

Construction: 1960 - 1967

Dam: 330 m long

Tide pool reservoir: 22 km²

Average tidal range: 8 m (max 13.5 m)

24 Bulb turbines (5.4 m diameter), rated at 10 MW_e each (**240 MW_e** total),
connected to the 225 kV French transmission network

La Rance (F) tidal power plant

The axial flow turbines allow generation on both floods and ebbs of the tide (**double effect**); they are also designed to **pump** water into the basin.

Number of units: 24 x 10 MW

Installed power: **240 MW_e**

Net power: **544 GWh/yr**

→ avg power: **60 MW_e**
(taking year-round operation)

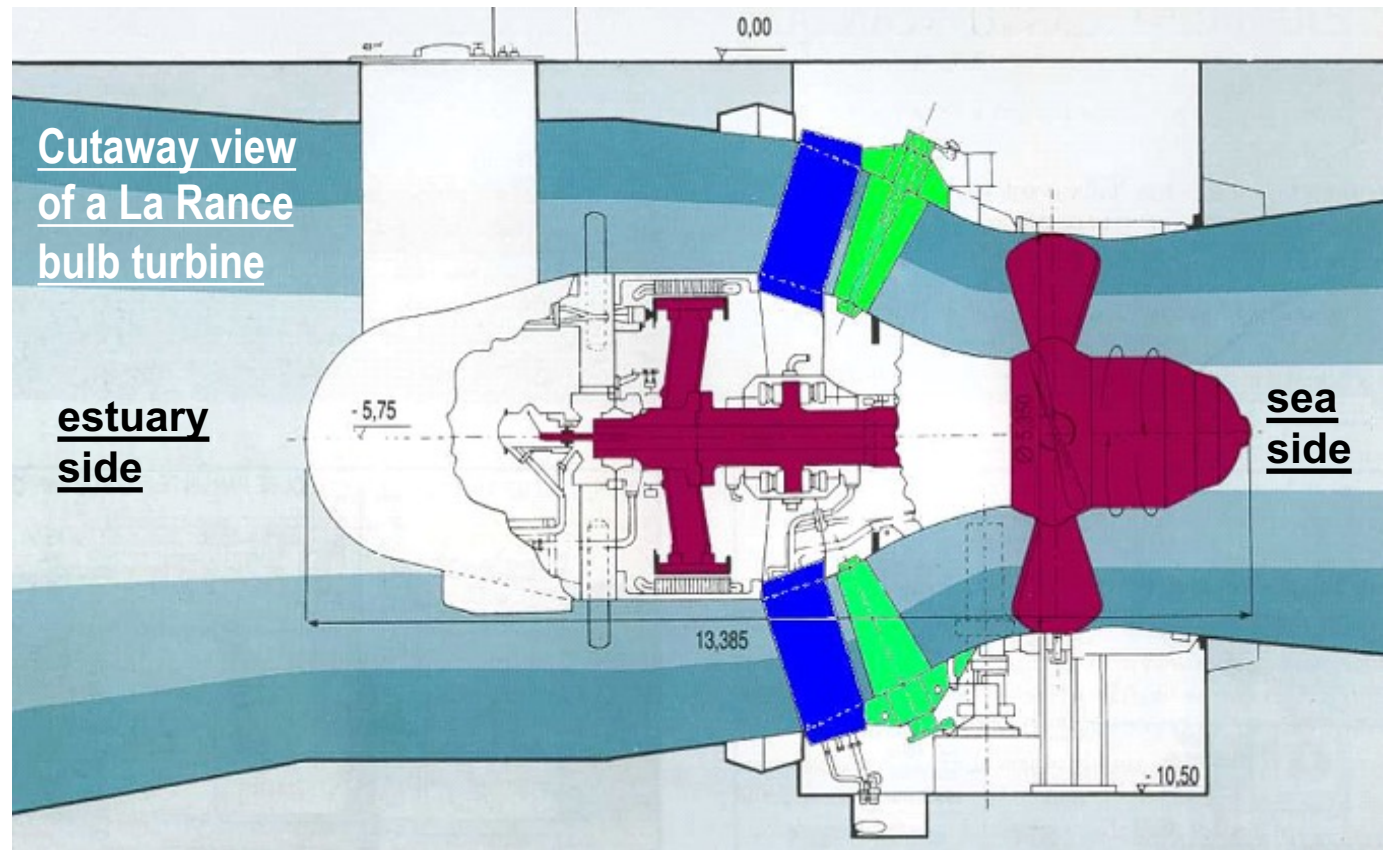
Turbines:

horizontal Kaplan

4 blades, blade inclination
-5° to + 35°

Alternators: rated power:
10 MW/unit

Output voltage: 3.5 kV

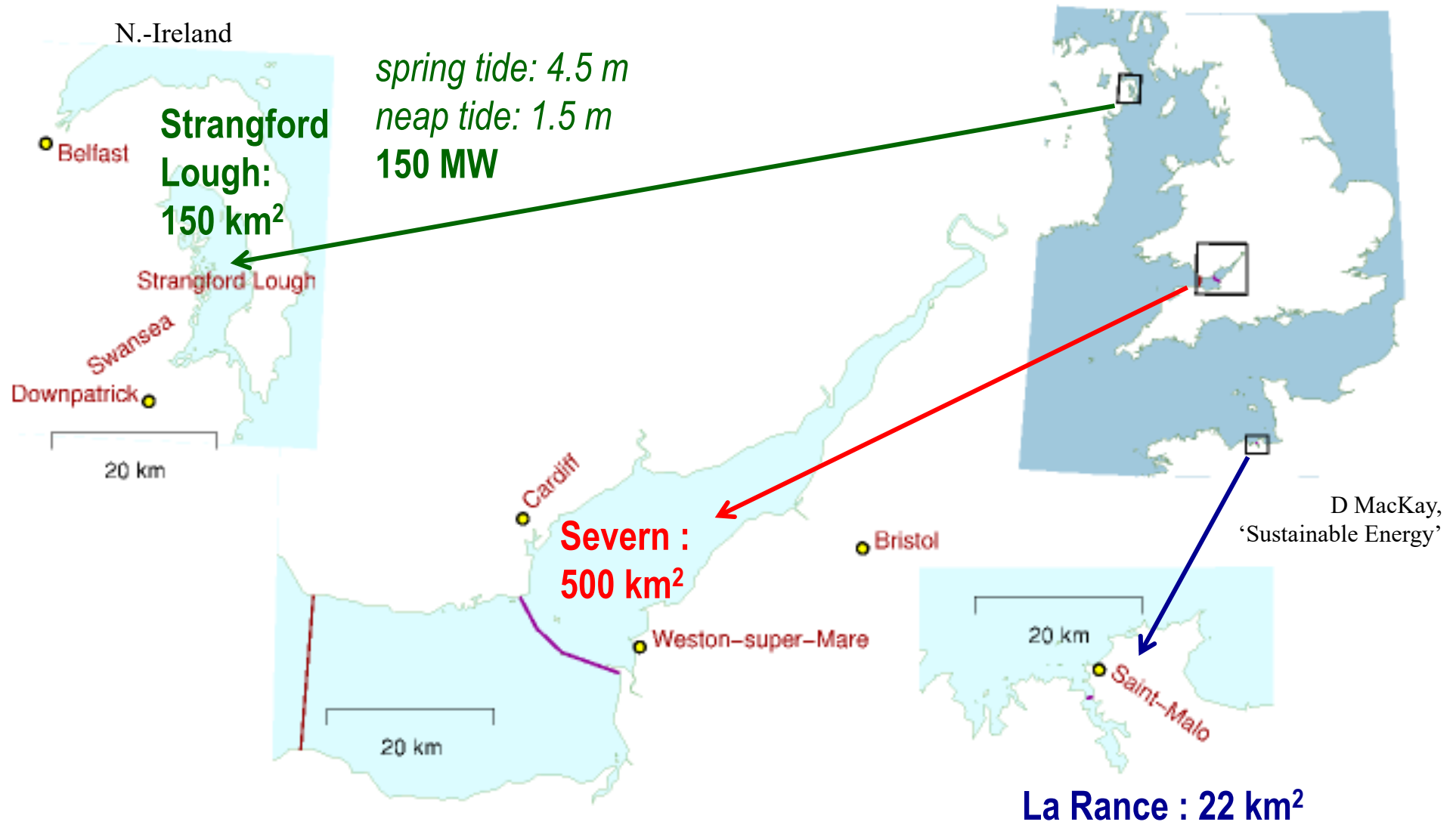


For comparison: Swiss electricity ~ 57 TWh ≈ 100 x La Rance

Example : UK

Other plants than La Rance (F) are operated in Canada and Russia, and others could be installed in certain other areas, e.g. the UK (Severn Barrage).

