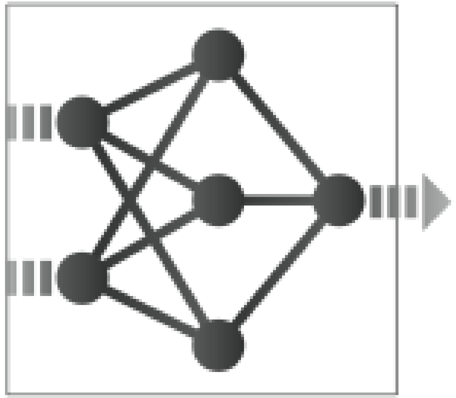
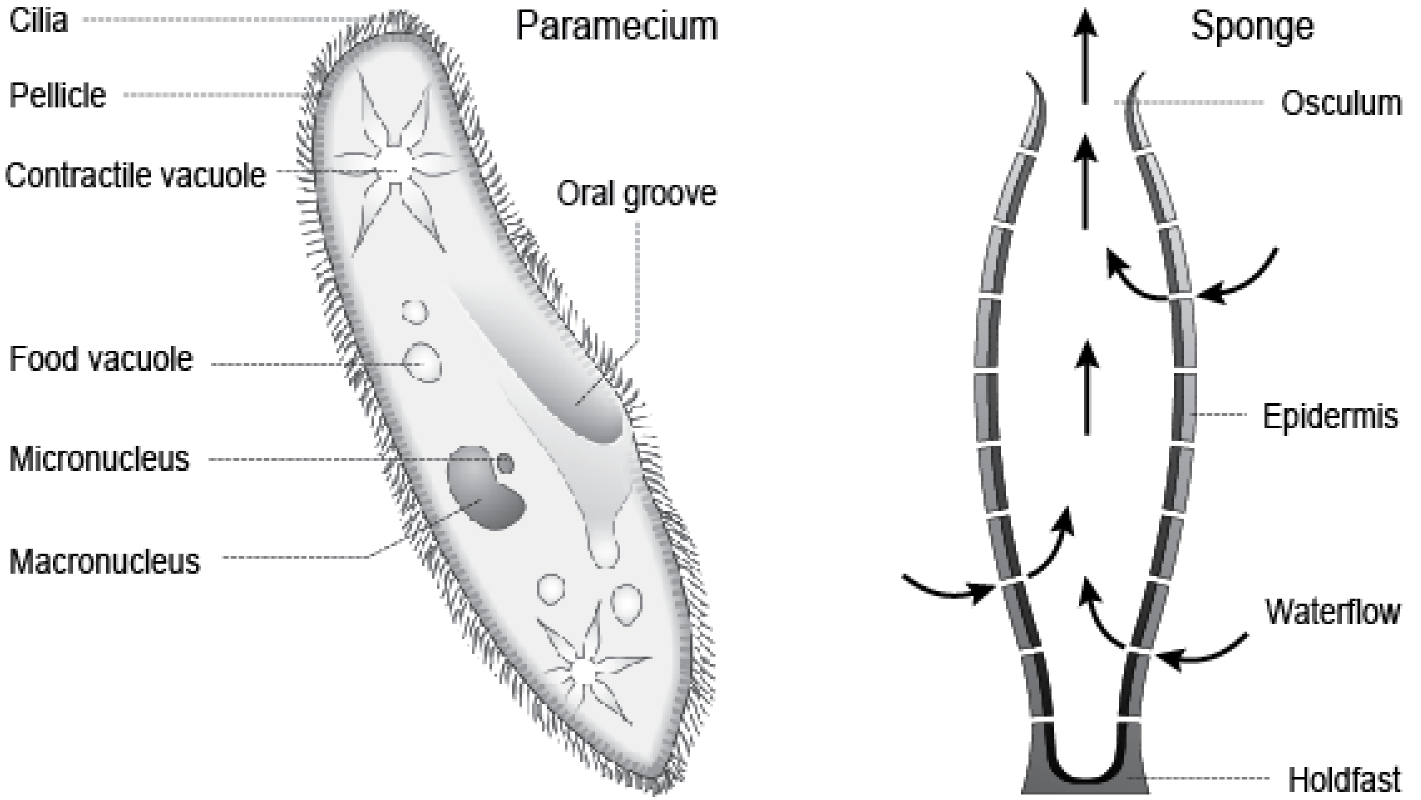
Neural Systems
Unsupervised and Supervised Learning



What you will learn today

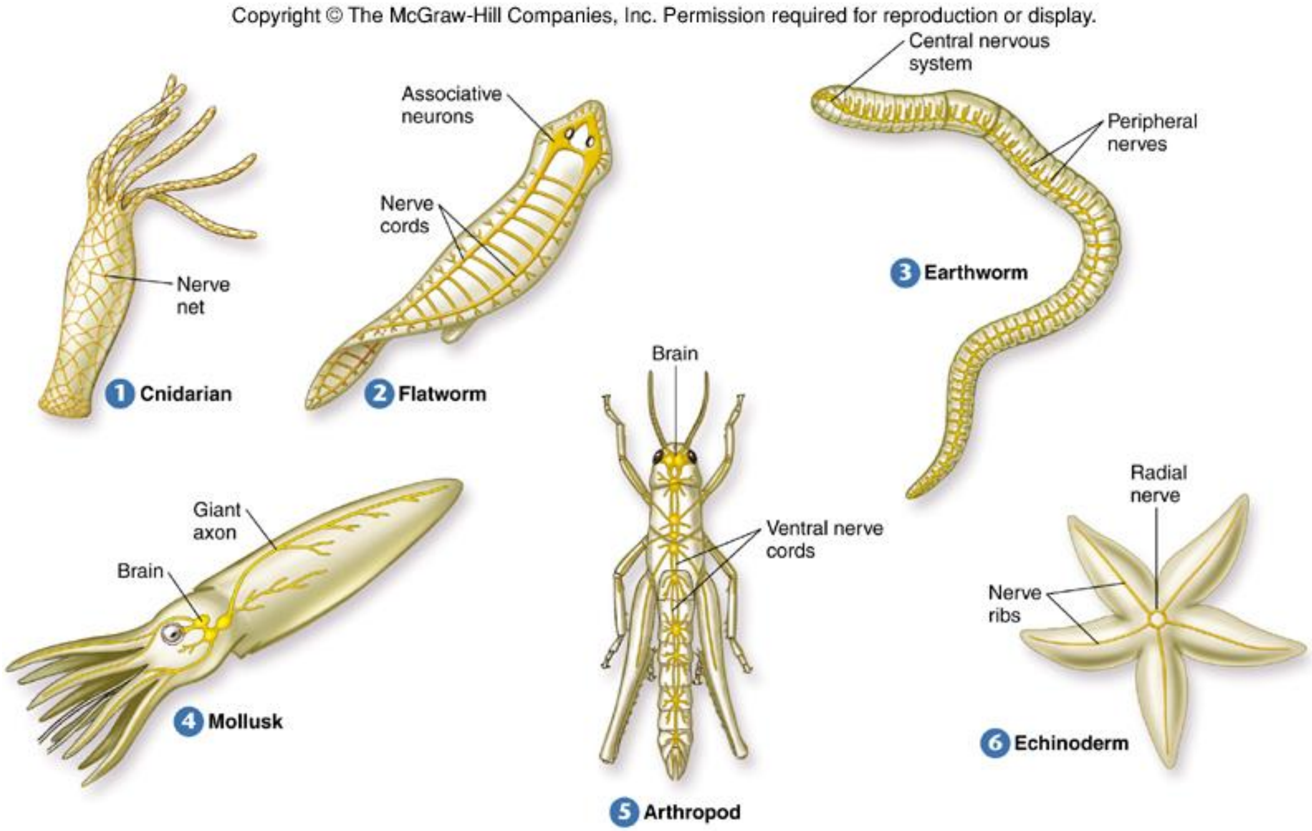
- Elements of biological nervous systems
- Artificial neuron models
- Neural architectures
- Input encodings
- Unsupervised learning
    - Feature extraction and representations
    - Topological Maps
- Supervised learning
    - From error correction to backpropagation
    - Learning time-dependent features

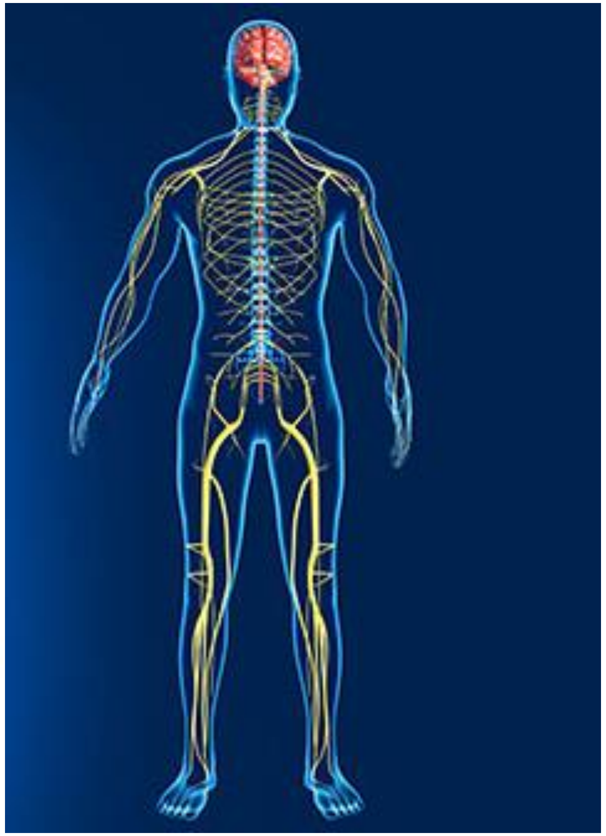
Do animals need nervous systems?



Not all animals have nervous systems; some use only chemical reactions
Paramecium and sponge move, eat, escape, display habituation

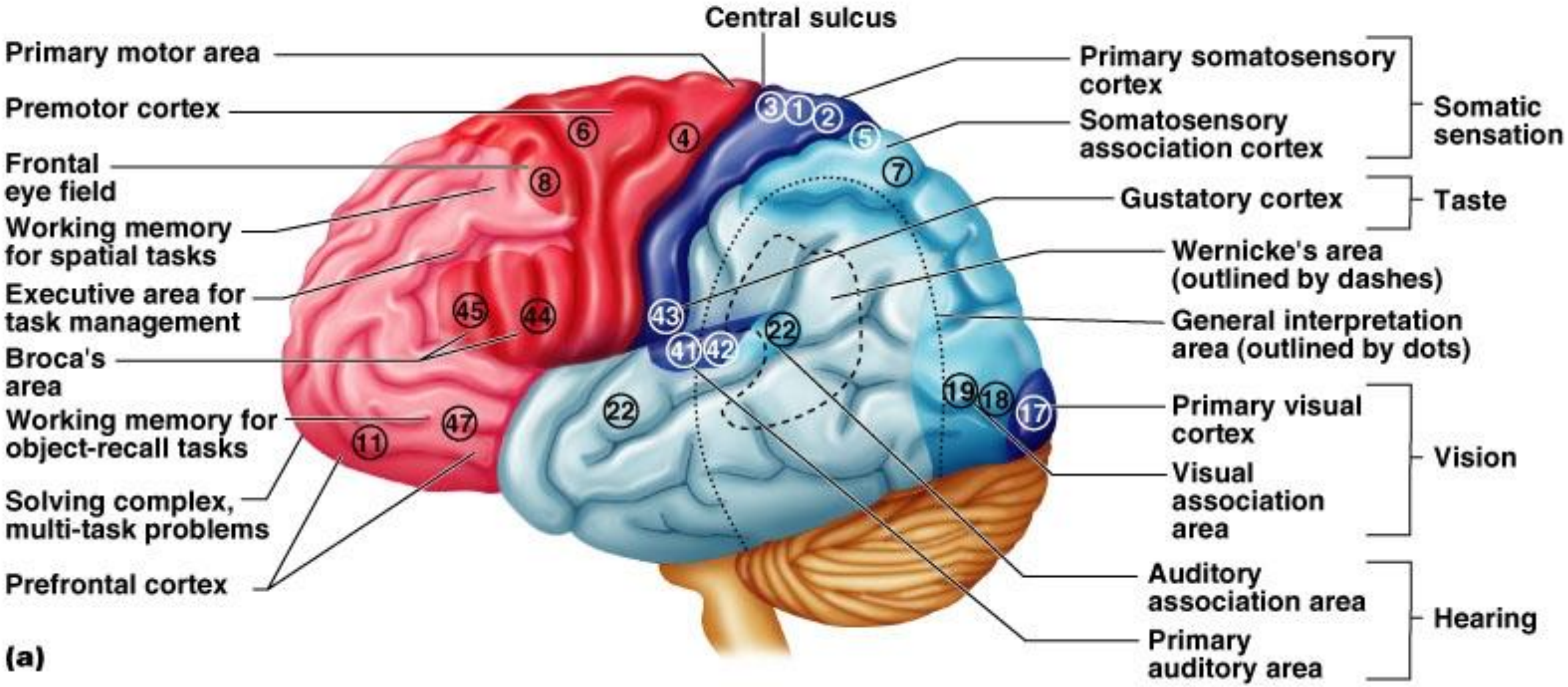
Why Nervous Systems?

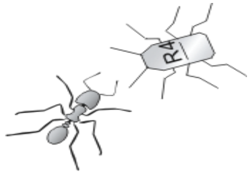




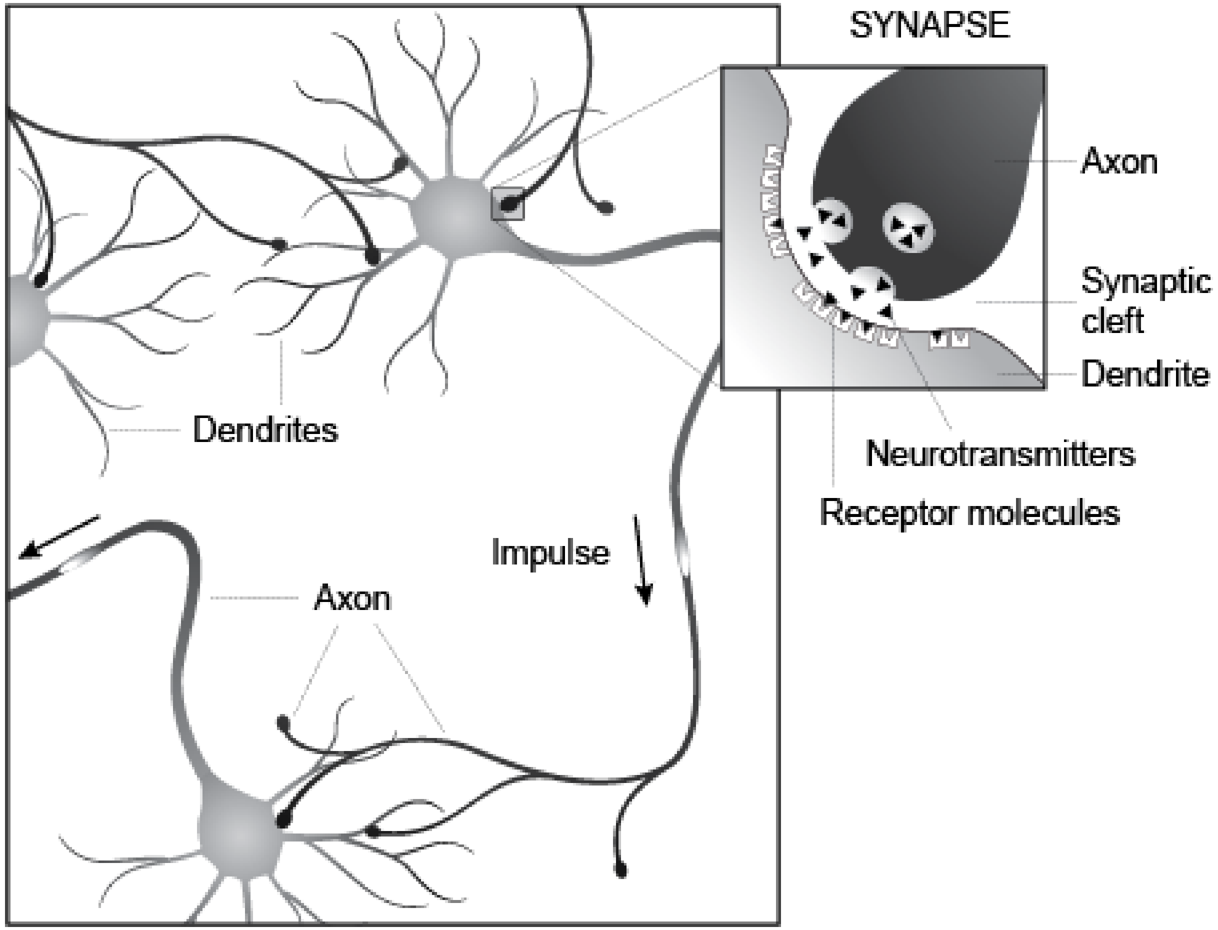
1) Faster reaction times = competitive advantage
2) Selective transmission of signals across distant areas = more complex bodies
3) Generation of non-reactive behaviors
4) Complex adaptation = survival in changing environments