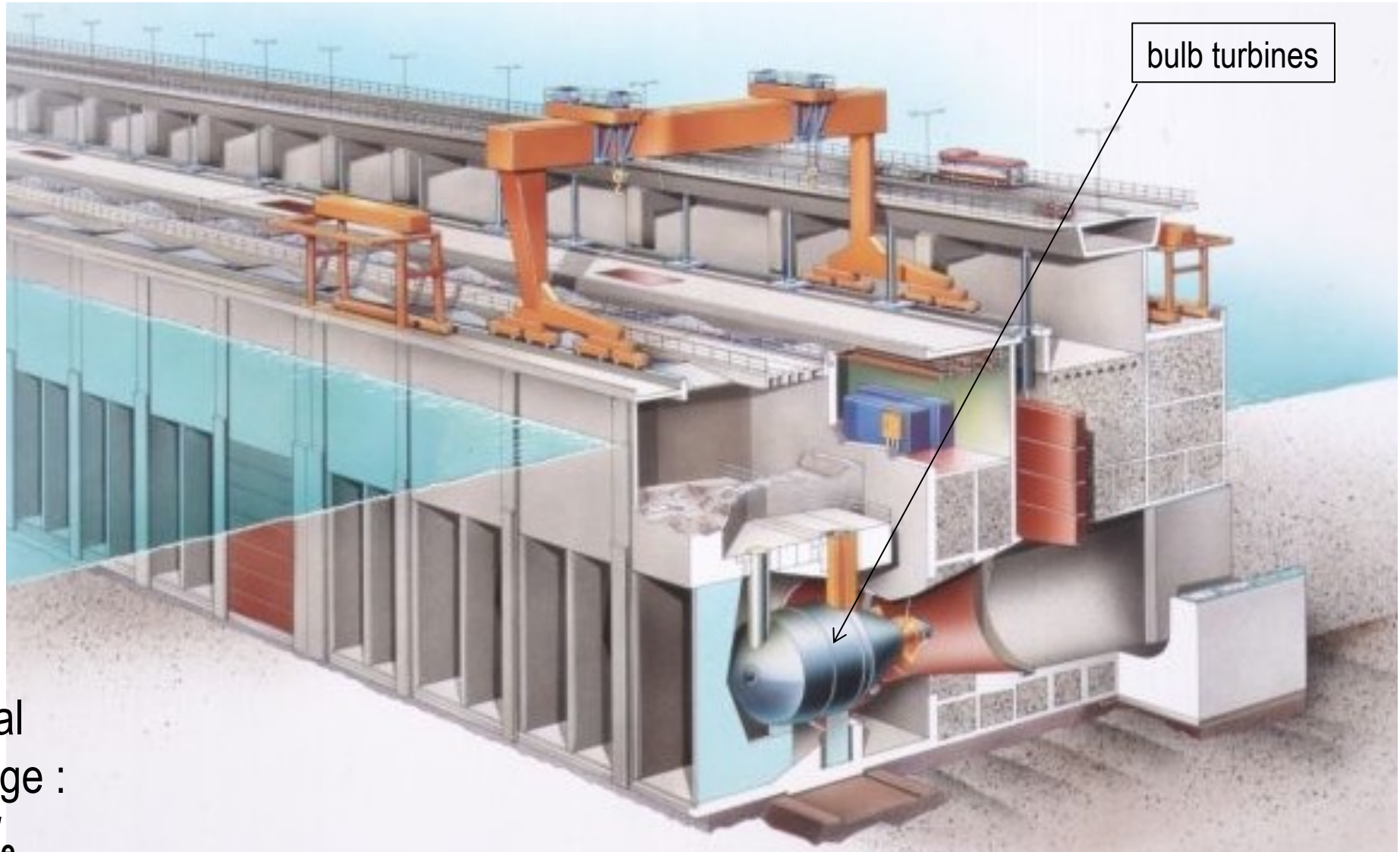
(Potential of 120 GW)



spring tide: 11 m \Rightarrow max **14.3 GW_e**

neap tide: 6 m \Rightarrow max **3.9 GW_e**

artist's impression of hypothetical Severn Barrage, UK



Annual
average :
2 GW_e

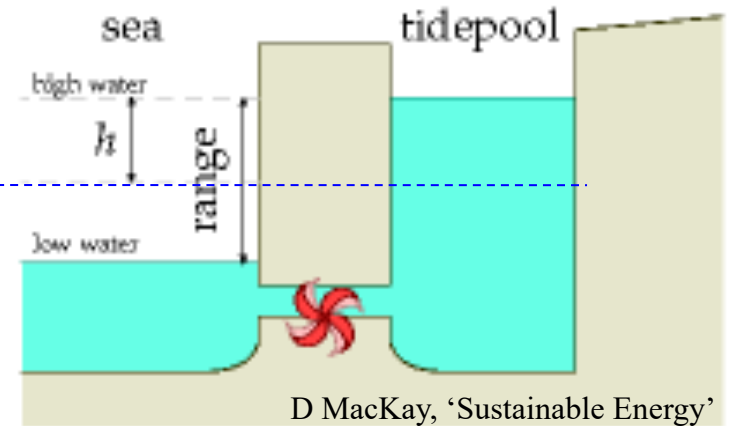
\rightarrow 17 TWhe / yr = 5% of UK electricity

Estimate of tidal range power

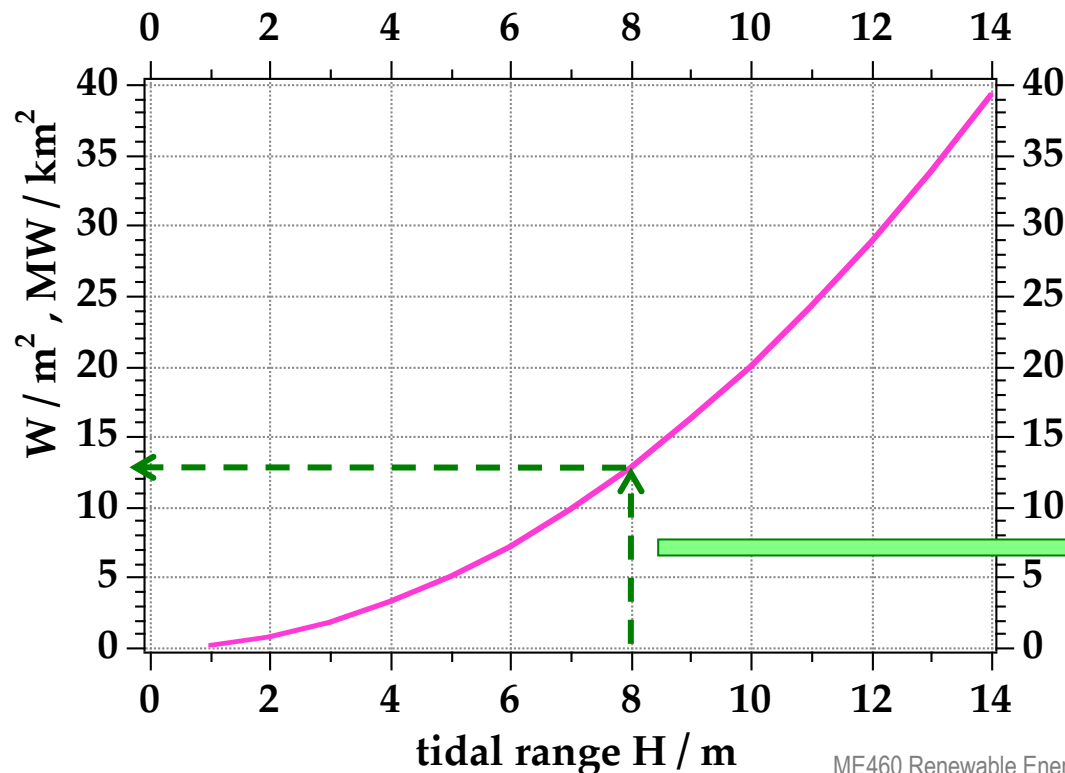
Potential energy: $E_{pot} = mgh$ mass centre

Mass per unit area (m^2)
in the tide pool :

$$m = \rho(2h)$$



Power per unit area: $P \left(\frac{W}{m^2} \right) = \frac{E_{pot}}{time} = \frac{\rho(2h)gh}{6.22 \text{ hours}} = \frac{19620 \left[\frac{J}{m^2} \right]}{22400 [s]} h^2 = 0.88 \left(\frac{H}{2} \right)^2 = 0.22 H^2 \left[\frac{W}{m^2} \right]$



turbining
efficiency
 $\approx 90\%$

$$0.2H^2 \left[\frac{MW_e}{km^2} \right]$$

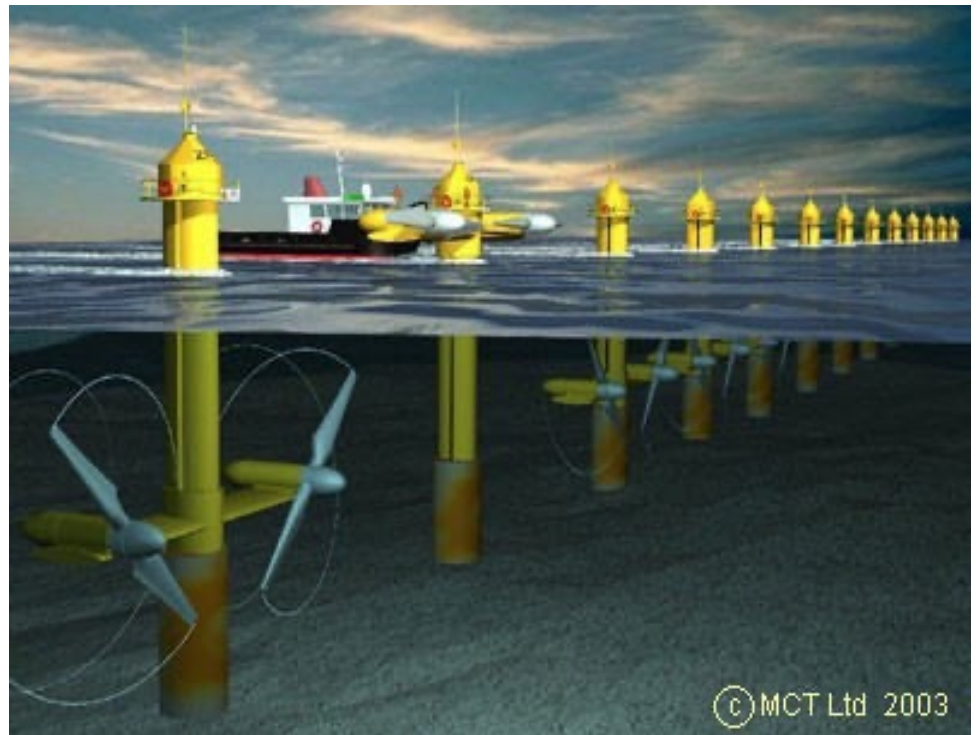
La Rance : 8 m \Rightarrow 12 MW_e / km²
reservoir 22 km² \Rightarrow \approx 260 MW_e

Tidal power 'on-stream'

Tidal stream generators draw energy from underwater **currents** in much the same way that **wind generators** are powered by the wind.

The much higher **density** of water vs. air gives significant power to a single generator.

The technology is at the early stage of development and requires significantly more research.