Central Nervous System with Cortex

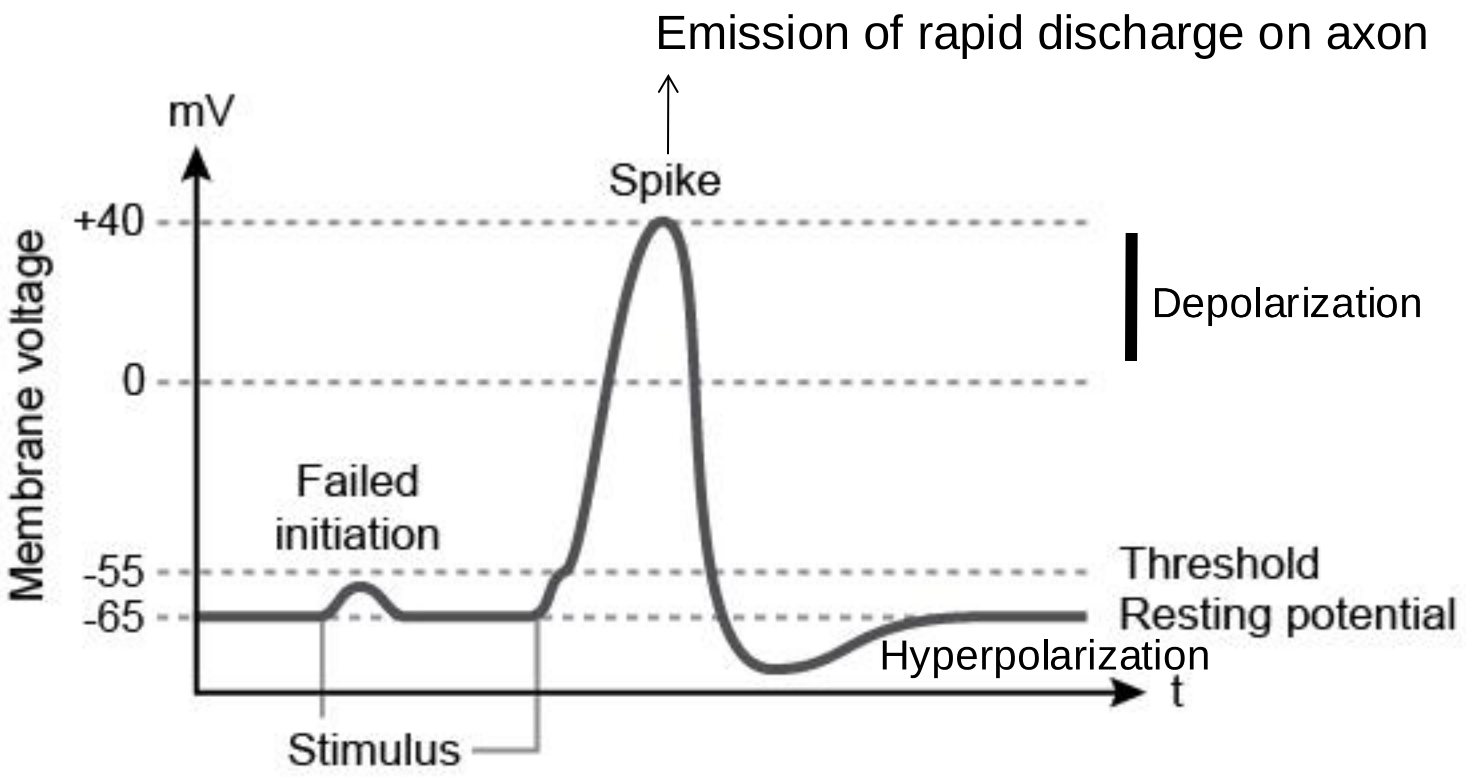




Biological Neurons

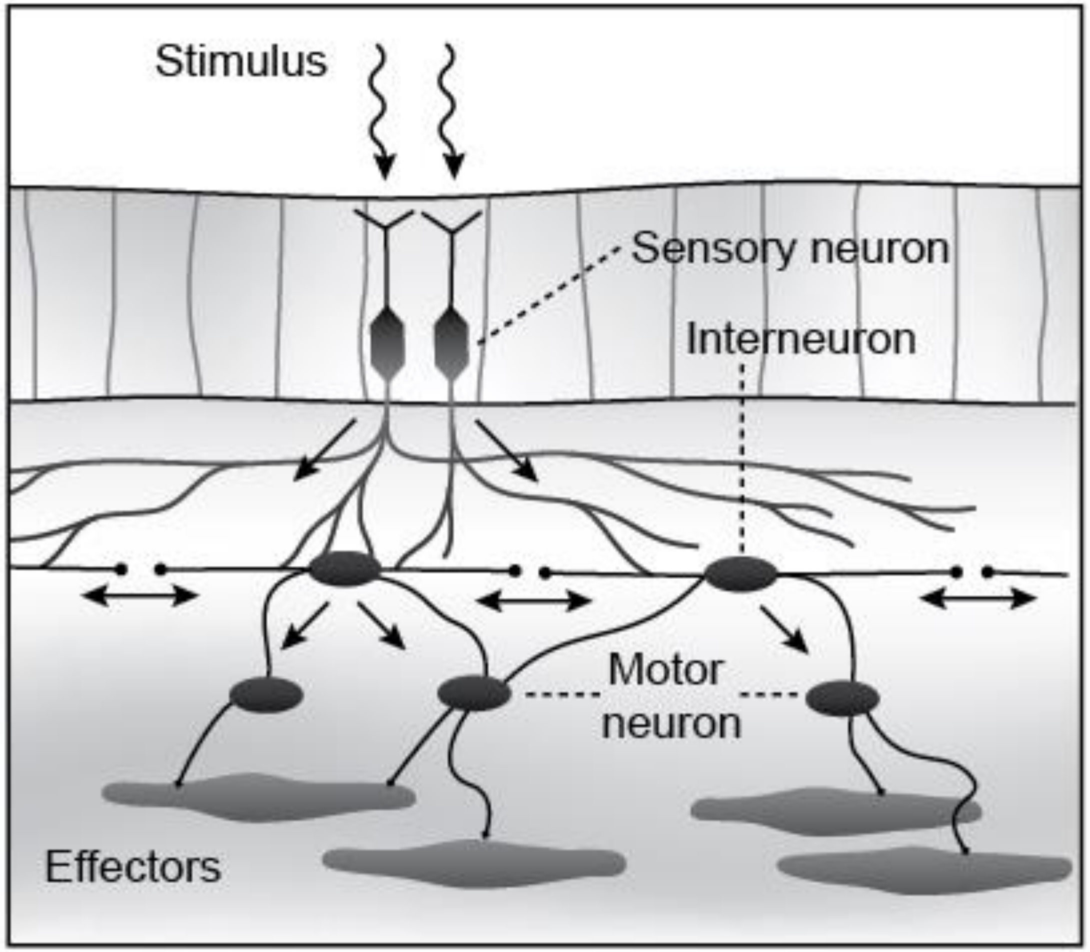


Dynamics of neural activation

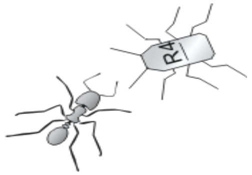


This cycle lasts approximately 3-50 ms, depending on type of ion channels involved (Hodgkin and Huxley, 1952)

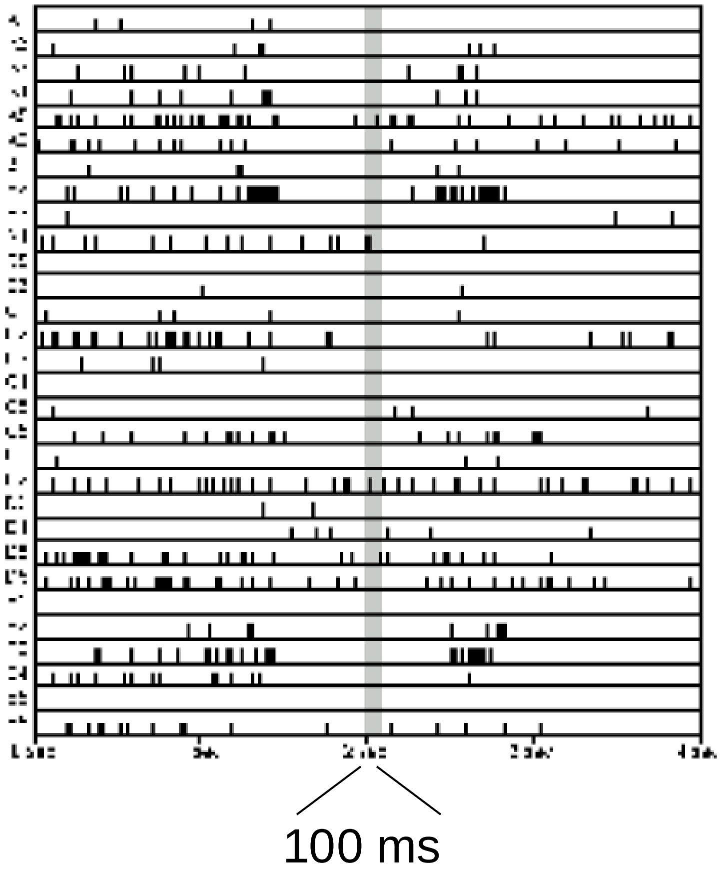
Types of Neurons



Interneurons can be
1- Excitatory
2- Inhibitory



How Do Neurons Communicate?

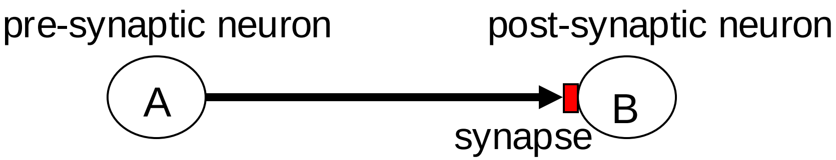


Firing rate → McCulloch-Pitts → Connectionism

Firing time → Spiking neurons → Computational Biology

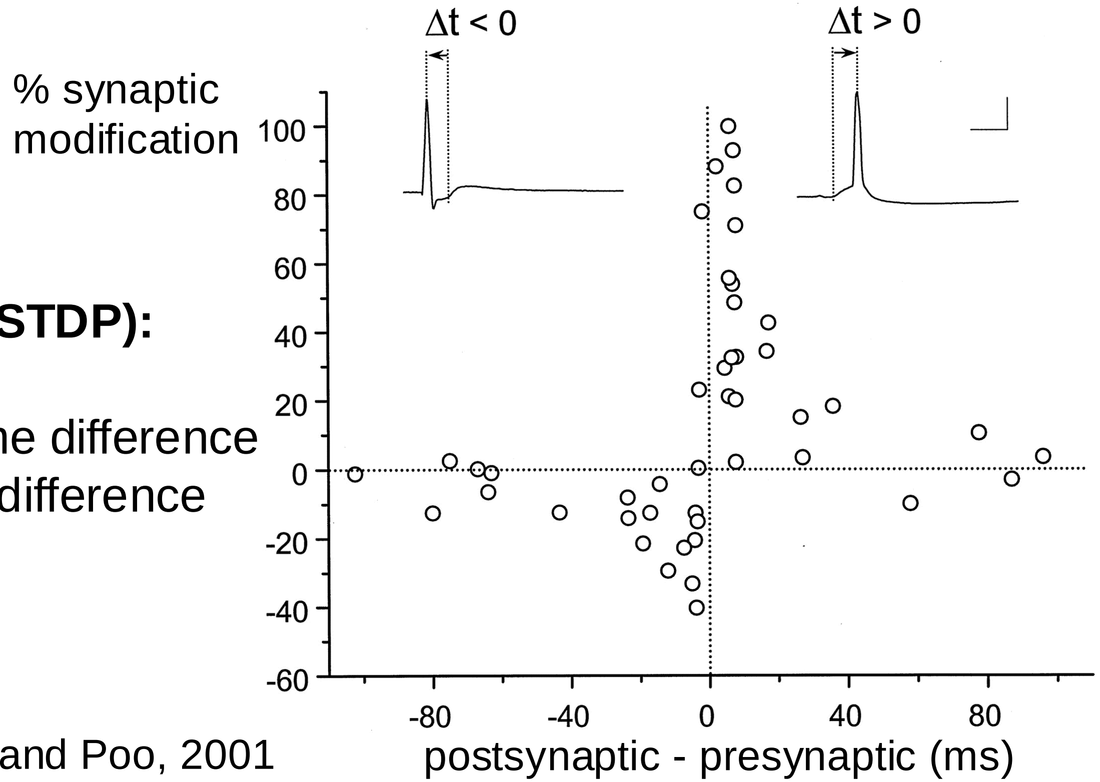
How Do Neurons Learn?

They learn by means of synaptic change



Hebb rule (1949):
Synaptic strength is increased if cell A consistently contributes to firing of cell B
This implies a temporal relation: neuron A fires first, neuron B fires second

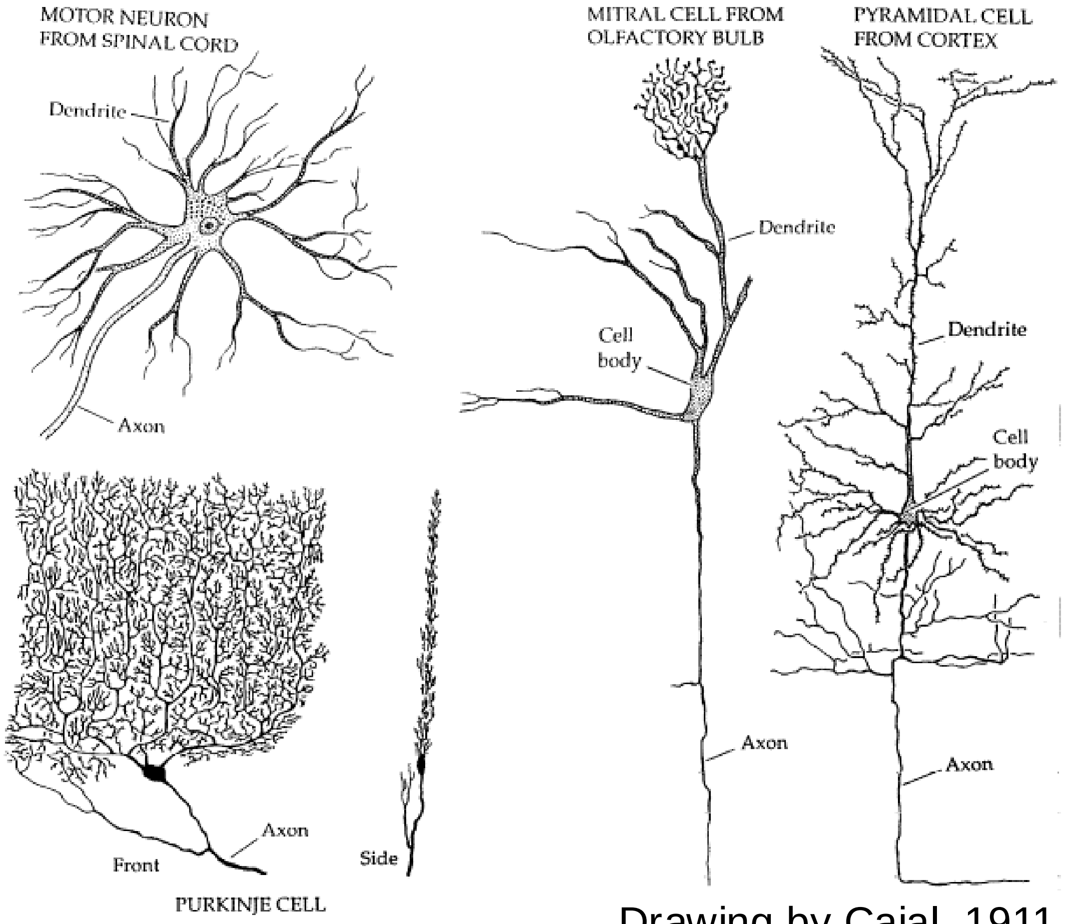
Spike Time Dependent Plasticity (STDP):
- Small time window
- Strengthening (LTP) for positive time difference
-Weakening (LTD) for negative time difference



From Bi and Poo, 2001

What Does Make Brains Different?

Components and behavior of individual neurons are very similar across animal species and, presumably, over evolutionary history (Parker, 1919)

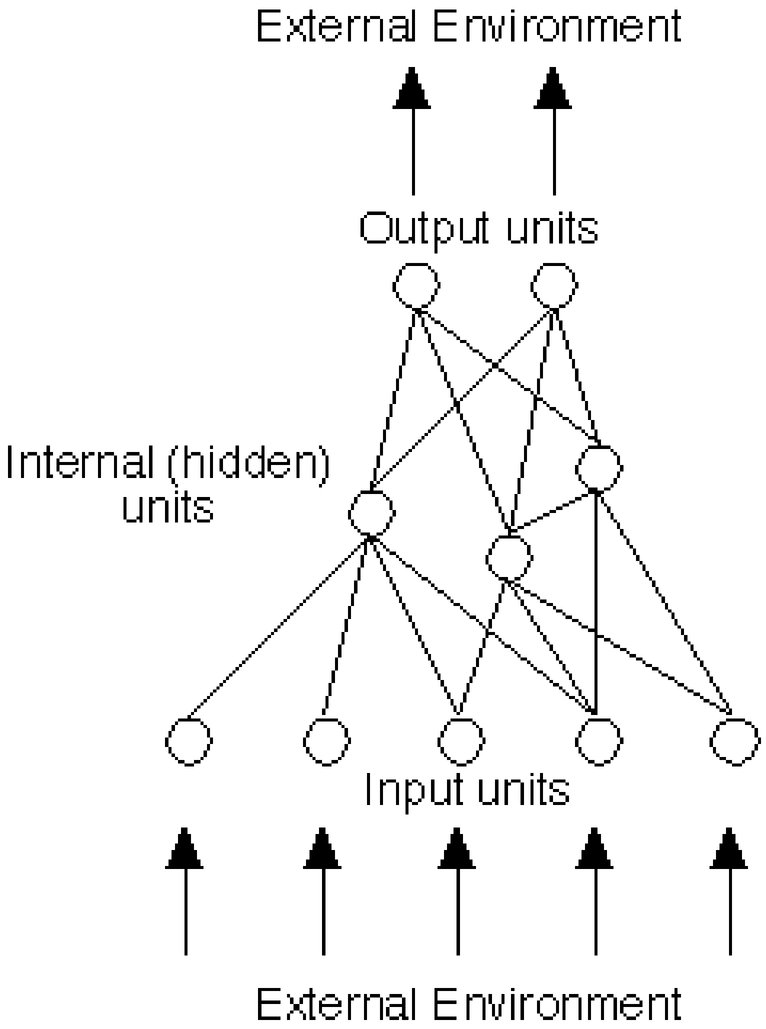


Drawing by Cajal, 1911

Evolution of the brain seems to occur mainly in the **architecture**, that is how neurons are interconnected.

First classification of neurons by Cajal in 1911 was made according to their connectivity patterns

An Artificial Neural Network

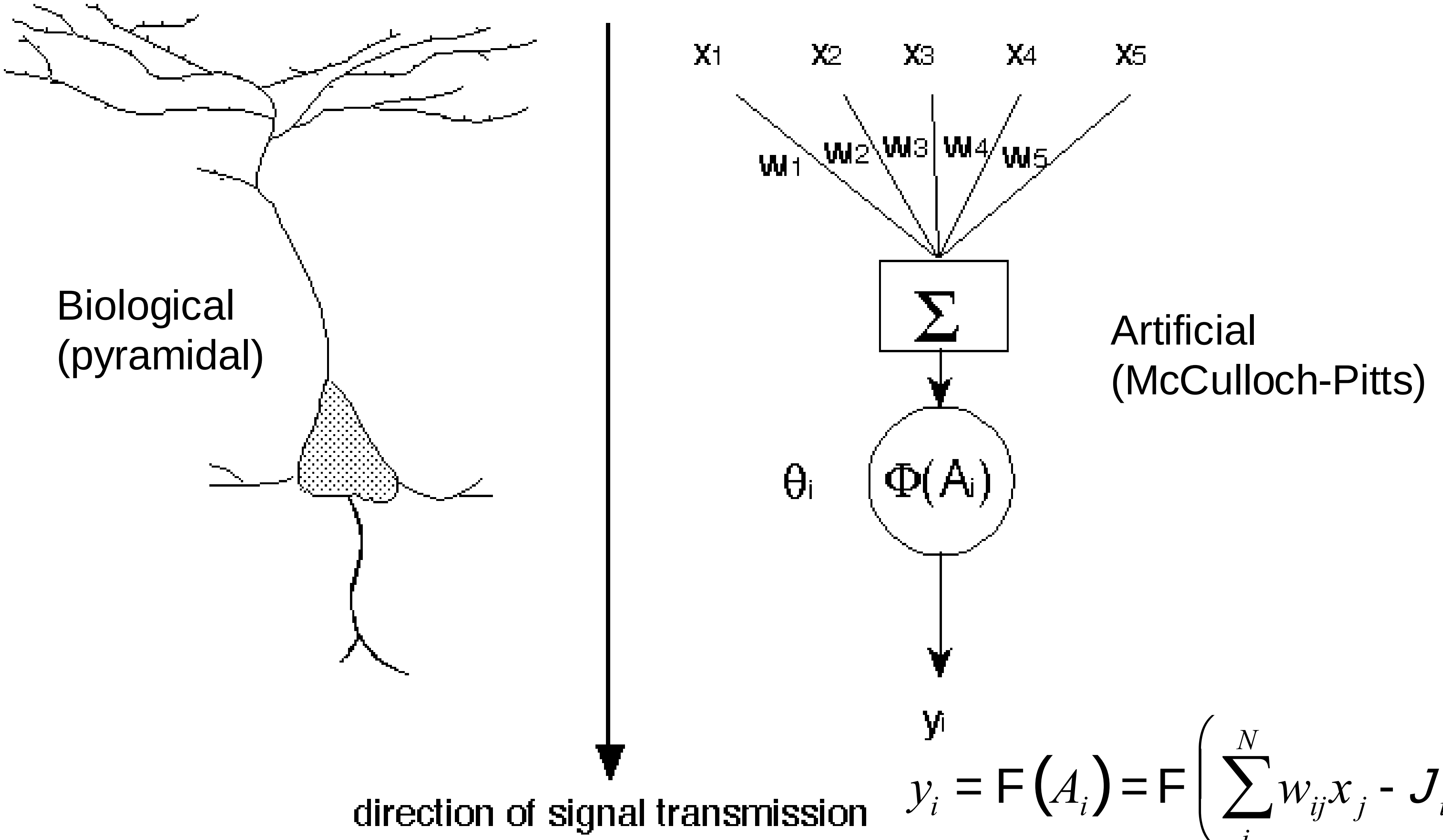


A neural network communicates with the environments through input units and output units. All other elements are called internal or hidden units.

Units are linked by uni-directional connections.

A connection is characterized by a weight and a sign that transforms the signal.

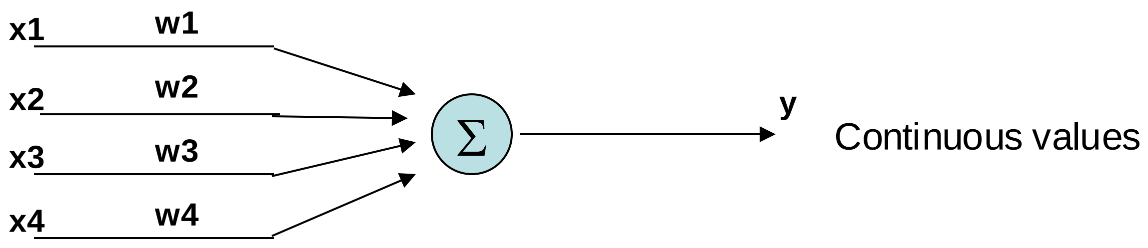
Biological and Artificial Neurons



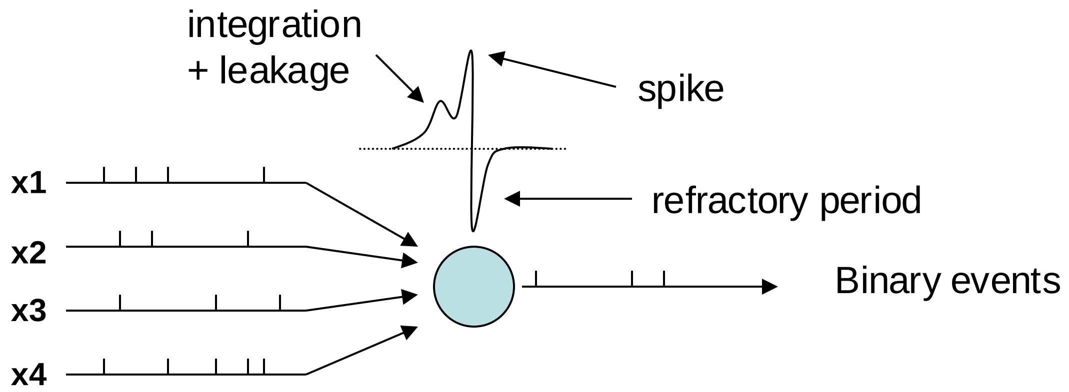
$$y_i = \mathrm{F}\left(A_i\right) = \mathrm{F}\left(\sum_j^N w_{ij} x_j - J_i\right)$$

Neuron models

McCulloch-Pitts



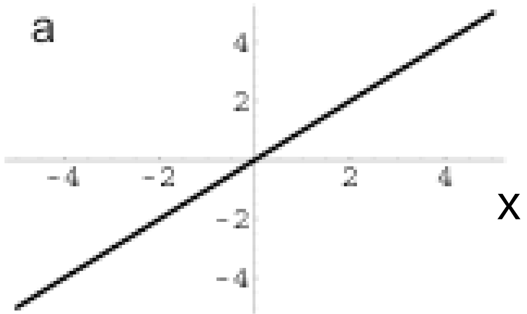
Spiking



Some output functions

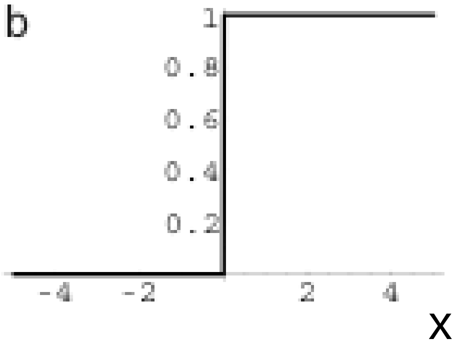
Linear

$\Phi(x)$



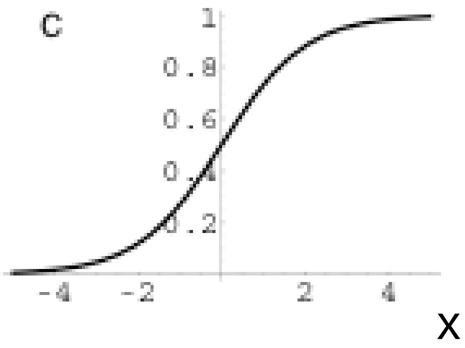
Step

$\Phi(x)$



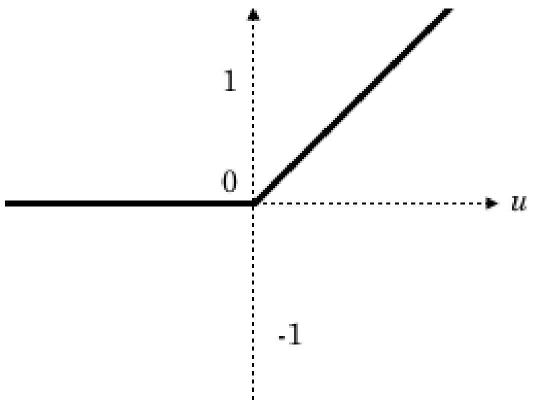
Sigmoid

$\Phi(x)$



Rectified Linear

$f(u) = \max(0, u)$



Sigmoid function:

- continuous
- non-linear
- monotonic
- bounded
- asymptotic

$$\Phi(x) = \frac{1}{1 + e^{-kx}}$$

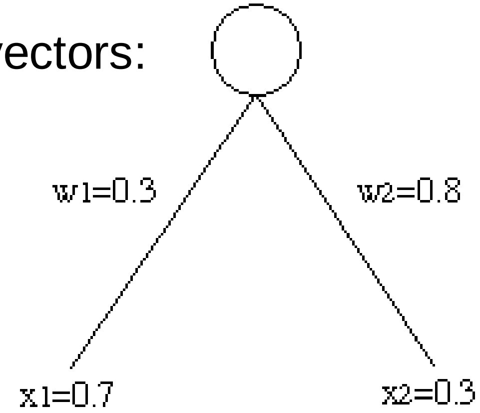
$$\Phi(x) = \tanh(kx)$$

Neurons signal "familiarity"

The output of a neuron is a measure of similarity between its input pattern and its pattern of connection weights.