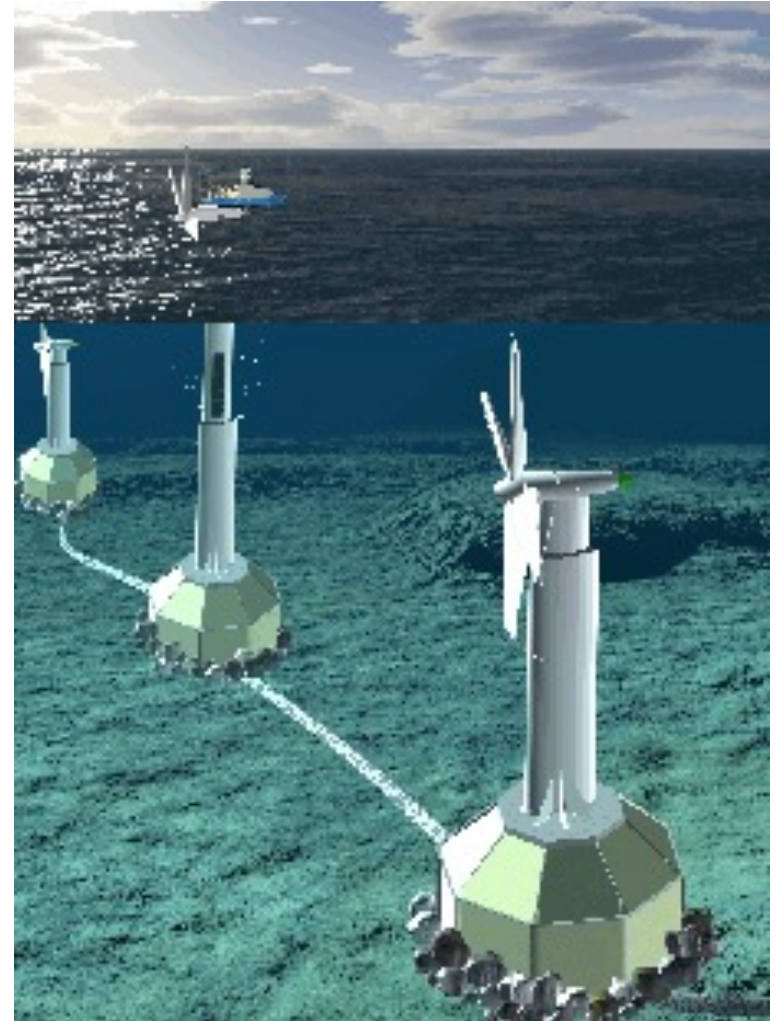
Tidal power 'on-stream' : Swan-turbine

Particular type of tidal stream turbine

Blades are connected directly to the electrical generator, without a gearbox.

A "gravity base" large concrete block holds it to the seabed.

The blades are **fixed pitch** (more reliable), rather than actively controlled.

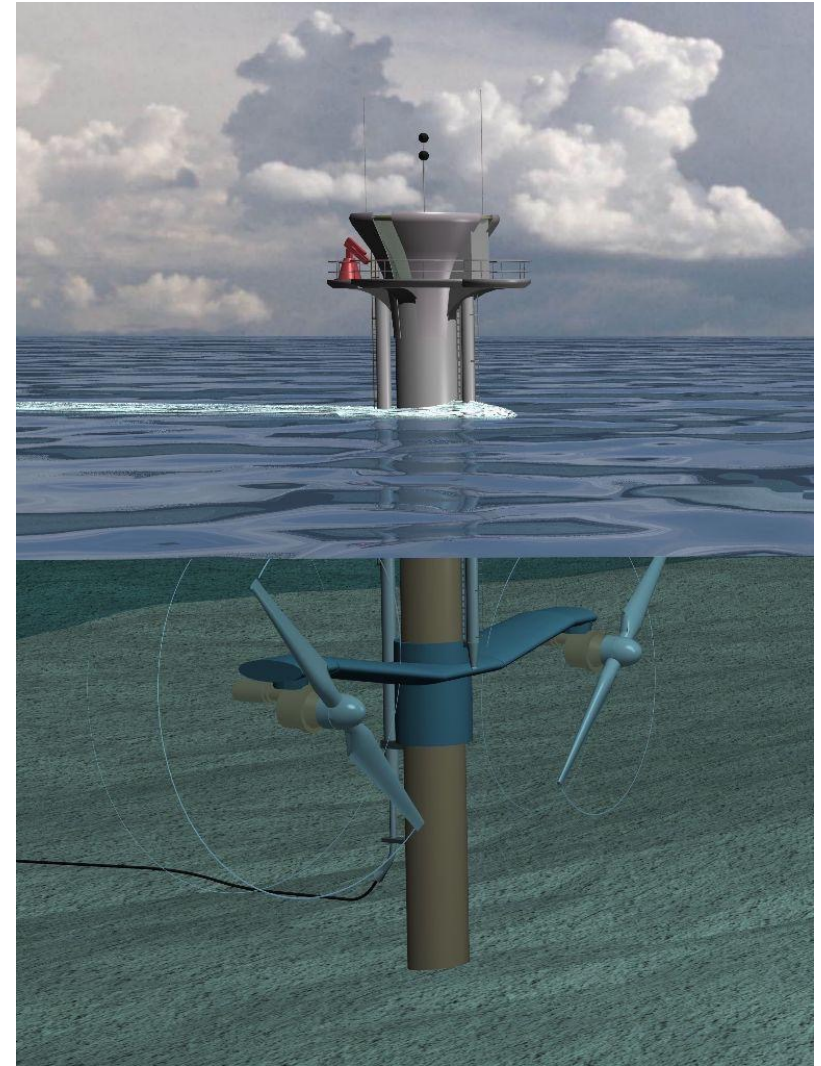


Tidal power 'on-stream' : sea-flow propeller

<http://www.youtube.com/watch?v=lzc9-V9DSew&feature=related>

Several prototypes have shown promise; in the UK, a **300 kW** Seaflow marine current propeller type turbine was first tested in 2003, followed by a **1.2 MW** SeaGen unit in Strangford Lough (Northern Ireland) in 2008.

'Blue Energy' (CAN) plans to install large array tidal current devices in 'tidal fences', based on a vertical axis turbine design



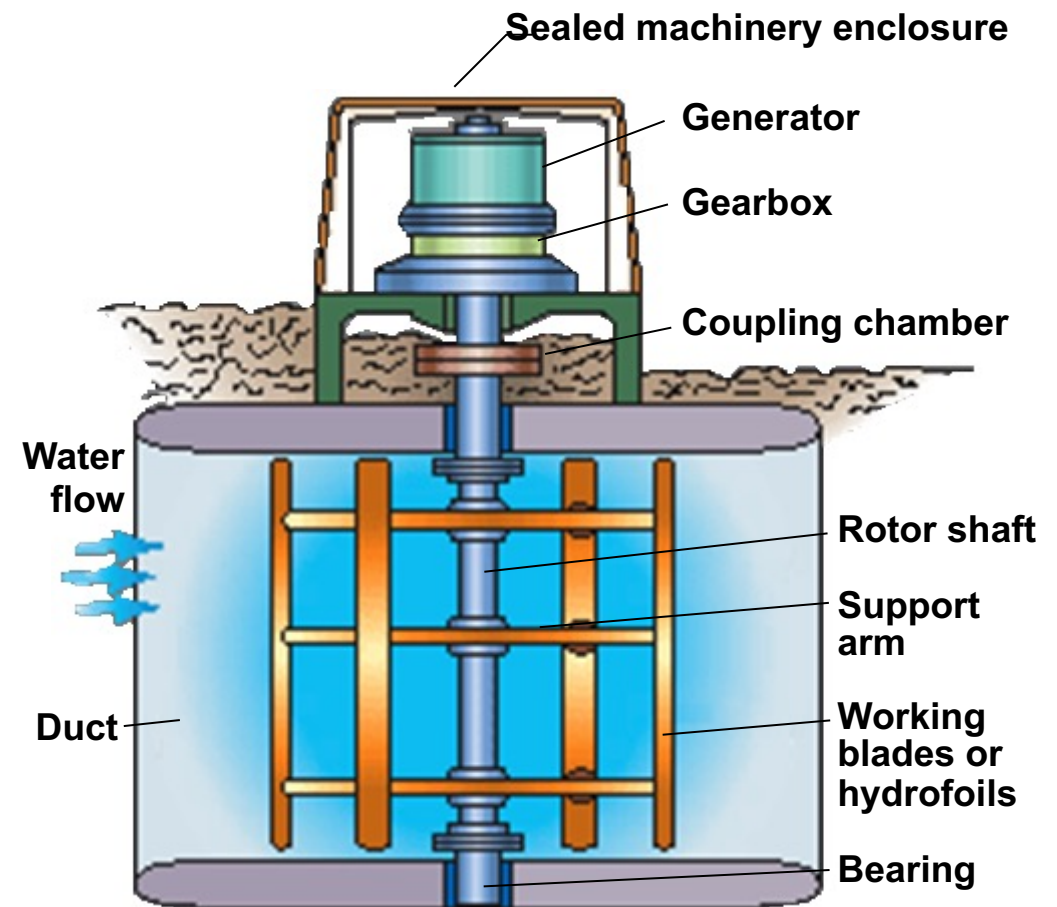
'Darrieus' tidal turbine

Four fixed vertical blades connect to a rotor shaft that drives an integrated gearbox/generator.

The turbine sits in a concrete marine casing; the casing directs water through the turbine and supports the generator and other machinery.

Water flowing either way spins the turbine, letting the unit generate power during a tide's ebb and flood.

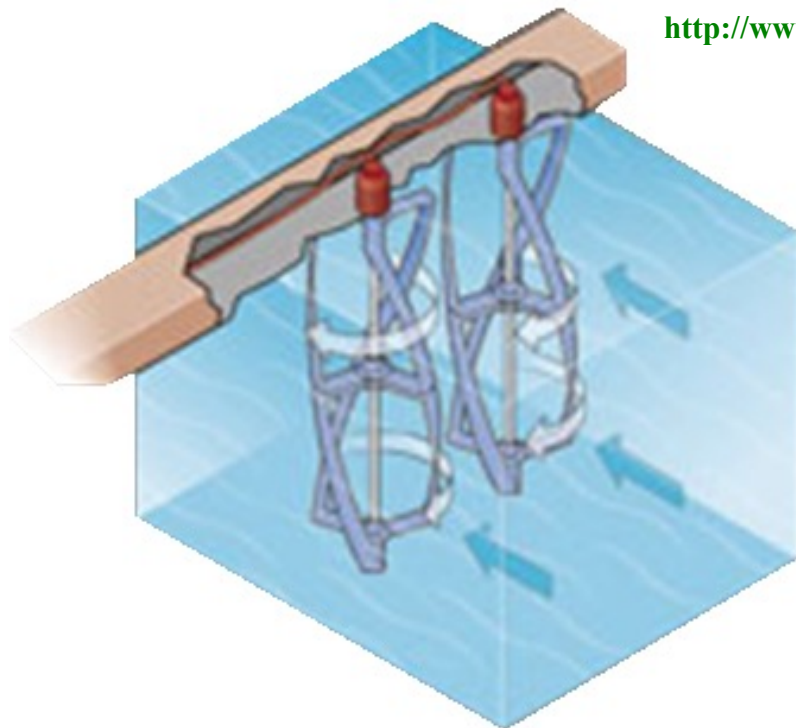
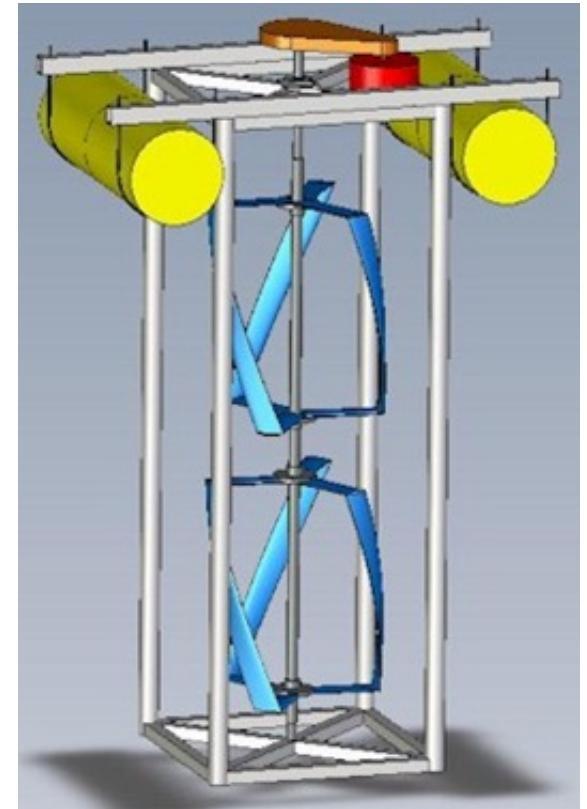
Darrieus-type device



'Gorlov' helical tidal turbine

The turbine rotates regardless of the direction of the water current.

The Gorlov turbine captures 35% ($=C_p$) of the water energy, compared to 23% for a straight Darrieus turbine and 20% for a conventional turbine.



<http://www.youtube.com/watch?v=zWu5auKHJzQ>

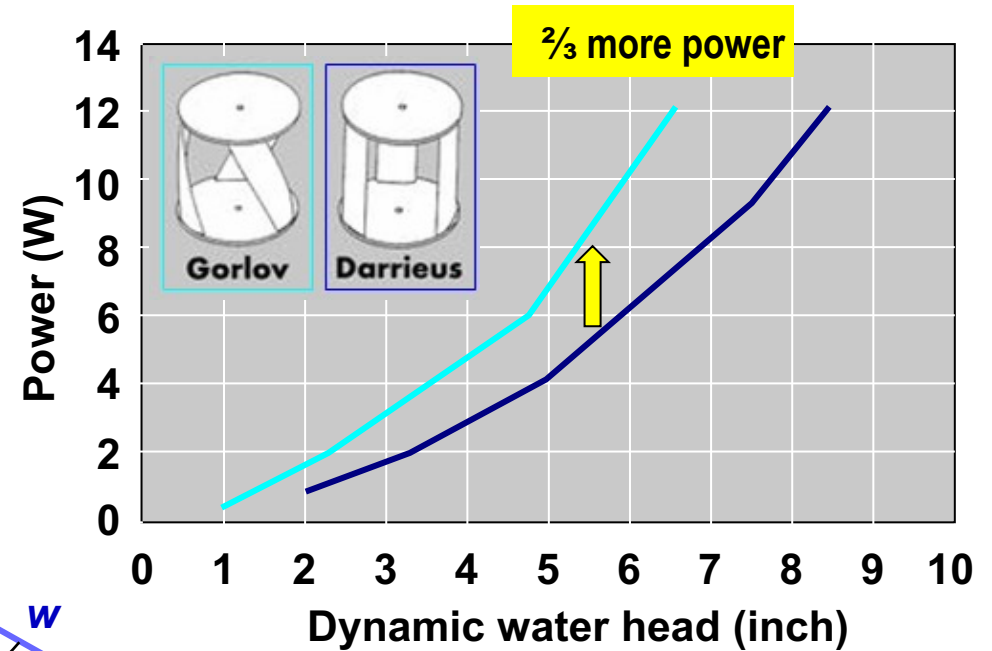
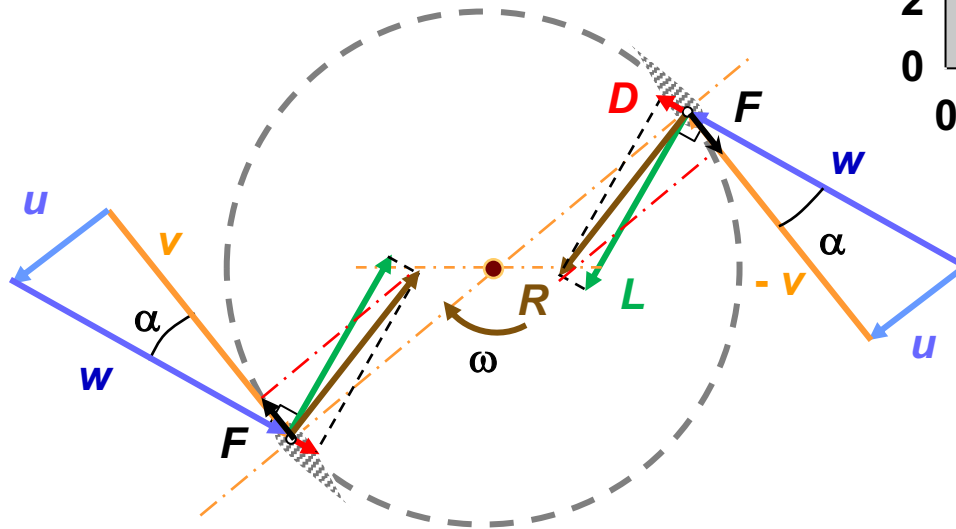


http://en.wikipedia.org/wiki/Gorlov_helical_turbine

Relation to wind turbines

v : linear blade speed
 U : water flow speed
 w : relative speed
 α : attack angle
 L : lift
 D : drag

} speed triangle



Propulsion force:

$$F = L \cdot \sin \alpha - D \cdot \cos \alpha$$

Estimate of tidal power (UK)



D MacKay, 'Sustainable Energy'

We want to estimate the power extractable from an underwater turbine farm that occupies a certain (sub)surface of the ocean.

In 1st approximation, we use the formula

$$P \left[\frac{W}{m_{Turbine_area}^2} \right] = \frac{1}{2} \rho_{H_2O} \cdot C_P \cdot A v_{H_2O}^3$$

assuming $C_P \approx 35\%$ and where v can be 0.5 m/s to 5 m/s (typically : 1 m/s).

Remark : 1 "knot" = 0.5 m/s

The turbines should be spaced apart by minimum 5 diameters to their neighbours, i.e. each turbine would occupy an area of $25 \cdot D^2$

$$= 100 \cdot A / \pi$$

(since $A = \pi \cdot D^2 / 4$)

This leads to the simplified expression:

$$P \left[\frac{W}{m^2_{ocean-subsurface}} \right] = \frac{0.5 \cdot 1000 \cdot 0.35 \cdot A \cdot v^3}{\frac{100 \cdot A}{\pi}} = 5\pi \cdot 0.35 \cdot v^3 \approx 5.5v^3$$

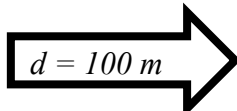
This is **substantial power** ! 

Placing a sea-turbine farm over an area of 1 km² (e.g. 100 rows by 100 columns of turbines (10'000 turbines!) of 4 m² each, D = 2 m, spaced 5D = 10 m apart) in a location where **ocean current** is 2 m/s (7 km/h), will generate 44 MW_e of electrical power (4.4 kW_e per turbine). (The equivalent **wind** turbine of 4 m², even at avg v³ = 1000 m³/s³, would generate <1 kW_e, i.e. **5 times less**)

Speed v (m/s)	Power W / m ²
0.5	0.7
1	5.5
2	44
3	150
4	350
5	687

It is moreover argued that **tidal current** in shallow water (i.e with depth *d* of the same magnitude as the length of the wave (λ), typically 100 m) can be derived to be (cf. « Wave-power » chapter):

$$P_{tidal} \left[\frac{W}{m} \right]_{shallow_water} = \frac{1}{2} \rho g^{3/2} d^{1/2} y_0^2 \approx 153'630 \cdot y_0^2$$