1. Output of a neuron is the dot product of the weight and input vectors:

$$y = a\left(\sum_{i}^{N} w_i x_i\right), \qquad a = 1 \longrightarrow y = \mathbf{w} \cdot \mathbf{x}$$



2. Distance between two vectors is:

$$\cos\theta = \frac{\mathbf{w} \cdot \mathbf{x}}{\|\mathbf{w}\|\|\mathbf{x}\|}, \qquad 0 \le \theta \le \pi$$

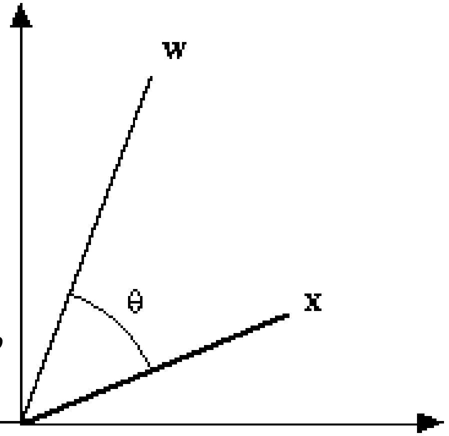
where the vector length is:

$$\|\mathbf{x}\| = \sqrt{\mathbf{x} \cdot \mathbf{x}} = \sqrt{x_1^2 + x_2^2 + \ldots + x_n^2}$$

3. Output signals vector distance (familiarity)

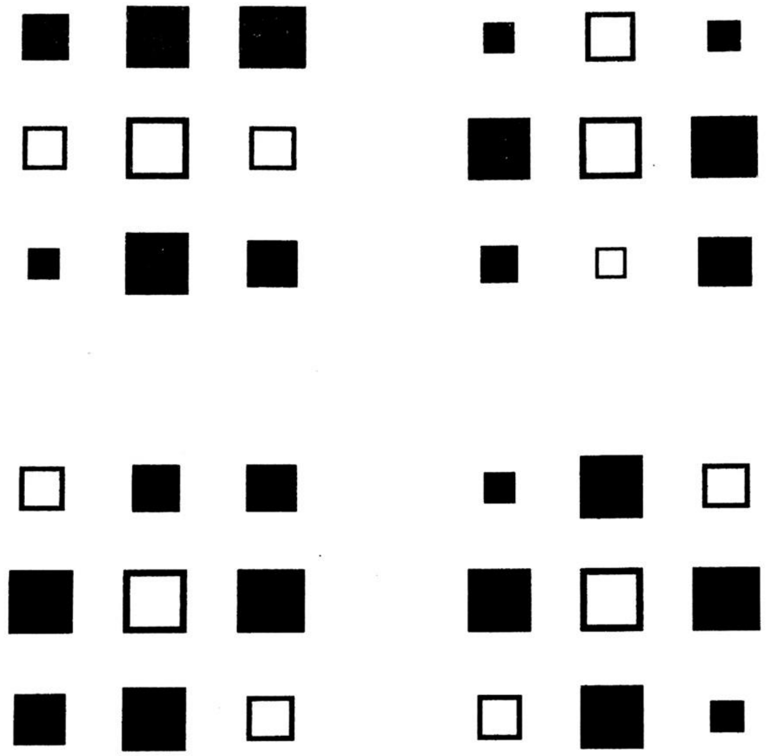
$$\mathbf{w} \cdot \mathbf{x} = \|\mathbf{w}\|\|\mathbf{x}\|\cos\theta$$

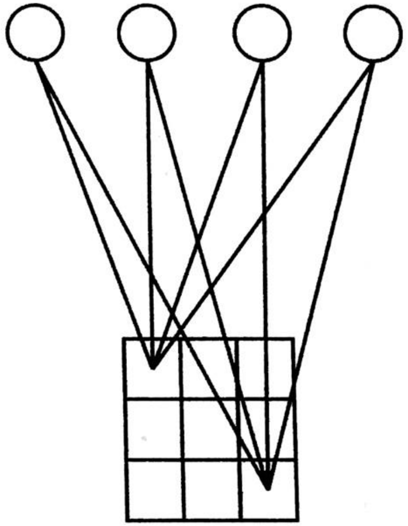
$$\theta = 0^{o} \rightarrow \cos\theta = 1,$$
$$\theta = 90^{o} \rightarrow \cos\theta = 0,$$
$$\theta = 180^{o} \rightarrow \cos\theta = -1,$$



Neural Receptive Fields

The **Receptive Field** indicates the input area subtended by a neuron *and* the input pattern that generates the strongest activation.
RF can be visualized by plotting the weight pattern in the input space.



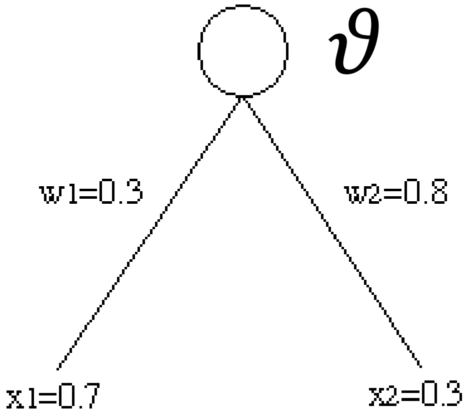


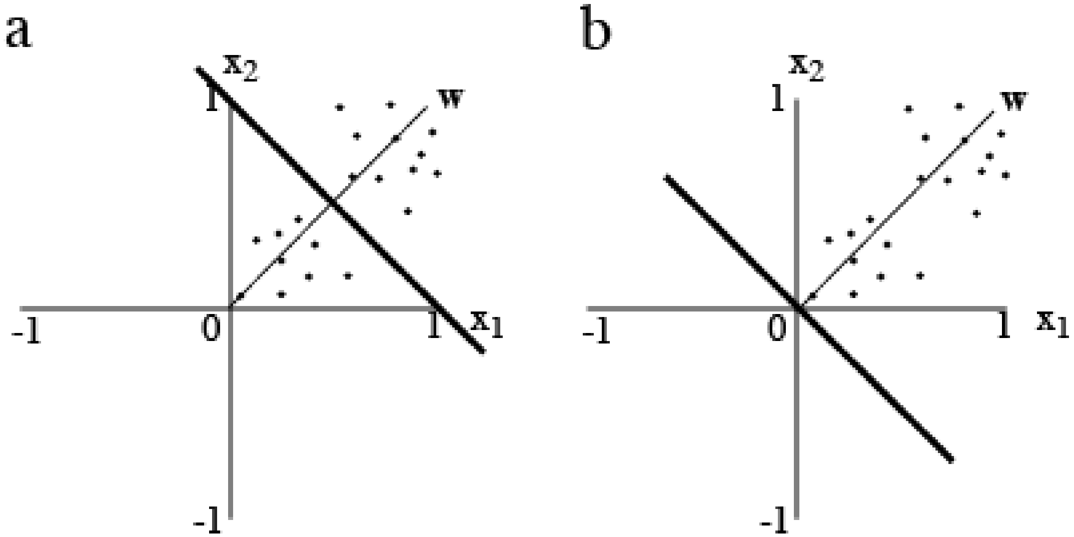
Fully connected
(only some connections are shown)

Neurons can act as classifiers

A neuron divides the input space in two regions, one where weighted input sum >=0 and one where weighted input sum <0. The separation line is defined by the synaptic weights

$$w_1 x_1 + w_2 x_2 - J = 0 \qquad x_2 = \frac{J}{w_2} - \frac{w_1}{w_2} x_1$$





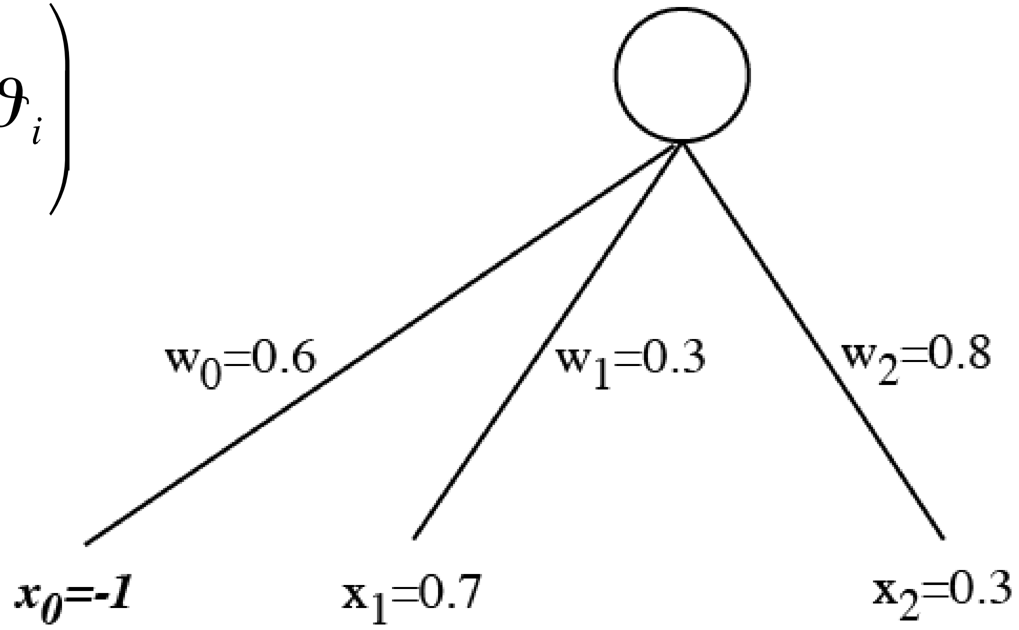
$\vartheta > 0$ $\quad$ $\vartheta = 0$

From Threshold to Bias unit

The threshold can be expressed as an additional weighted input from a special unit, known as bias unit, whose output is always -1.

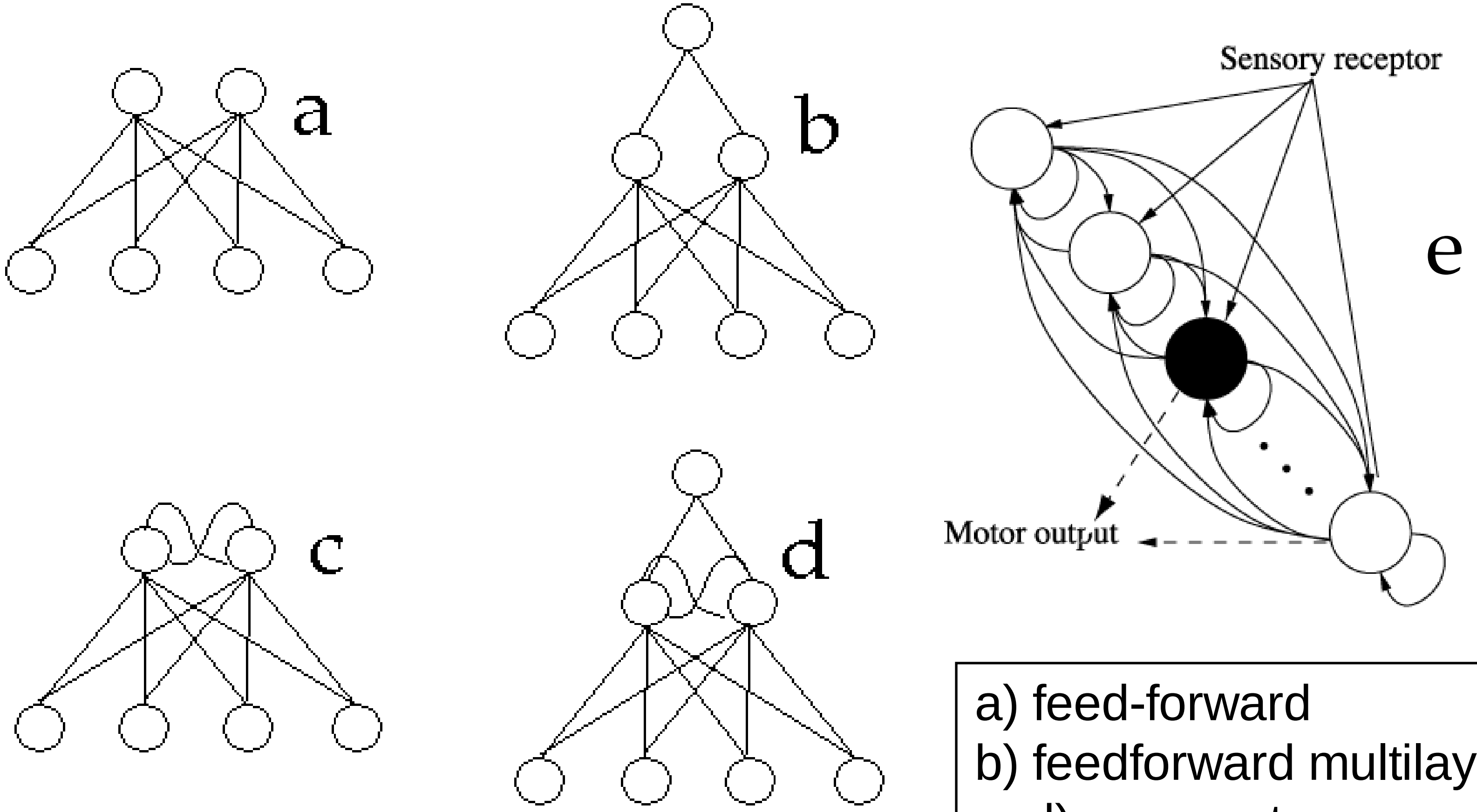
$$y_i = \Phi(A_i) = \Phi\left(\sum_{j=1}^{N} w_{ij} x_j - \vartheta_i\right)$$

$$y_i = \Phi(A_i) = \Phi\left(\sum_{j=0}^{N} w_{ij} x_j\right)$$



- Easier to express/program
- Threshold is adaptable like other weights

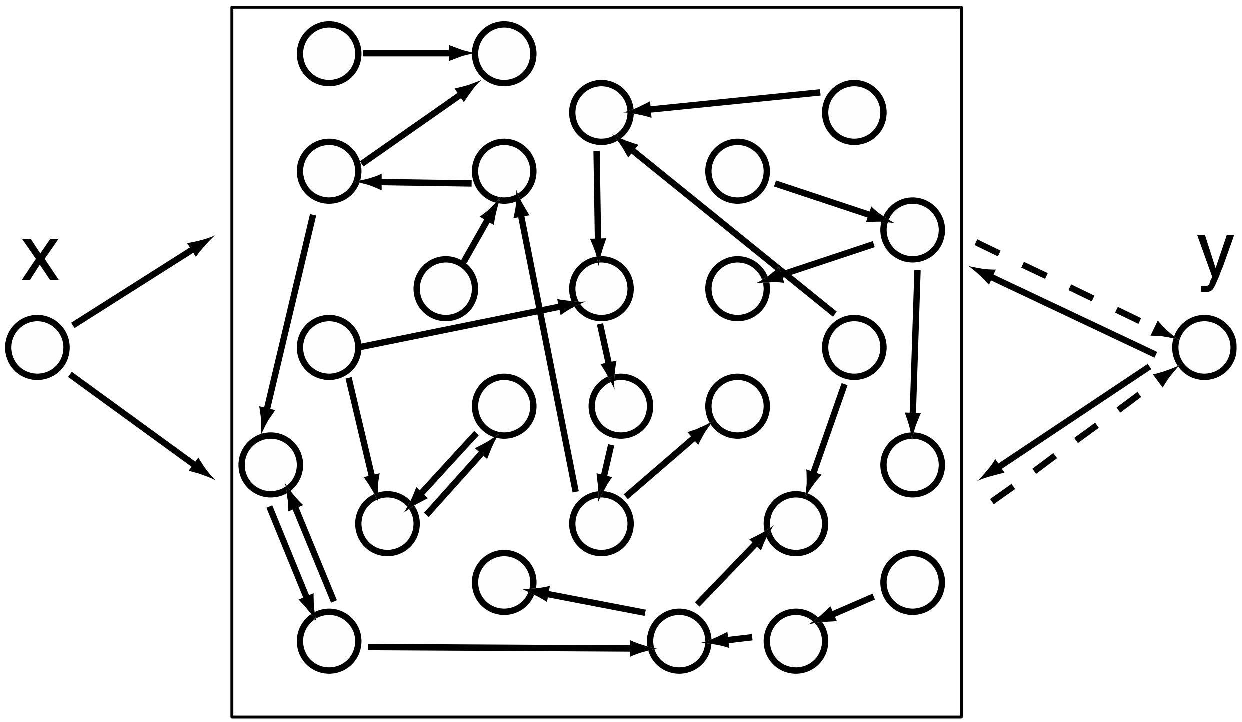
Architectures



a) feed-forward
b) feedforward multilayer
c, d) recurrent
e) fully connected

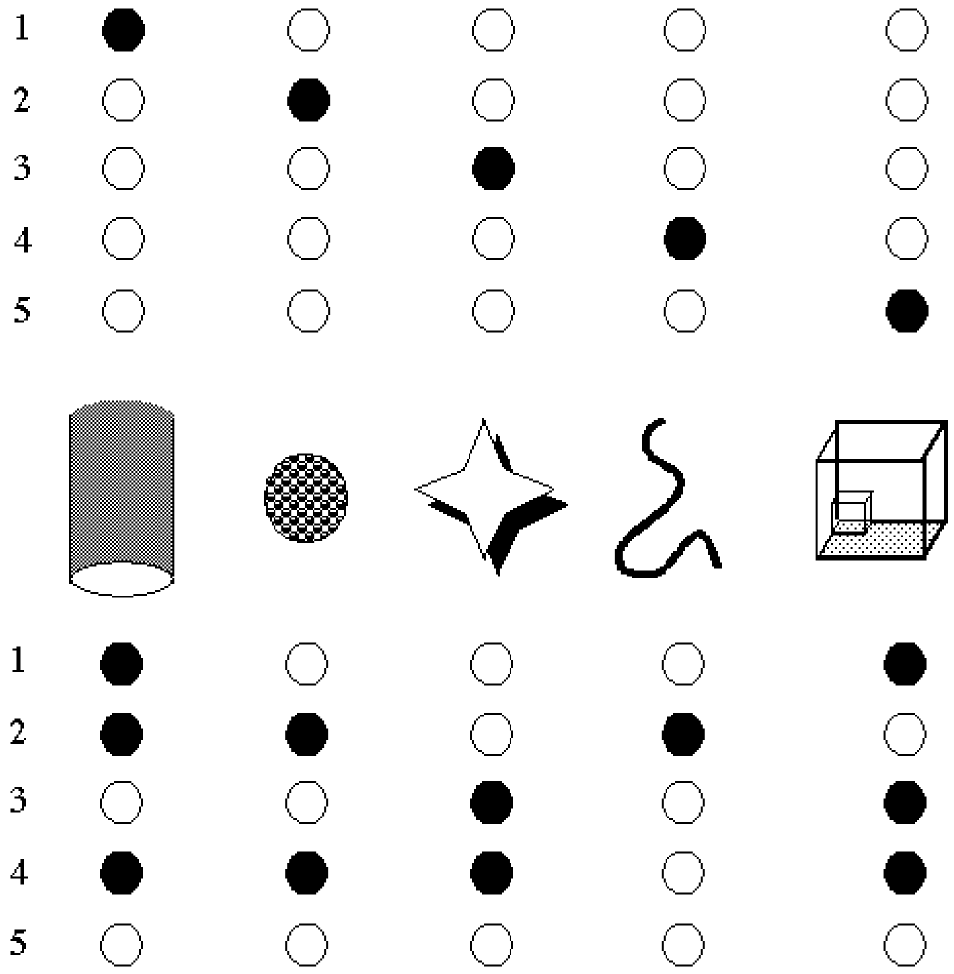
Reservoir Architectures

Exploit rich dynamics in the reservoir of hundreds of randomly interconnected neurons with low connectivity (0.01, e.g)