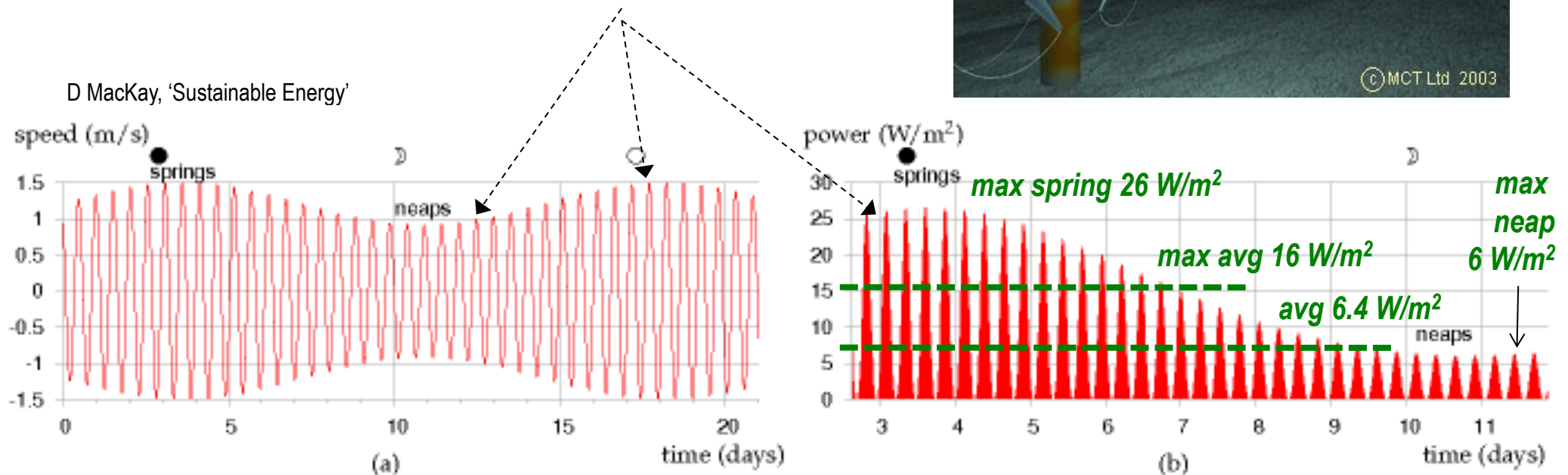
 $\approx \lambda$ wave amplitude, typically 1 m (waveheight = 2y₀)

This is again **substantial power**, some **5 times bigger than wave-power** (which, again, is **wind-derived**, not tidal).

y ₀ , Wave amplitude (m)	Power kW / m wavelength
0.5	38
1	153
1.5	345
2	615

Tidal stream farm

The sea-turbine power values (W per m² of seafloor) even with only 1-2 m/s ocean current are only **peak** values.



Due to the sinusoidal back-and-forth rocking of tides every 6h, the average effective current is $\approx 0.4 \cdot \text{peak current}$.

At neap tide, power is further reduced.

$$\begin{aligned} \text{max spring power} &= 4 \times \text{max neap power} \\ \text{average max power} &= (\text{spring}_{\text{max}} + \text{neap}_{\text{max}}) / 2 \\ \text{average power (W/m}^2\text{)} &= 0.4 \cdot \text{avg max power} \end{aligned}$$

Advantages of tidal power

- entirely predictable; reliable; year-round operation
- known hardware; but with challenges:
 - ◆ salt corrosion
 - ◆ silt accumulation
 - ◆ entanglement of floating matter
 - ◆
- in particular when comparing **tidal** turbines **vs. wind** turbines:
 - ◆ no visual impact !
 - ◆ low turbine safety factor needed
(there are no underwater tidal 'storms' as opposed to wind storms)
 - ◆ smaller turbines for the same power, due to $\rho_{H_2O} \gg \rho_{air}$

Wave power

Wave power: origin and characteristics

Ocean surface wave motion is **3rd generation sun power**: sun heat creates the winds, and **winds create the waves**.

The **wavelength** λ (m) and **period** T (s) depend on the **windspeed**.

The **height** h of the wave depends on the **time** the wind has been blowing!

Like with wind, you can extract power only once.

The potential is especially interesting on the Atlantic;
the feasibility of wave power is especially investigated in the UK, Sweden, ..

Power Generators :

= either coupled to ***floating*** devices (oscillating buoys)

= or turned by ***air, displaced*** by waves in a hollow concrete structure

Numerous practical problems exist, however.

- Total theoretical wave power potential estimated to 2000 GW_e, the recoverable part to 300-500 GW_e
- Total theoretical energy potential estimated to 8000 – 80'000 TWh/year, the actual recoverable part to 1200 TWh/yr

Source: *Sustainability* 2020, 12, 2178

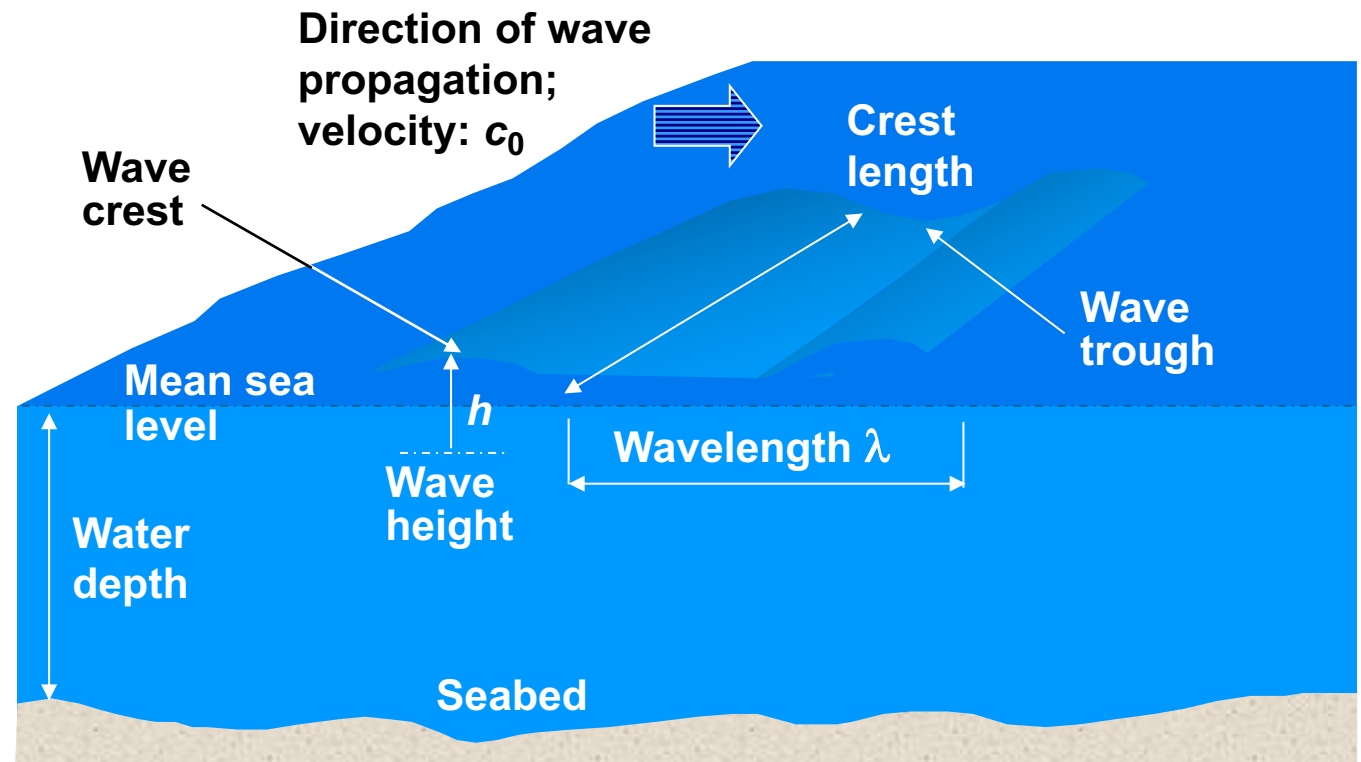
Electrical Power Generation from the Oceanic Wave for Sustainable Advancement in Renewable Energy Technologies

RESs	Year-2017	Year-2018
Hydropower (GW)	1112	1132
Wind power (GW)	540	591
Solar PV (GW)	405	505
Bio-power (GW)	121	130
Geothermal power (GW)	12.8	13.3
Concentrating solar thermal power (GW)	4.9	5.5
Ocean power (GW)	0.5	0.5
Total (GW)	2196.2	2377.3

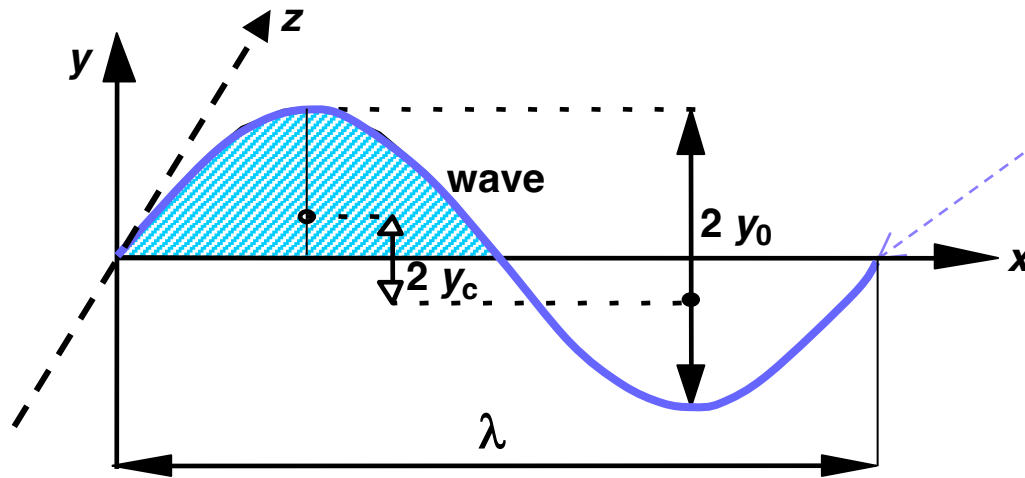
Energy stored in ocean surface waves: calculation

Two components:

- *kinetic* (horizontal: water flow)
- *potential* (vertical: height difference between crest and trough of the wave)



The energy that can be extracted from a wave corresponds roughly to the **potential energy liberated by the collapse of the wave** crest, i.e. the energy obtained when the mass M of the “half-wave” falls from a **height $2 y_c$** , where y_c represents the mass centre of the “half-wave”.