Liquid State Machines (Maas et al, 2002)
Echo State Networks (Jaeger et Haas, 2004)

Local vs Distributed Input Encoding



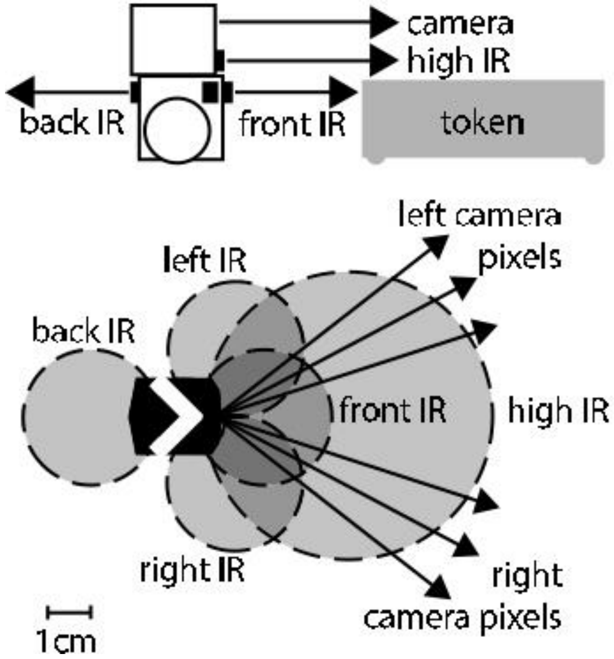
LOCAL
One neuron stands for one item
a.k.a. «Grandmother neurons»
Scalability problem

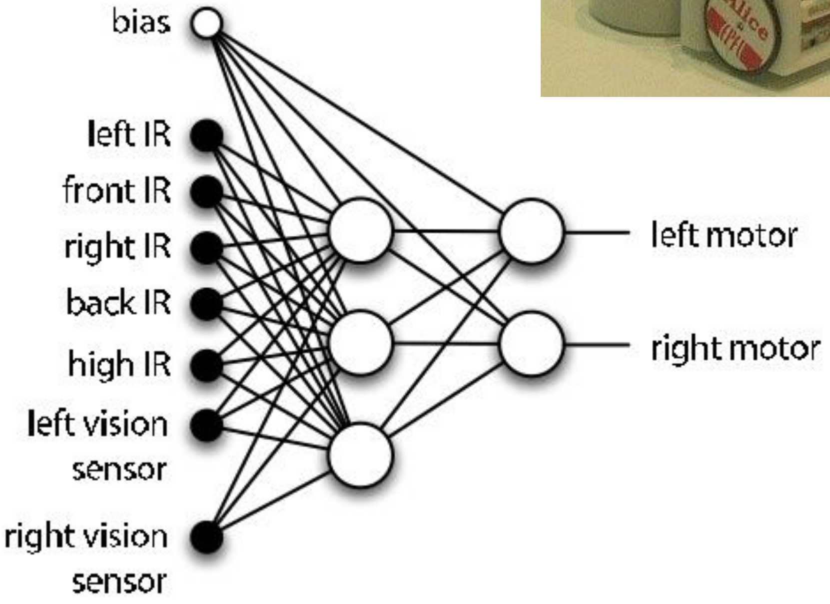
DISTRIBUTED
Neurons encode features (not items)
One neuron can represent >1 item
One item may activate >1 neuron
Robust to damage

Normalisation of sensory input

Input signals from different sensory sources can have different amplitudes that must be normalised to enable comparisons by receiving neurons

$$x_i' = \frac{x_i}{\sqrt{\sum_{j=1}^{N} x_j^2}}$$



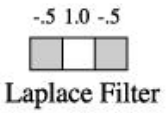


Convolution to capture spatial relationships

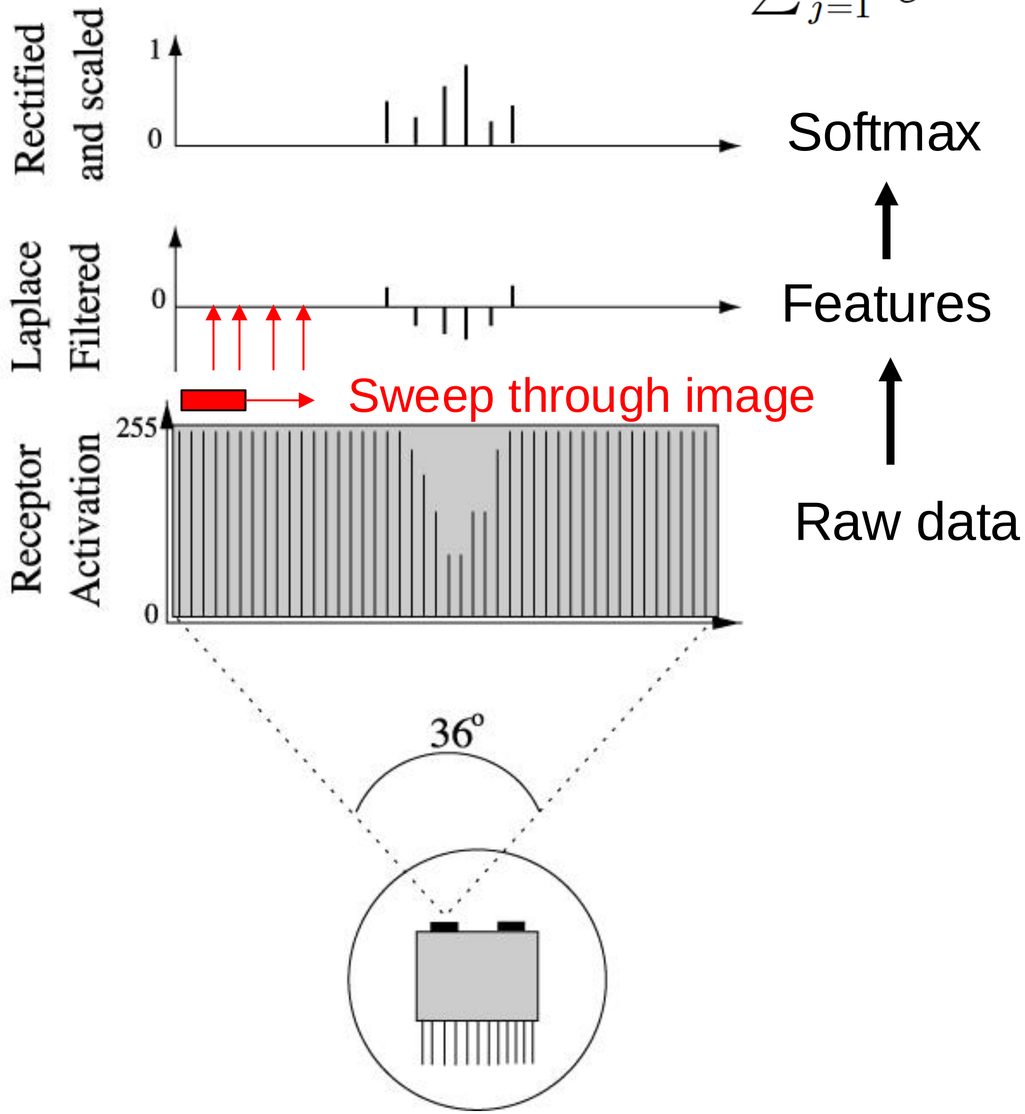
$$\sigma(\vec{z})_i = \frac{e^{z_i}}{\sum_{j=1}^{K} e^{z_j}}$$

We want to find in the image locations with features of interest, for example a brightness contrast

Design a filter (vector) that captures the feature, for example a Laplace filter

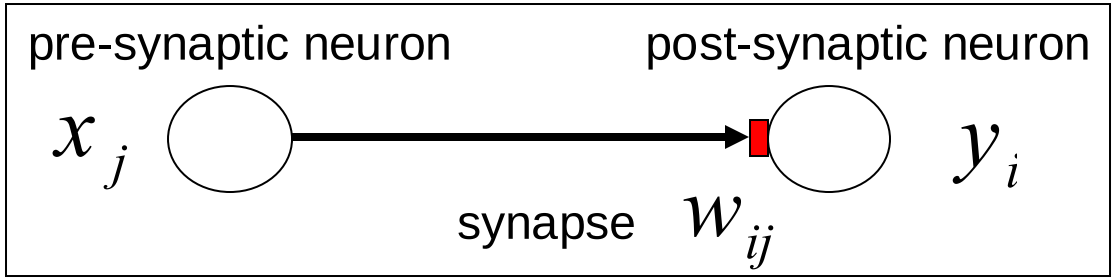


Convolve the image with the filter



Learning

Learning is experience-dependent modification of connection weights



$$\Delta w_{ij} = x_j y_i$$

Hebb’s rule (1949)

Learning is a gradual process and requires many input-output comparisons

Learning cycle

1. Initialize weights (e.g., random values from normal distribution)
2. Present randomly selected input pattern to network
3. Compute values of output units
4. Compute weight modifications
5. Update weights

Standard weight update

$$w_{ij}^{t} = w_{ij}^{t-1} + \eta \Delta w_{ij}$$

learning rate [0,1] (for η)

6. Repeat from 2. until weights do not change anymore

Learning modalities

Unsupervised learning

Supervised learning

Reinforcement learning

Evolution

Evolution and learning

Unsupervised learning: what for?

Input: *x* (images, signals, text, etc.)

Categories (labels): none

Goal: learn compact structure (features) that describes input

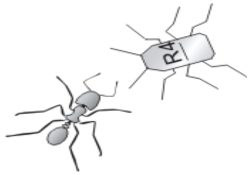
Unsupervised learning

The weight change depends on the activity of the pre-synaptic and of the postsynaptic neurons:

$$\Delta w_{ij} = x_j y_i$$

Unsupervised learning is used for

- Detecting statistical features of the input distribution
- Data compression and reconstruction
- Detect topological relationships in the input data
- Memorization

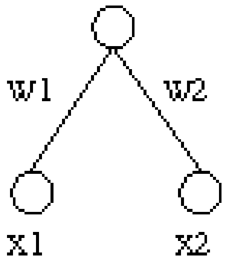


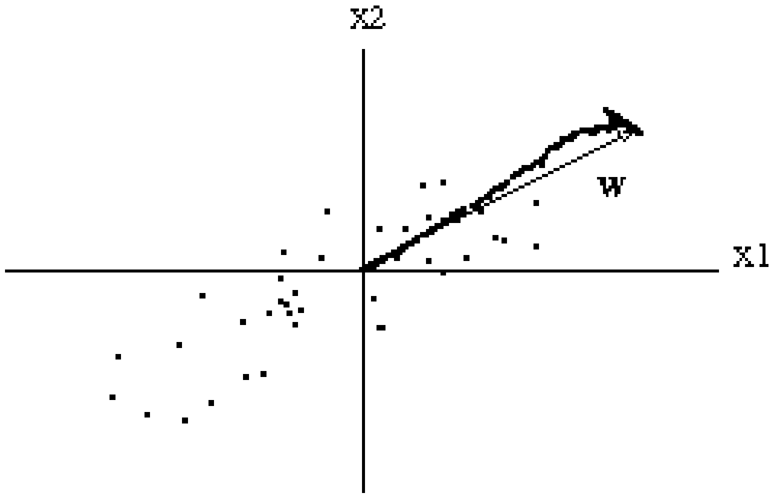
Oja's learning rule

Hebb's rule suffers from **self-amplification** (unbounded growth of weights)**, but b**iological synapses cannot grow indefinitely

Oja (1982) introduced self-limiting growth factor in Hebb rule

$$\Delta w_j = \eta y\left(x_j - w_j y\right)$$

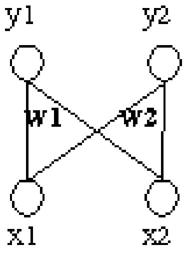




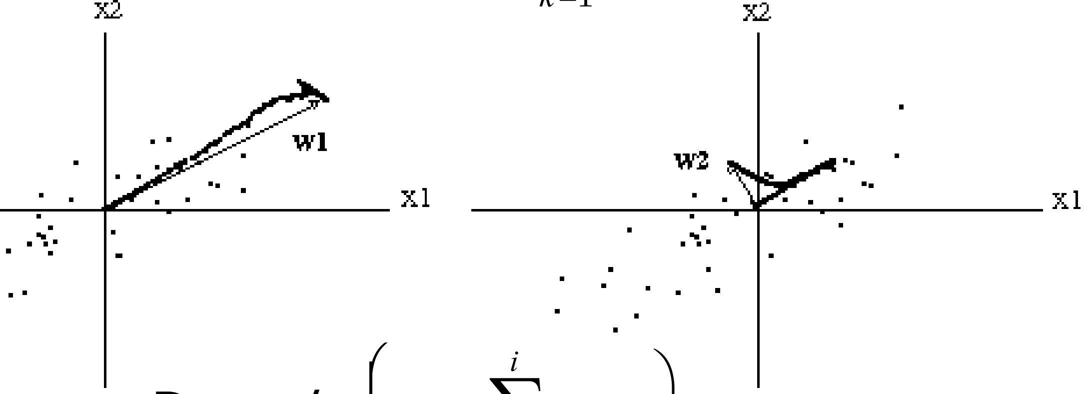
As a result, the weight vector develops along the direction of maximal variance of the input distribution.

Neuron learns **how familar** a new pattern is: input patterns that are closer to this vector elicit stronger response than patterns that are far away.