assume $y = y_0 \cdot \sin(2\pi x/\lambda)$

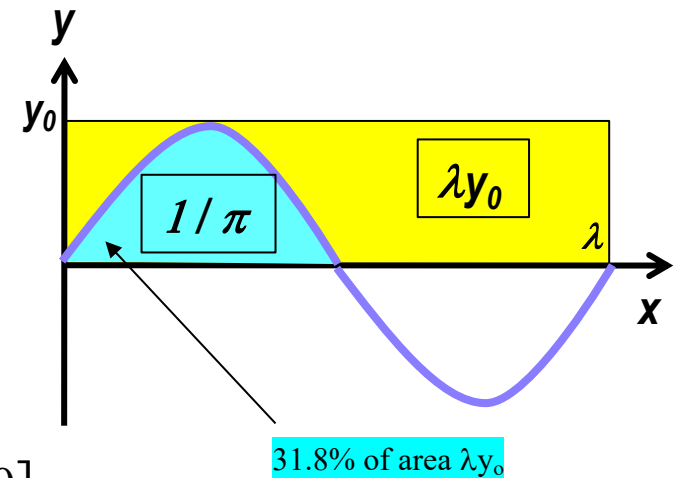
$2 y_0$: double amplitude of the wave (= h)

$2 y_c$: height difference between the half-waves mass centres

Mass M_z per unit length of the wave front in z-direction (kg/m):

$$M_z = \rho \int_0^{\lambda/2} \int_0^{y_0 \sin(2\pi x/\lambda)} dy dx = \rho \int_0^{\lambda/2} y_0 \sin(2\pi x/\lambda) dx = \boxed{\rho \frac{\lambda y_0}{\pi}}$$

$$\rho y_0 \left[-\cos\left(\frac{2\pi x}{\lambda}\right) \frac{\lambda}{2\pi} \right]_0^{\lambda/2} = \rho y_0 \frac{\lambda}{2\pi} \underbrace{[-\cos \pi + \cos 0]}_{=2}$$



Position y_c of the mass centre = ? :

$$M_z = \frac{\rho y_0 \lambda}{\pi}$$

$$y_c = \frac{\int_0^{\lambda/2} \int_0^{y_0 \sin(2\pi x/\lambda)} y dy dx}{\int_0^{\lambda/2} \int_0^{y_0 \sin(2\pi x/\lambda)} dy dx} = \frac{\int_0^{\lambda/2} \int_0^{y_0 \sin(2\pi x/\lambda)} y dy dx}{M_z} = \frac{\rho}{\frac{\rho y_0 \lambda}{\pi}} \int_0^{\lambda/2} \frac{y^2}{2} \Big|_0^{y_0 \sin(2\pi x/\lambda)} dx$$

$$y_c = \frac{\pi}{y_0 \lambda} \int_0^{\lambda/2} \left(\frac{y_0^2}{2} \right) \sin^2(2\pi x/\lambda) dx = \frac{\pi y_0}{2\lambda} \int_0^{\lambda/2} \sin^2(2\pi x/\lambda) dx$$

$\int \sin^2(x) dx = \frac{1}{2} \int (1 - \cos(2x)) dx$ because: $\cos(2x) = \cos^2(x) - \sin^2(x)$
 $\cos^2(x) + \sin^2(x) = 1$

$\int \sin^2\left(\frac{2\pi x}{\lambda}\right) dx = \frac{1}{2} \int \left(1 - \cos\left(\frac{4\pi x}{\lambda}\right)\right) dx$

Substitute: $t = \frac{4\pi x}{\lambda}, \quad dt = \frac{4\pi}{\lambda} dx, \quad dx = \frac{\lambda}{4\pi} dt$

$\longrightarrow \frac{1}{2} \int dx - \frac{1}{2} \int \frac{\lambda}{4\pi} \cos(t) dt = \frac{x}{2} - \frac{\lambda}{8\pi} \int \cos(t) dt$

$$\left(\frac{x}{2} - \frac{\lambda}{8\pi} \sin(t) \right) \Big|_0^{\lambda/2} = \left(\frac{x}{2} - \frac{\lambda}{8\pi} \sin\left(\frac{4\pi x}{\lambda}\right) \right) \Big|_0^{\lambda/2} = \frac{\lambda}{4} - 0 - \frac{\lambda}{8\pi} (\sin 2\pi - \sin 0) = \frac{\lambda}{4}$$

$$y_c = \frac{\pi y_0}{2\lambda} \cdot \frac{\lambda}{4}$$

$$y_c = \frac{\pi y_0}{8} \approx 0.39 y_0$$

→ **Energy** stored in ocean surface waves: result

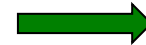
Total energy W ($=m \cdot g \cdot h$) *per unit length* ($=\text{Joule/m}$) of the wave *front* ($=z$) due to the collapse of the wave, generated during time of passage of the wave (period T):

$$W_{\text{wave}} = M_z \cdot g \cdot 2y_c = 2 \frac{\rho \cdot y_0 \cdot \lambda}{\pi} \cdot g \cdot \frac{\pi \cdot y_0}{8} = \frac{\rho \cdot g \cdot y_0^2 \cdot \lambda}{4}$$

Wave characteristics, e.g.:

$$\lambda = 100 \text{ m}$$

$$y_0 = 1 \text{ m}$$



$$W = 250 \text{ kJ / m}$$



How to compute the **power** in a wave?

($=W_{\text{wave}} / T$) (T =period)

- The water particles move themselves (= **kinetic** energy) but at much lower speed, v , than the wave itself, c_0 (as demonstrated by the slow movement of objects floating on water, 'under' which the waves travel faster).
- ⇒ we need to know this **speed v** with which the water itself moves (**\neq wave speed $c_0 = \lambda / T$!**), in order to find an expression for T
- ⇒ a relation between the water velocity v and the wavelength λ can be physically derived (next slide)

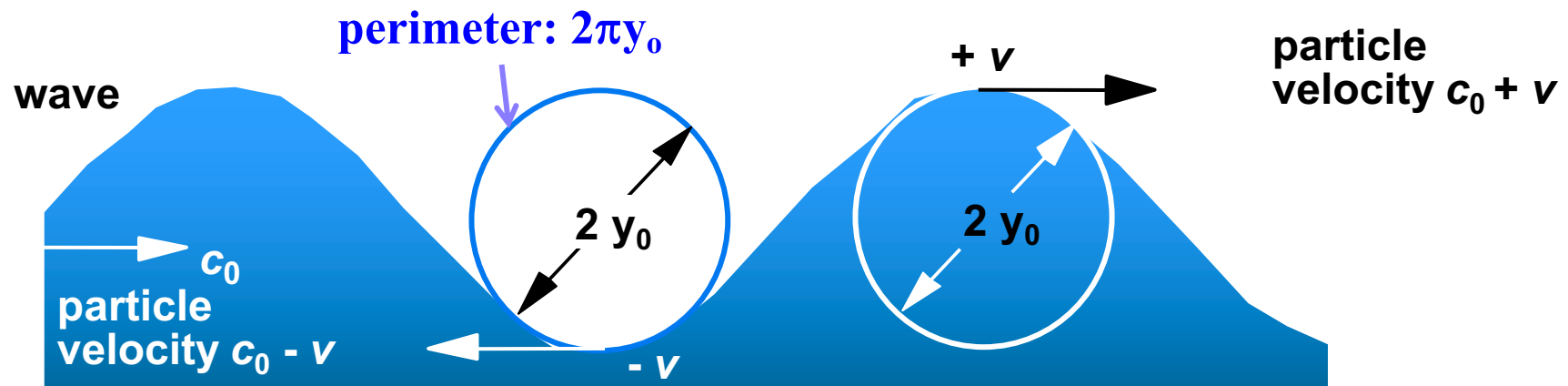
Correlation between water velocity v and wavelength λ :

Consider a **reference frame** in movement with the wave: **cylindrical surfaces**, **tangential to the trough and crest of the wave** respectively, **rotating with the angular frequency $\nu = c_0/\lambda = 1/T = \nu / (2\pi y_0)$** (\Rightarrow speed **$\nu = 2\pi y_0 \nu$**) *

(The speed ratio of 'wave' vs. 'water')

$= c_0/\nu = \lambda/2\pi y_0 = \text{wavelength vs. } \approx (\pi \cdot \text{waveheight}))$

We then calculate the difference in **kinetic** energy of the water particles at the wave crest ('in') and in the trough ('out'): $E_{\text{kin}} (\text{out} - \text{in}) = 2.v$



*physical explanation: During the time that 1 wave travels through, i.e. $T(\text{in seconds}) = (1/\text{angular frequency}) = \text{wave length } \lambda / \text{wave_speed } c_0$, the cylinder has rolled once around its perimeter $\pi \cdot 2y_0$

Kinetic & potential energy of the water in the wave:

Variation of the **kinetic** energy of the water particles:

$$c_0/\lambda$$

speed $v = 2\pi y_0 v$

$$\Delta W_{\text{kin, water}} = m \cdot c_0 \cdot 2v = m \cdot c_0 \cdot 2 \cdot 2\pi \cdot y_0 \cdot v = (4 \cdot m \cdot c_0^2 \cdot \pi \cdot y_0) / \lambda$$

$$E_{\text{kin,out}} - E_{\text{kin,in}} = \left[\frac{1}{2} m (c_0 + v)^2 - \frac{1}{2} m (c_0 - v)^2 \right] = \frac{1}{2} m (2c_0 v - (-2c_0 v)) = 2mc_0 v$$

cf. Euler equation : specific mass energy w (J/kg) = $v_e \cdot \Delta v_t$

= entrainment velocity * (tangential velocity difference out-in) = $c_0 * 2v$

Corresponding variation of the **potential** energy (height difference)

of the water particles: $\Delta W_{\text{pot, water}} = m \cdot g \cdot 2 \cdot y_0$

Since $\Delta W_{\text{kin, water}} = \Delta W_{\text{pot, water}}$ (like for all oscillating movement),

we can derive:

$$\frac{4mc_0^2 \pi y_0}{\lambda} = 2mgy_0 \Rightarrow c_0^2 = \frac{g\lambda}{2\pi} \Rightarrow c_0 = \left(\frac{g\lambda}{2\pi} \right)^{1/2}$$

$$\frac{2\pi}{g} c_0 = \frac{\lambda}{c_0} = T = 0.64 c_0$$

wave period T (s)

$$T = \frac{\lambda}{c_0} = \frac{\lambda}{\sqrt{g\lambda/2\pi}} = \left(\frac{2\pi\lambda}{g} \right)^{1/2} = \sqrt{0.64 \cdot \lambda}$$

=> **power** P in the wave (W_{wave} / T) : result

The **energy** per unit length of the wave front (J/m) was found before as:

$$W_{\text{wave}} = \frac{\rho \cdot g \cdot y_0^2 \cdot \lambda}{4}$$

The **power** per unit length of the wave front (W/m) is then this wave **energy** (J/m) divided by the period T (s), found on the previous slide:

$$\Rightarrow P_{\text{wave}} = \frac{W_{\text{wave}} [J]}{T [s]} = \frac{\rho g y_0^2 \lambda / 4}{\sqrt{\frac{2\pi\lambda}{g}}} = \frac{\rho g^{3/2}}{4\sqrt{2\pi}} y_0^2 \sqrt{\lambda} \approx 3064 \cdot y_0^2 \sqrt{\lambda}$$

const: 3064

**power per unit length (W/m)
of the wave front**

=> numerical application:

$$P \text{ [W/m]} = 3064 \cdot y_0^2 \cdot \lambda^{1/2} \quad (y_0, \lambda \text{ in m})$$

Wave characteristics:

$$\lambda = 100 \text{ m}$$

$$y_0 = 1 \text{ m}$$

$$\text{waveheight} = 2 \text{ m}$$

$$T = 8 \text{ s}, \nu = 1 / 8$$

$$c_0 = 12.5 \text{ m/s}$$

$$\nu = 0.8 \text{ m/s}$$



$$P = 30 \text{ kW / m}$$





30 GW ?

$$T = \frac{2\pi}{g} c_0 = 0.64 c_0$$

$$\lambda = \frac{2\pi}{g} c_0^2 = 0.64 c_0^2$$

$\lambda = 100 \text{ m}$

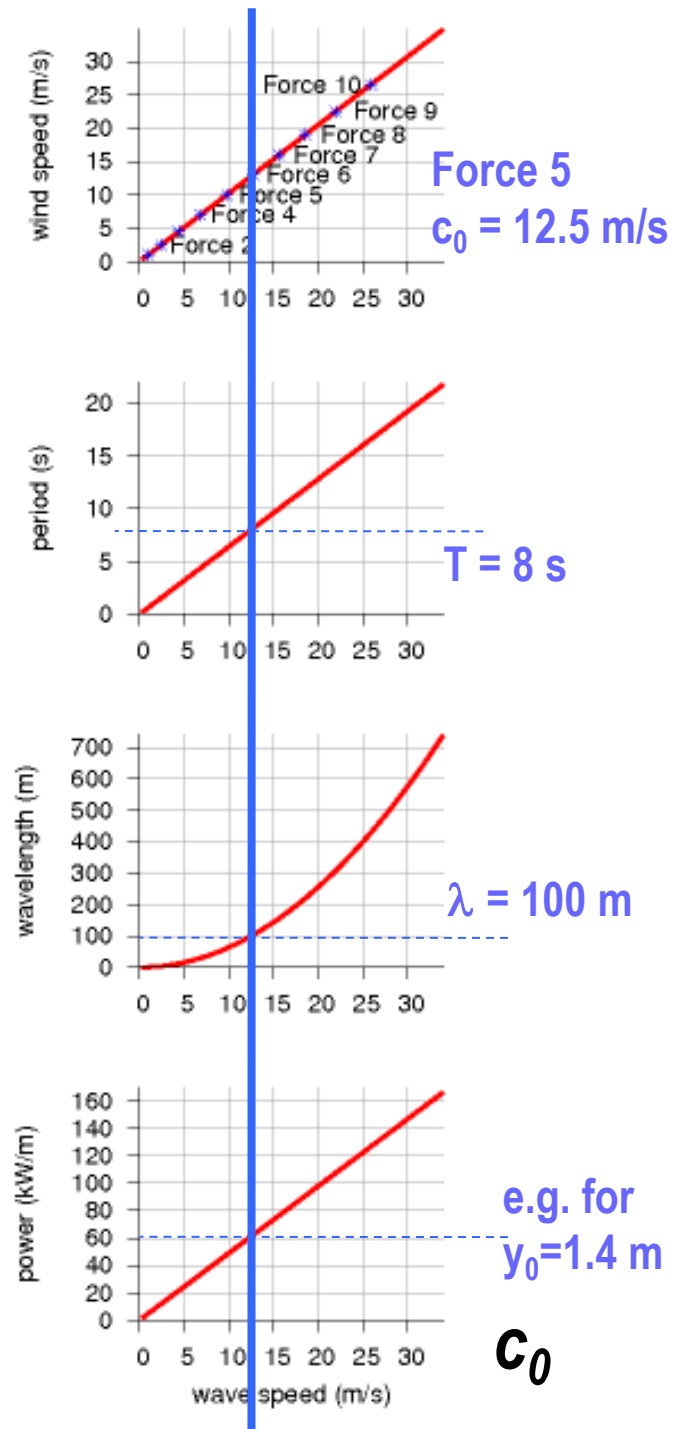
y_0	P (kW / m)
1 m	30
1.4 m	60
1.7 m	90
2 m	120

$$P(W/m) = 3064 y_0^2 \sqrt{\lambda} = 3064 y_0^2 \sqrt{0.64 c_0^2}$$

$$P(kW/m) = 3.064 y_0^2 \cdot 0.8 c_0 = 2.45 y_0^2 c_0$$



The **wavelength** λ (m) and **period** T (s) depend on the **windspeed**.
The **height** h ($=2y_0$) of the wave depends on the **time** the wind is blowing

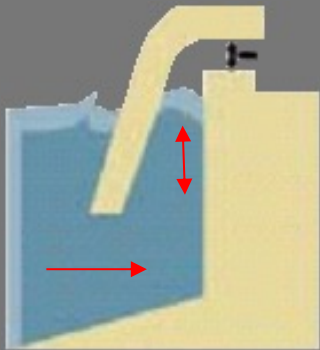


Wave power inspires ingenuity...

The oscillating up-down motion of waves is captured by different approaches, in a rotary or piston motion to drive a generator.

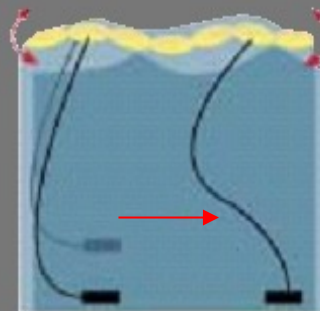
Oscillating Water Column

Waves push air through turbine, then suck it back, as they advance and recede.



Pelamis

Serpentine devices flex in oncoming waves. Pivoting of segments drives pistons.



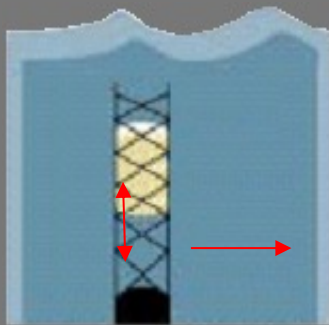
McCabe Wave Pump

Bobbing of outer barges, hinged to central one, stabilized by underwater plate, drives pumps.



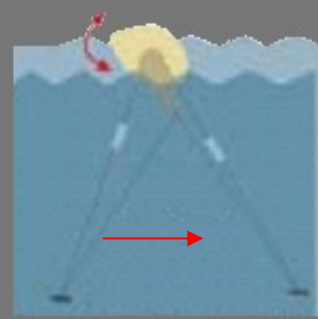
Archimedes Wave Swing

Air tank in fixed, submerged tower rises and falls with passing waves, driving generator.



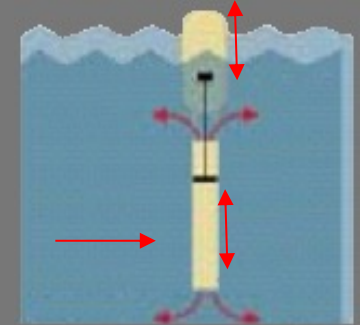
Nodding Duck

Waves push beak of floating device. Its rotation relative to central shaft pumps oil → generator

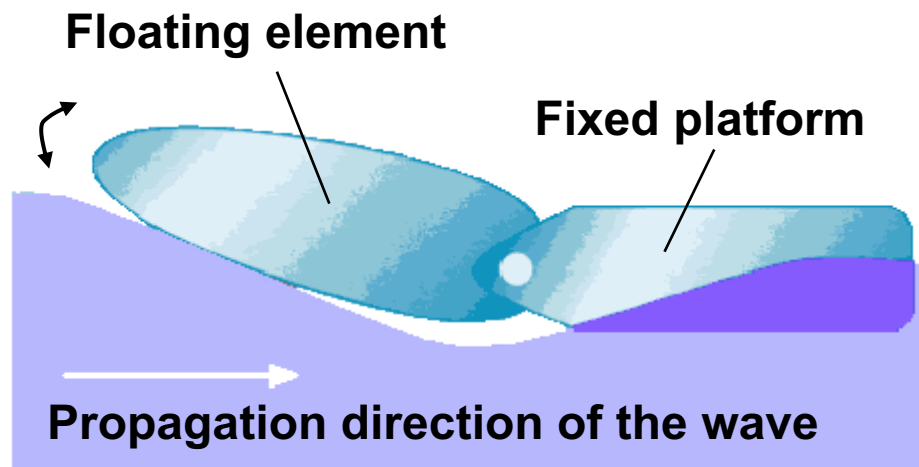


IPS Buoy

Motion of bobbing buoy relative to piston shaft drives generator.