Principal Component Analysis

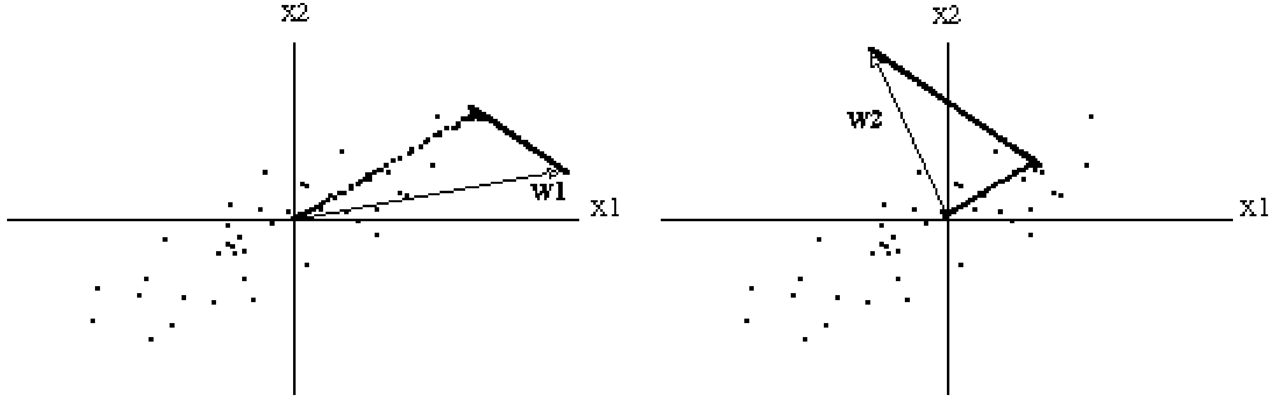


$$Dw_{ij} = hy_i\left(x_j - \sum_{k=1}^{N} w_{kj} y_k\right)$$



Oja rule for N output units develops weights that span the sub-space of the N principal components of the input distribution.

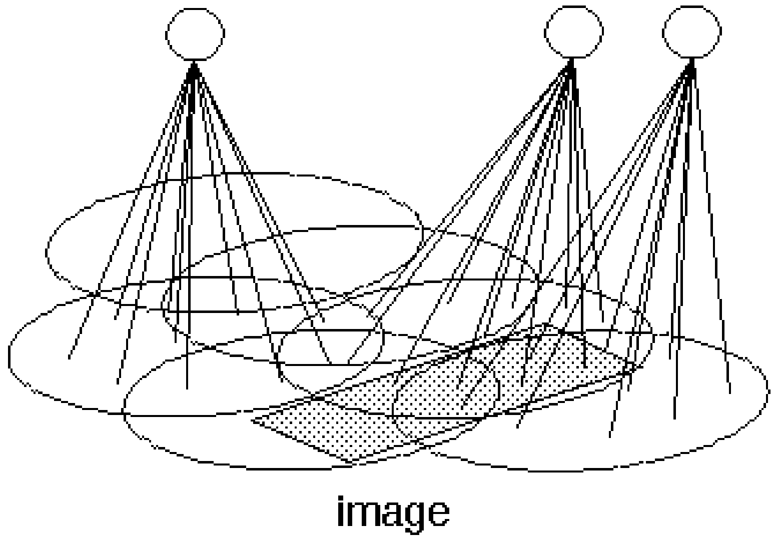
$$Dw_{ij} = hy_i\left(x_j - \sum_{k=1}^{i} w_{kj} y_k\right)$$

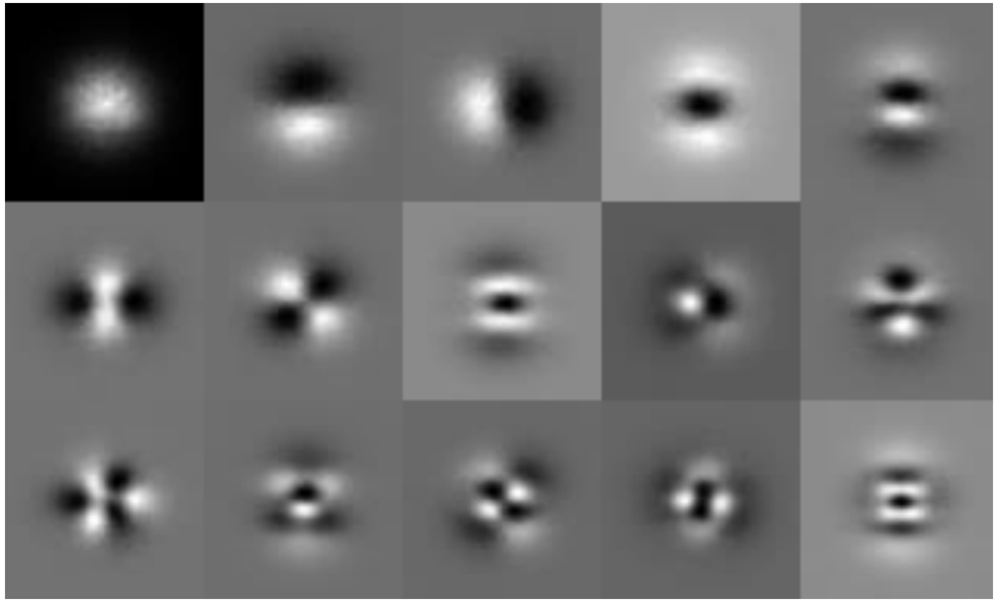


Sanger rule for N output units develops weights that correspond to the N principal components of the input distribution.

Do brains compute PCA?

An Oja network with multiple output units exposed to a large set of natural images develops *receptive fields* similar to those found in the visual cortex of all mammals [Hancock et al., 1992]

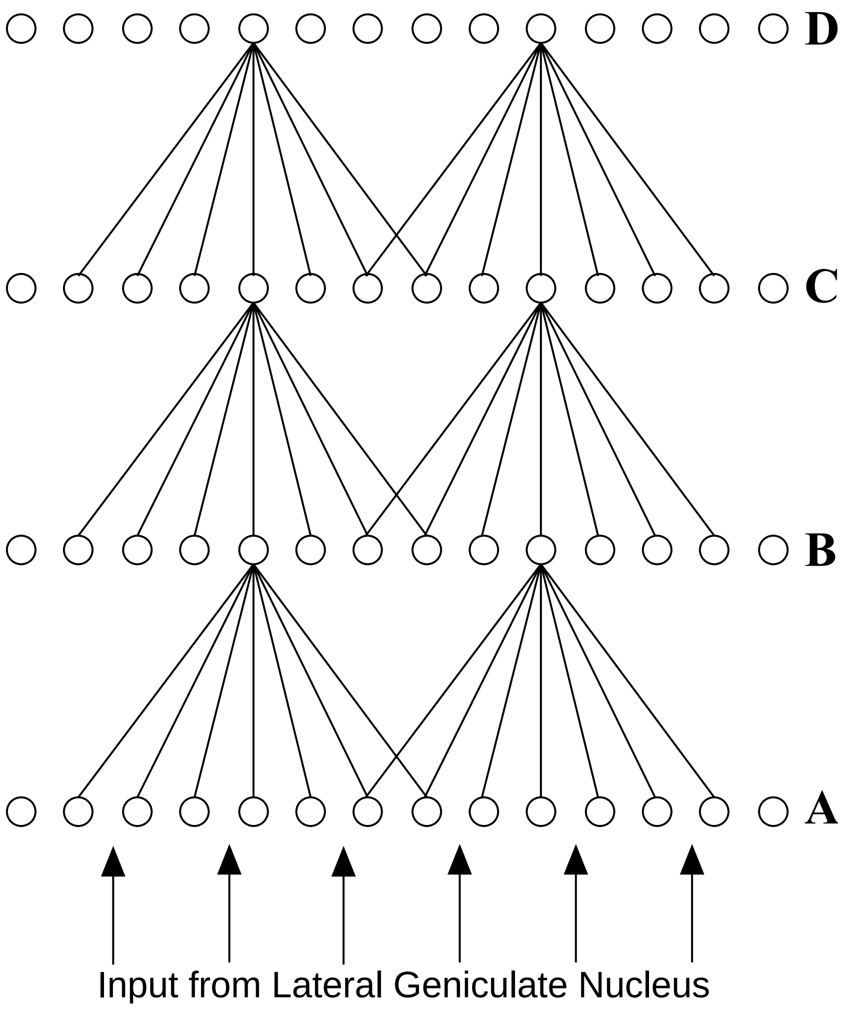




Mammals are born with pre-formed hierarchically-organized feature detectors. But they never saw anything in the womb: how can it be?

Multilayer Feature Detection

Linsker (1986)

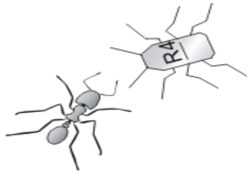


Topologically restricted connectivity

Linear activation function

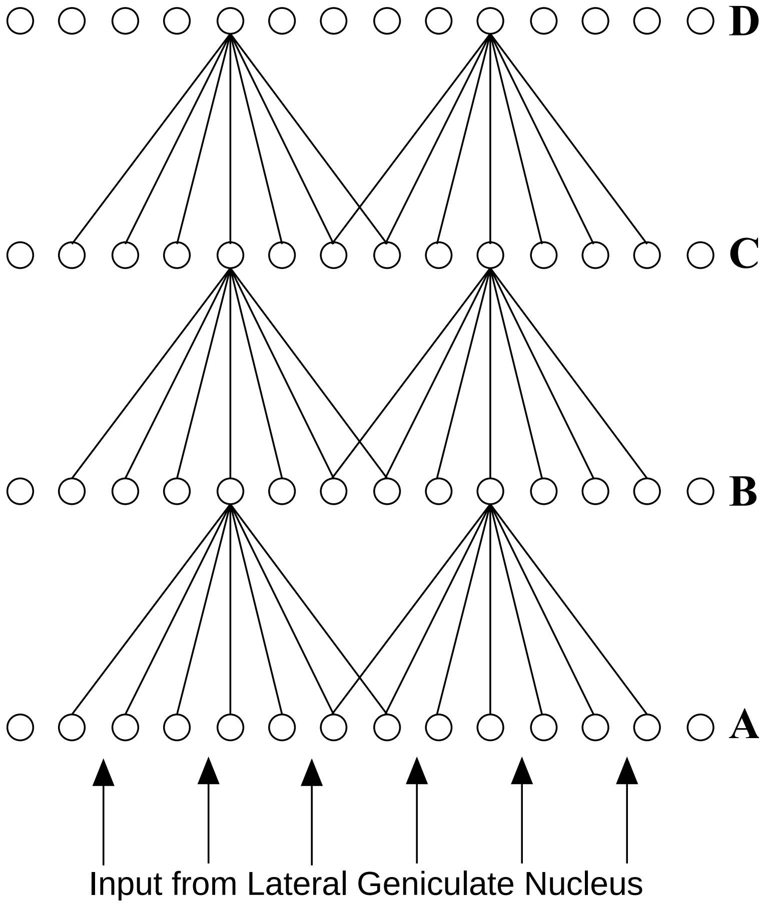
Plain Hebbian learning with weight clipping at *w*+ and *w*-

Learn one layer at a time, starting from lower layer

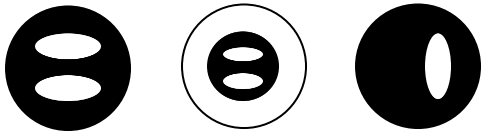


Emerging Receptive Fields

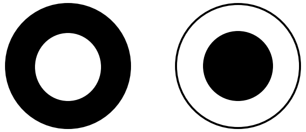
Linsker (1986)



Response: Complex feature detectors



Response: Simple feature detectors



Response: Average luminosity in RF

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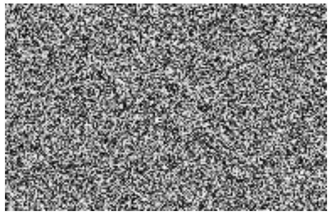
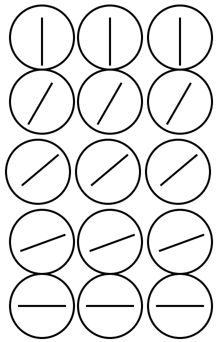


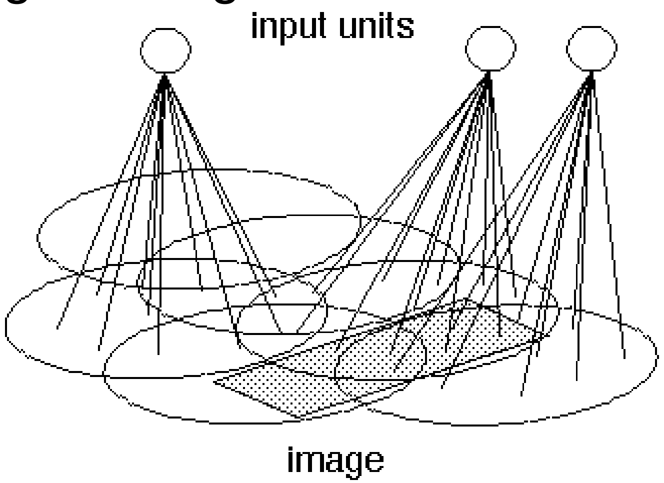
Sensory maps

Neighbouring neurons respond to similar patterns with gradual transitions

The <u>visual cortex</u> is organized in specialized modules. Each module is composed by a series of columns of neurons. For example, neurons in early modules respond to bars at different orientation

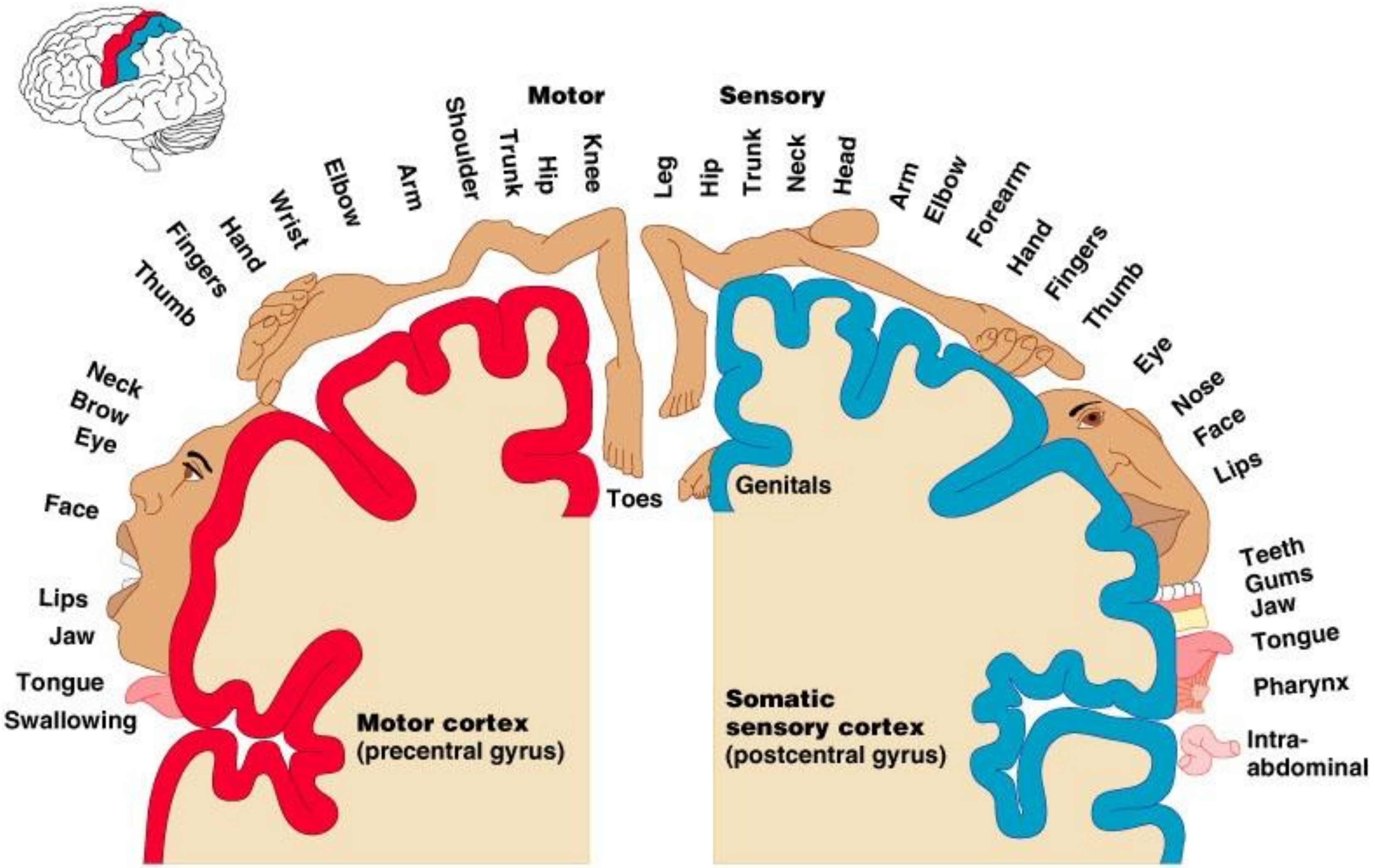


1. The bar orientation gradually varies along the column.
2. Neighbouring columns correspond to neighbouring areas of the retina (**retinotopic maps**).