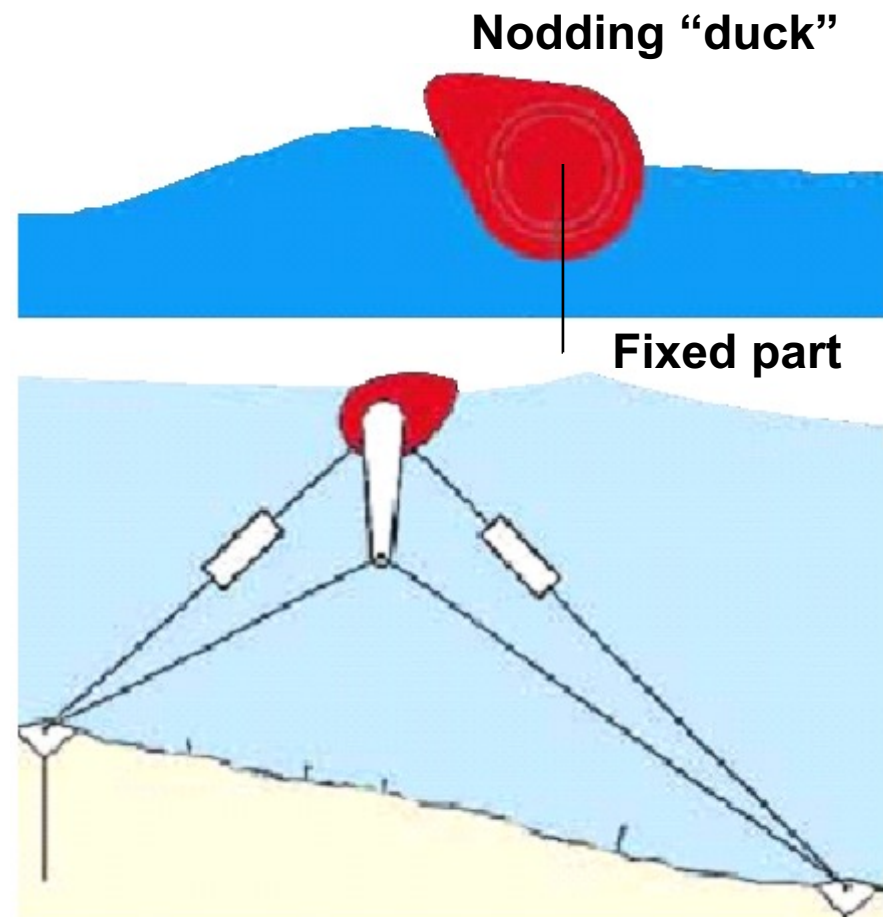


'Nodding duck' oscillating device



In shallow water (approaching the shore), the wave energy is lost in sea-bottom friction: from 100 m depth to 15 m depth, 70% is lost.

→ a 30 kW/m wave becomes a 10 kW/m wave



Buoy piston oscillating device

<http://www.youtube.com/watch?v=90AcxxwoPu0&feature=fvsr>

http://www.youtube.com/watch?v=tt_lqFyc6Co&feature=related

In the permanent magnet linear generator buoy, waves cause the coil to move up & down relative to the fixed magnetic shaft. Voltage is induced and electricity generated. A buoy could potentially generate 250 kW power.

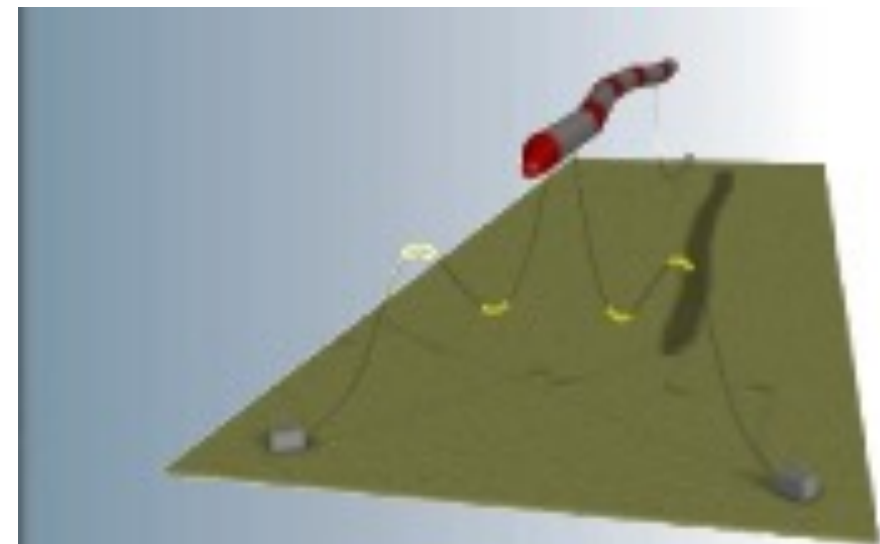


'Pelamis' - device

Semi-submerged, articulated structure composed of cylindrical sections linked by hinged joints.

The wave-induced motion of the joints is resisted by hydraulic rams, which pump high-pressure oil through hydraulic motors via smoothing accumulators.

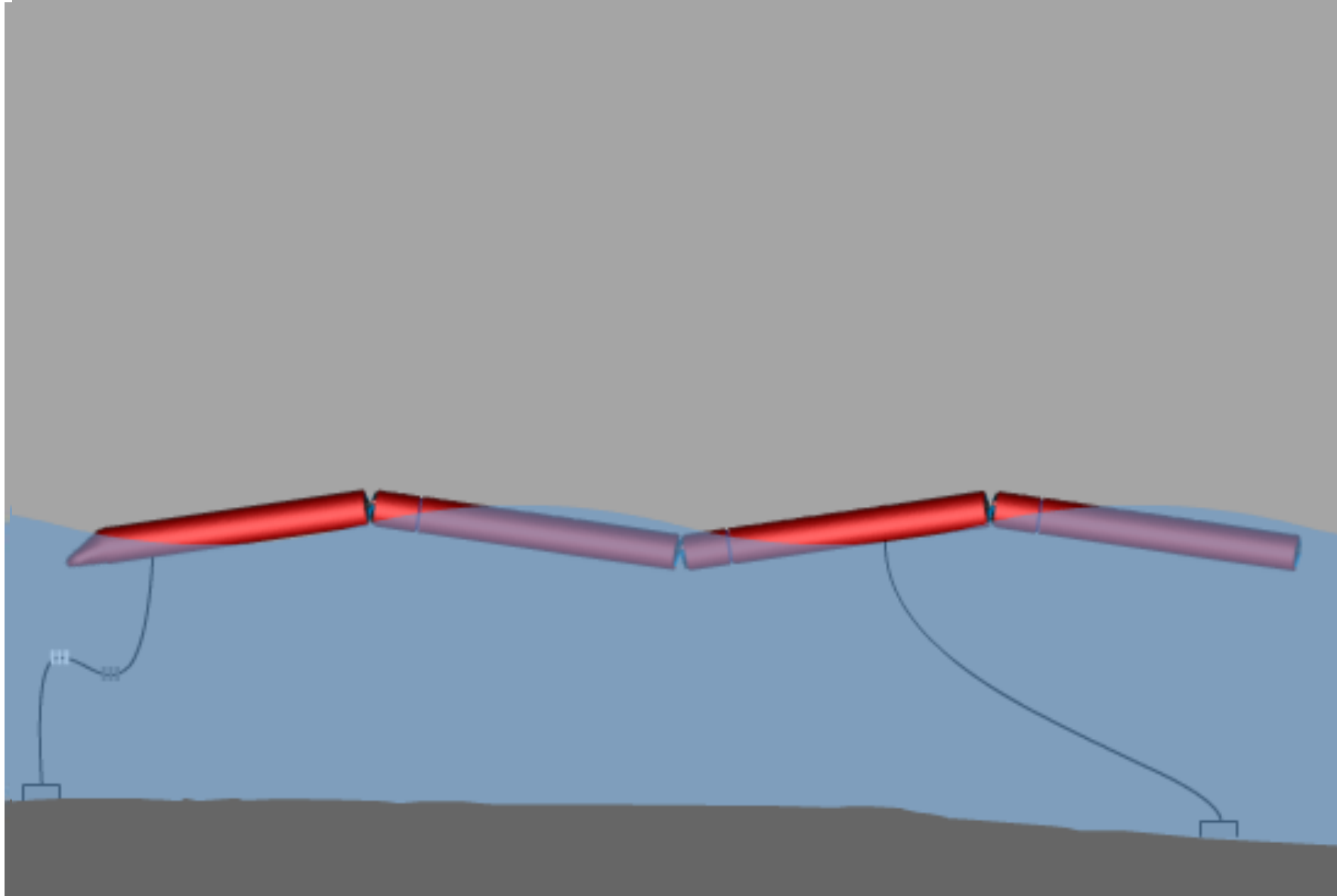
The hydraulic motors drive electrical generators to produce electricity. Power is fed down a single umbilical cable to a junction on the sea bed.



attached ca. 50 m above the sea-bottom

'Pelamis'-device

<http://www.youtube.com/watch?v=F0mzrbfzUpM>



Application example

1 Pelamis-snake : **130 m long**; **3.5m** diameter D.

750 kW max wave power (i.e. it can take up to 200 kW/m waves!); 350 ton steel (**500 kg/kW!**); *vastly overdimensioned* !

1 km² area = 400 m long x **2.5 km wide incoming wave**, 3 rows of 40 Pelamis (120 Pelamis).

Assume **30 kW / m** waves ($c_0 = 12.5$ m/s, $y_0 = 1$ m) \rightarrow 2.5 km wide equals 75 MW_e

1 every 60 m in each row, 1 every 20 m in the wave front, staggered from row 1 to row 3

\Rightarrow *effective wave-front use* = **3.5m** / 20m = 17%

Incoming power used =

3.5 m * 30 kW/m (1 Pelamis) * 120 = 12.6 MW_e

\Rightarrow **take 70% efficiency** : 8.8 MW_e

\Rightarrow 38.6 GWh_e per year gross electricity, assuming
50% load factor (4380 h/yr) (320 MWh_e/yr for 1)

**The same result with same wind (12.5 m/s), at
2500h/yr load), could be achieved with a 18.5 m
diameter 130 kW_e wind turbine for 1 Pelamis!**



Real potential... ?



30 GW_e ?

- assume the UK can 'block' 10% of its coast with wave generators ...
- the effective wave front use could be $\approx 17\%$
- assume conversion efficiency $\approx 70\%$
 $\rightarrow 0.1 * 0.17 * 0.7 \approx 1.2\%$
- a net real power of 360 MW_e would result (\approx of the same magnitude as the **La Rance tidal scheme alone(!)**, 240 MW_e)
- again one notices the discrepancy between '*potentiality*' and '*reality*', and the huge installation scales involved.
- wave power is ~marginal, **yet interesting for islands** ✓