A similar structure exists in the auditory cortex (**tonotopic maps**).

Sensory-Motor Body Map

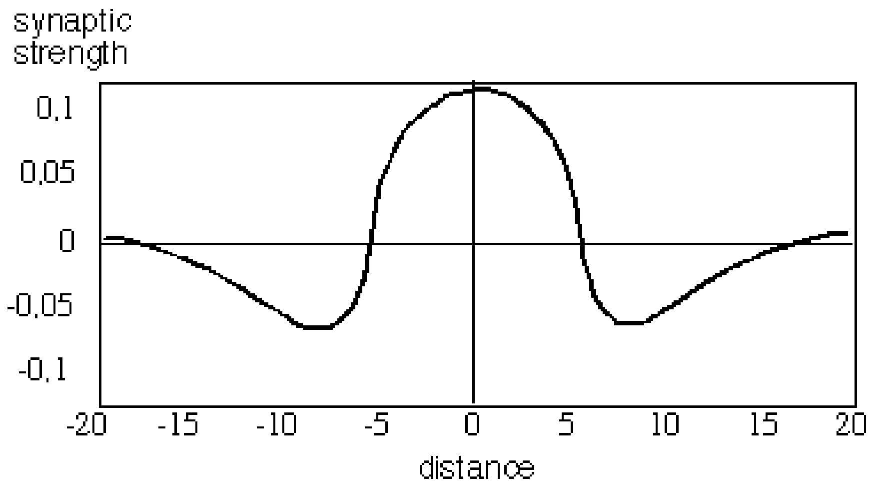


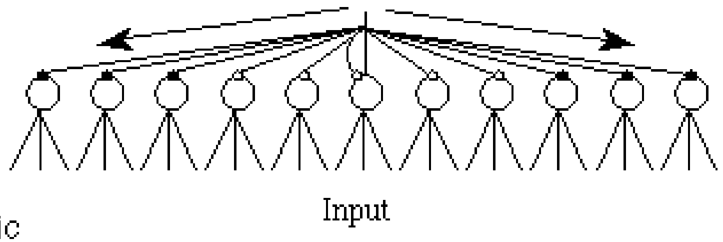
Lateral connections to neighbouring neurons

Cortical neurons display the following pattern of *projective* connectivity:

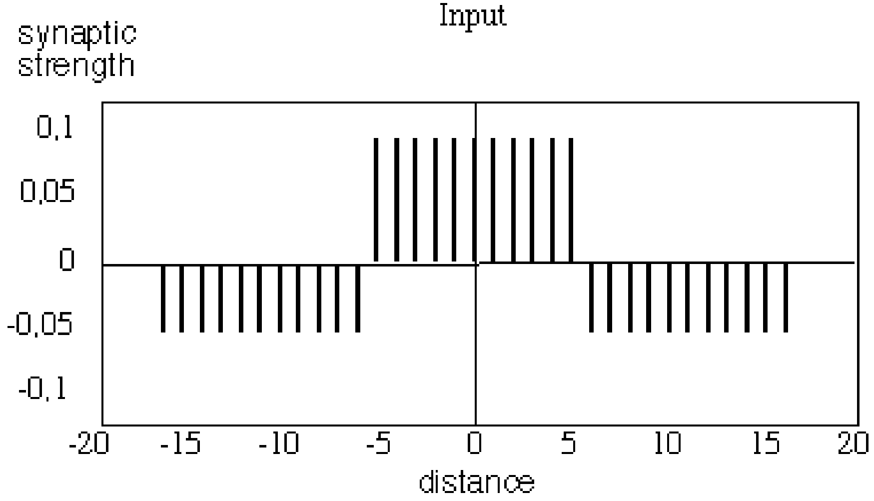
- up to 50-100 μm radius = excitatory
- up tp 200-500 μm radius = inhibitory
- up to few cm radius = slightly excitatory

Also known as *Mexican Hat distribution*

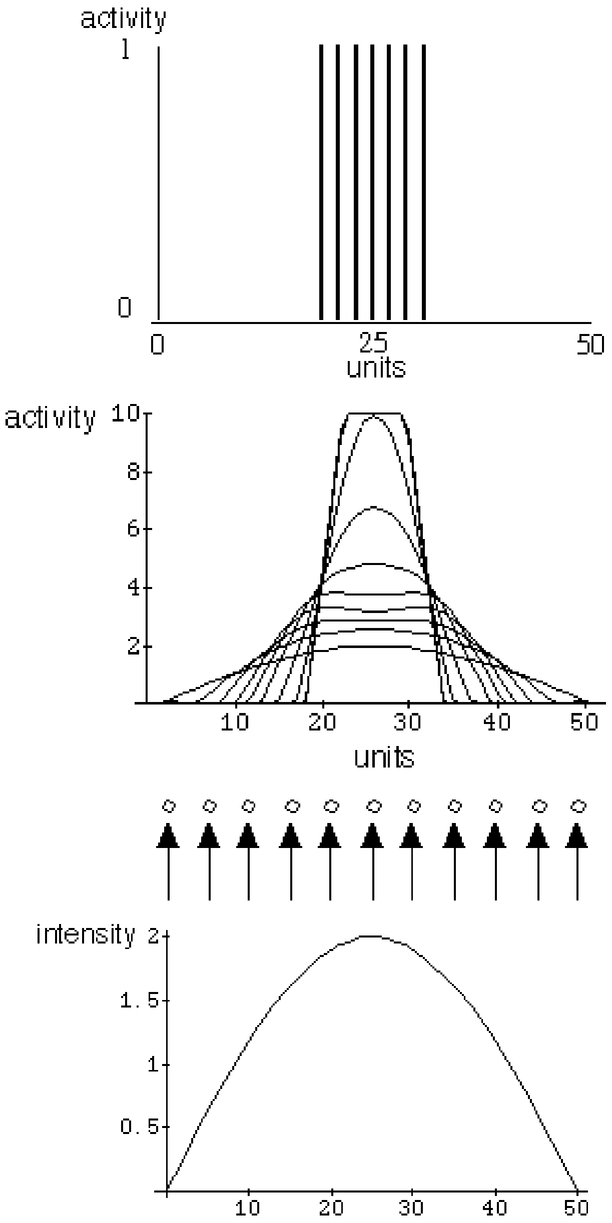




In a neural network, we can approximate the Mexican hat with a bipolar weight distribution.



Formation of neural bubbles around strongest input



Simplification: set output of unit with highest activation and its *n* neighbors to 1, and all other units to 0

Gradual emergence of bubble centered around unit with strongest activation

Laterally connected neurons

Input pattern

Self-Organizing Topological Maps

Kohonen (1982)

Let's apply Hebb rule to a layer of laterally connected neurons

$$y_i = \Phi(A_i) = \begin{cases} 1 & \text{if within neighbourhood } \Psi(y) \text{ of neuron } y \text{ with highest activation} \\ 0 & \text{otherwise} \end{cases}$$

$$\Delta w_{ij} = \eta y_i x_j - \Psi(y_i) w_{ij}$$

$$\Psi(y_i) = \begin{cases} \psi & \text{if } y_i = 1 \\ 0 & \text{if } y_i = 0 \end{cases}$$

If we set $\Psi(y_i)$ equal to η, then the learning rule becomes:

$$\Delta w_{ij} = \begin{cases} \eta(x_j - w_{ij}) & \text{if } y_i = 1 \\ 0 & \text{if } y_i = 0 \end{cases}$$

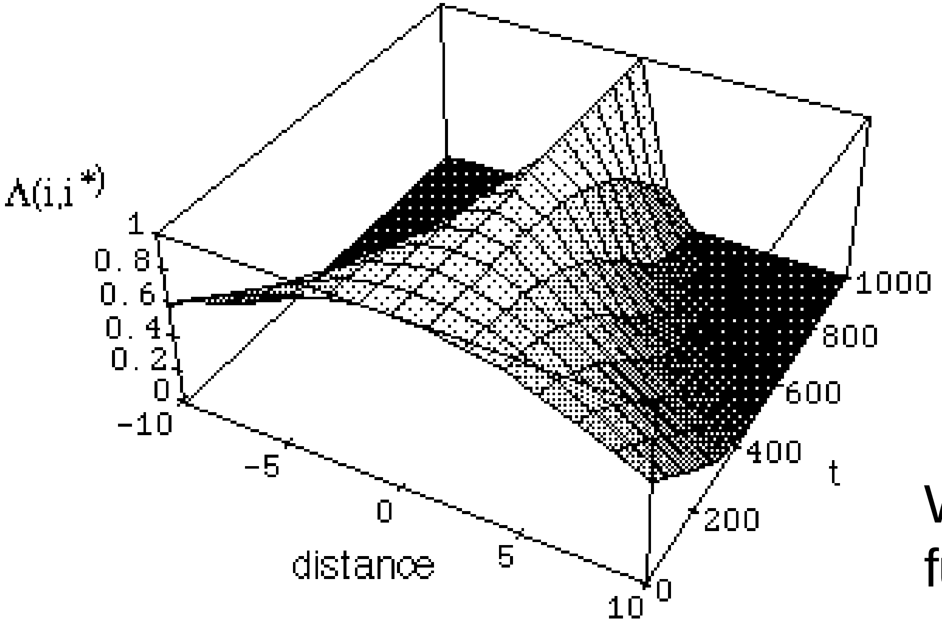
and

$$\mathbf{w}_i^{t+1} = \begin{cases} \mathbf{w}_i^t + \eta(\mathbf{x} - \mathbf{w}_i^t) & \text{if } y_i = 1 \\ \mathbf{w}_i^t & \text{if } y_i = 0 \end{cases}$$

1. The weights are changed only for the neurons that are geographically near the neuron with the highest activity,
2. The change moves the weight vector towards the input pattern.

Neighborhood function

The neighbourhood size $\Psi(y)$ is a critical aspect of map self-organization. It should be large at the beginning of training to give a chance to all neurons to change weights and gradually shrink

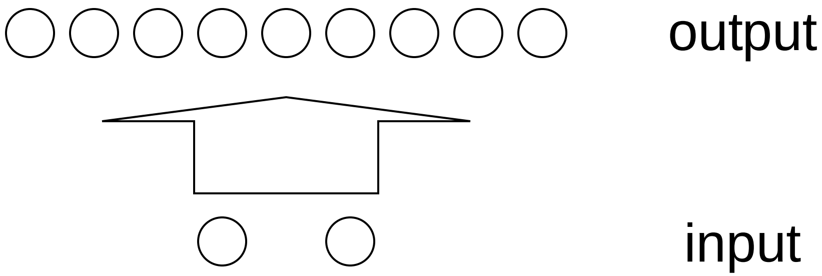


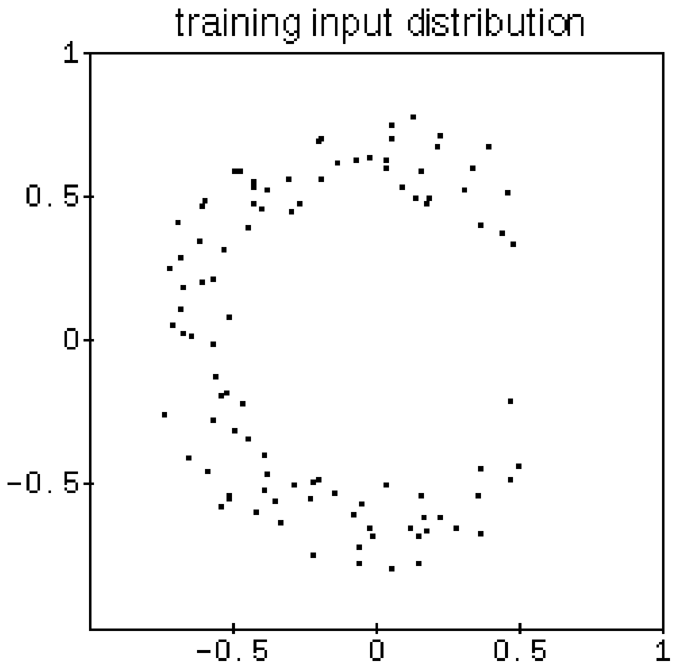
$$\Lambda(i,i^*) = \begin{cases} 1 & \text{if } \|\mathbf{c}_i - \mathbf{c}_{i^*}\| <= r \\ 0 & \text{otherwise} \end{cases}$$

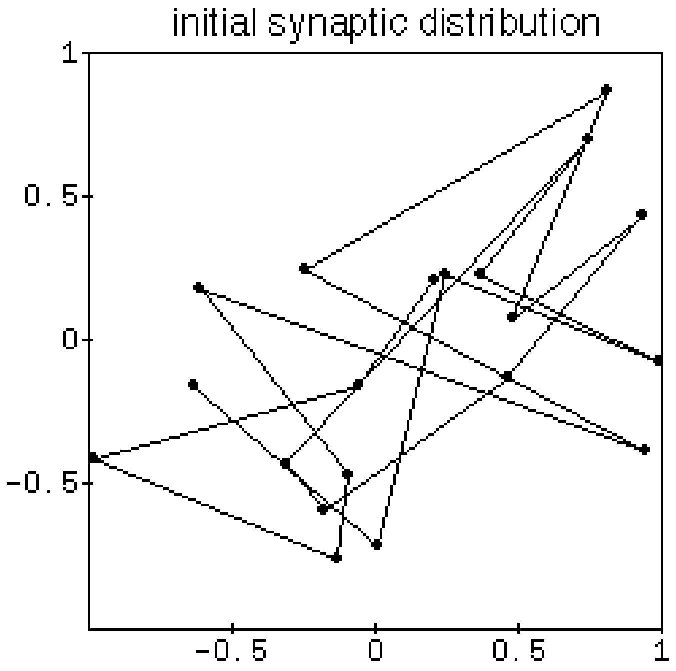
We can incorporate the neighborhod function in the learning rule

$$\Delta w_{ij} = \eta \Lambda(i, i^*)\left(x_j - w_{ij}\right)$$

Example of self-organizing map



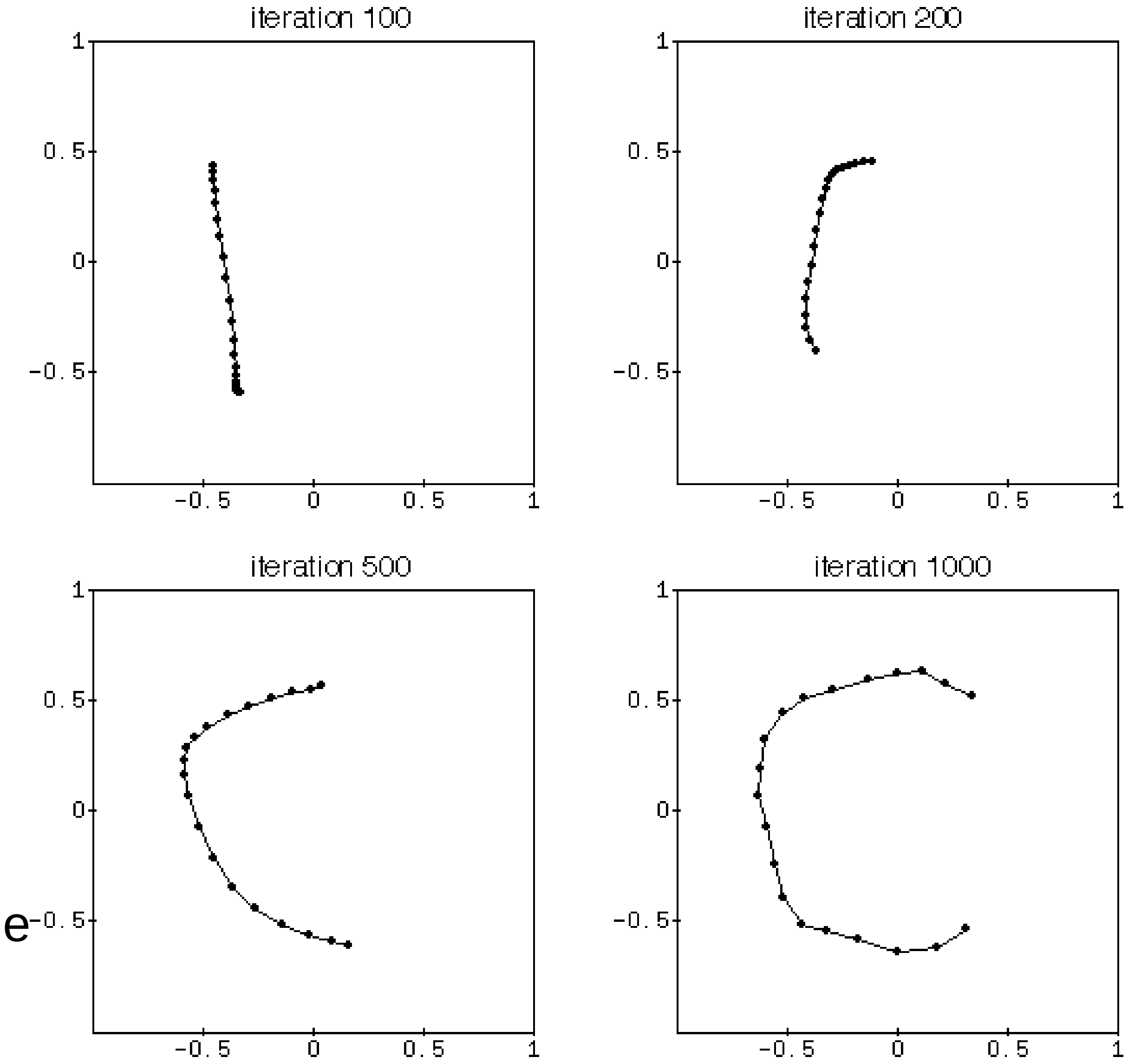




Self-organization phases

Ordering phase:
Fast
Neighborhood change

Convergence phase:
Slow
No neighborhood change



Supervised learning: what for

Input: *x* (images, signals, text, etc.)

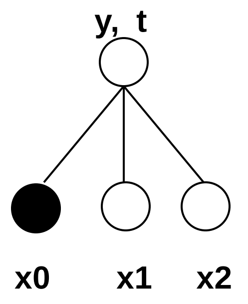
Category (label): *y* (eat, wear, wear, eat, wear, eat)

Goal: learn mapping between input data and labels

Supervised Learning

- **Teacher** provides desired responses for a set of training patterns
- Synaptic weights are modified in order to reduce the **error** between the output *y* and its desired output *t* (a.k.a. teaching input)

Widrow-Hoff defined the error with the symbol delta: $\delta_i = t_i - y_i$
(a.k.a. delta rule)



linear units

$$w_{ij} = rnd(\pm 0.1)$$ initialize weights to random values

$$y_i = \sum_{j=0} w_{ij} x_j$$ present input pattern and compute neuron output

$$\Delta w_{ij} = \eta (t_i - y_i) x_j$$ compute weight change using difference between desired output and neuron output

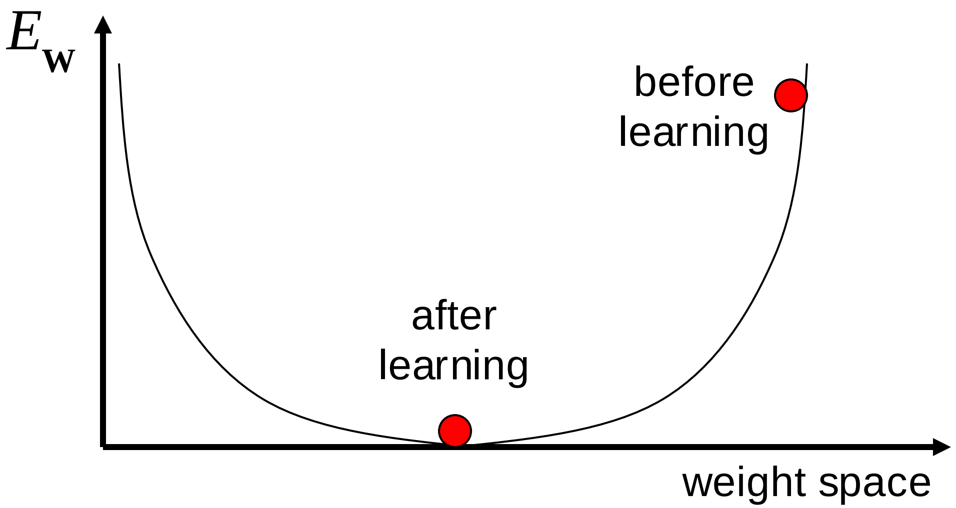
$$w_{ij} = w_{ij}^{t-1} + \Delta w_{ij}$$ get new weights by adding computed change to previous weight values

repeat for every input/output pair until error is 0

Error (loss) function

The delta rule modifies the weights to descend the gradient of the error function

$$E_{\mathbf{w}} = \frac{1}{2}\sum_{\mu}\sum_{i}\left(t_i^{\mu} - \sum_{j=0} w_{ij} x_j^{\mu}\right)^2$$



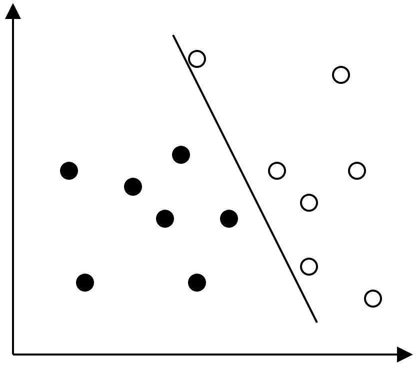
Error space for a network with a single layer of synaptic weights
*(**perceptron**, Rosenblatt, 1962)*

Linear Separability

Perceptrons can solve only problems whose input/output space is **linearly separable**.

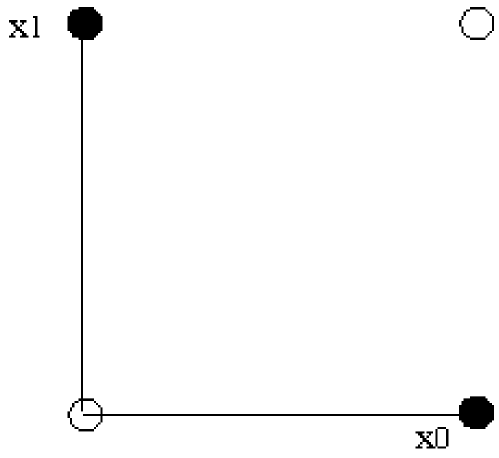
Several real world problems are not linearly separable.

input/output space



● class A
○ class B

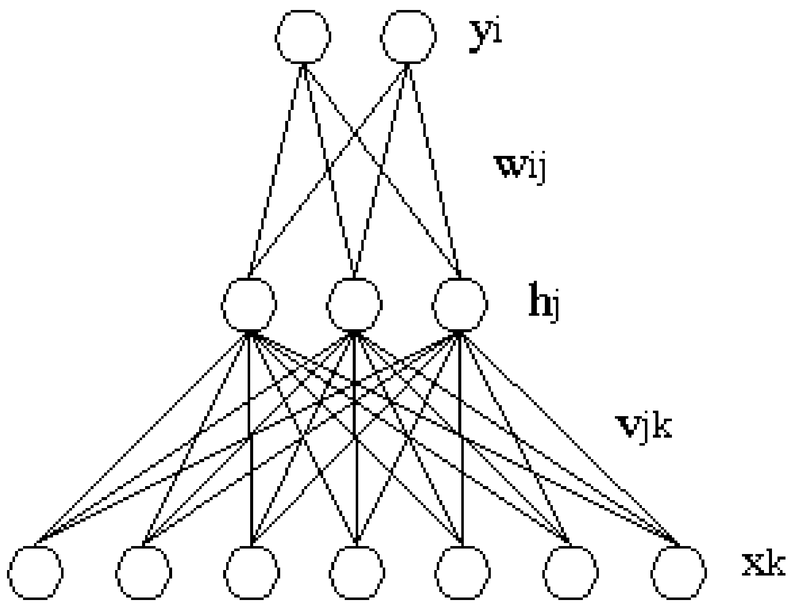
Example of XOR problem



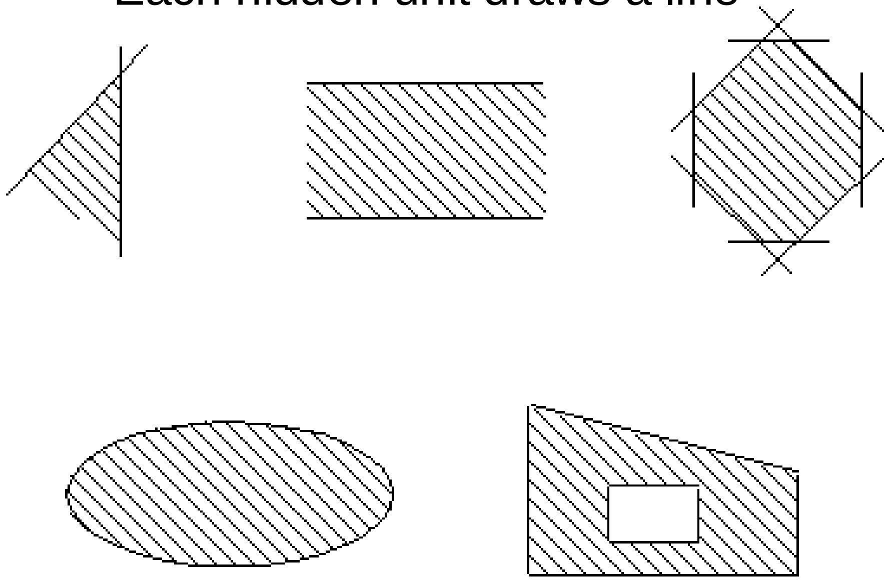
x0	x1	t	
0	0	0	○
1	1	0	○
1	0	1	●
0	1	1	●

Multi-layer Perceptron (MLP)

- Multi-layer neural networks can solve problems that are not linearly separable
- Hidden units re-map input space into a space which can be linearly separated by output units.



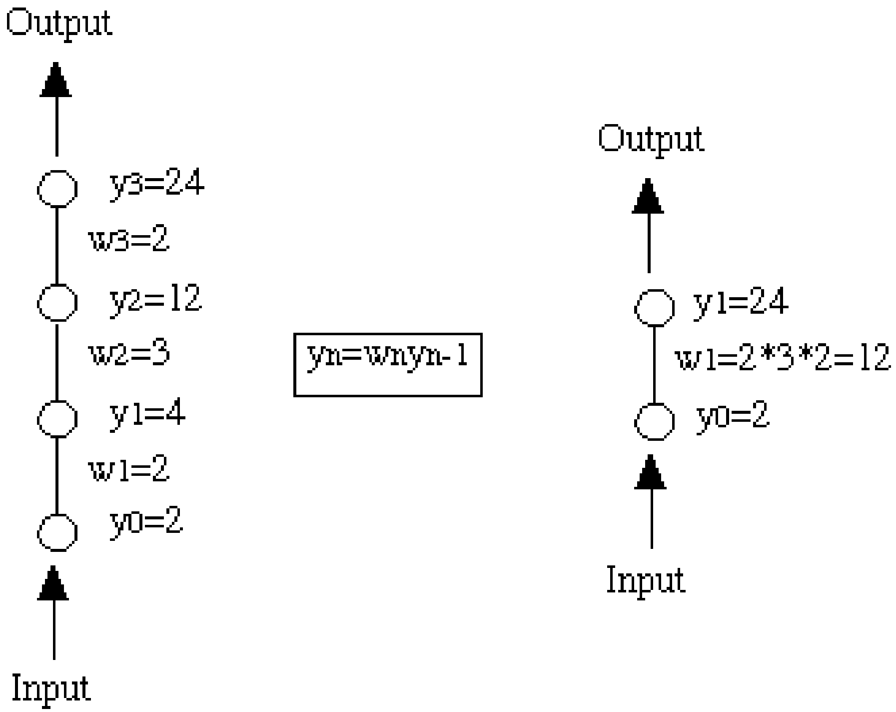
Each hidden unit draws a line



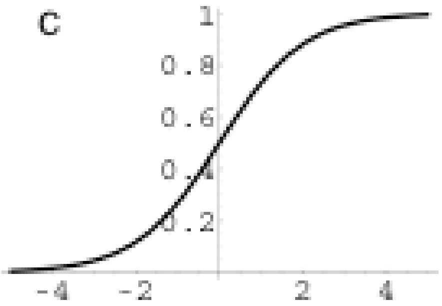
Output units “look” at regions (in/out)

Output Function in MLP

• Multi-layer networks should not use linear output functions because a linear transformation of a linear transformation remains a linear transformation.
• Therefore, such a network would be equivalent to a network with a single layer



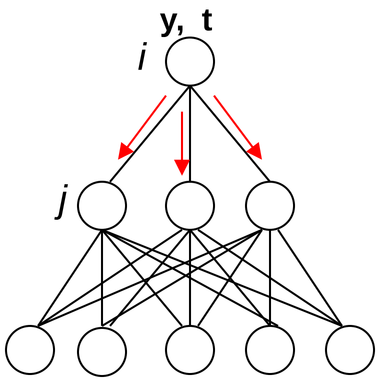
For example, sigmoid function



$$\Phi(x) = \frac{1}{1+e^{-kx}}$$

Back-propagation of Error

In a simple perceptron, it is easy to change the weights to minimize the error between output of the network and desired output.



$$\delta_i = t_i - y_i \qquad Dw_{ij} = hd_i x_j$$

$$\delta_i = (t_i - y_i)\, \dot{\Phi}(A_i)$$

in the case of non-linear output functions, add derivative of output

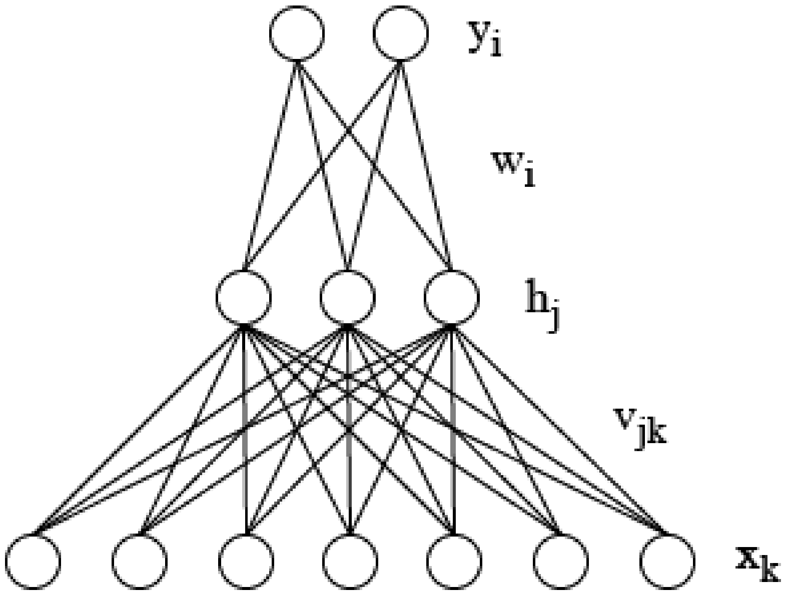
In a multilayer network, what is the error of the hidden units? This information is needed to change the weights between input units and hidden units.

The idea suggested by Rumelhart et al. in 1986 is to propagate the error of the output units backward to the hidden units through the connection weights:

$$\delta_j = \dot{\Phi}(A_j) \sum_i w_{ij} \delta_i$$

Once we have the error for the hidden units, we can change the lower layer of connection weights with the same formula used for the upper layer.

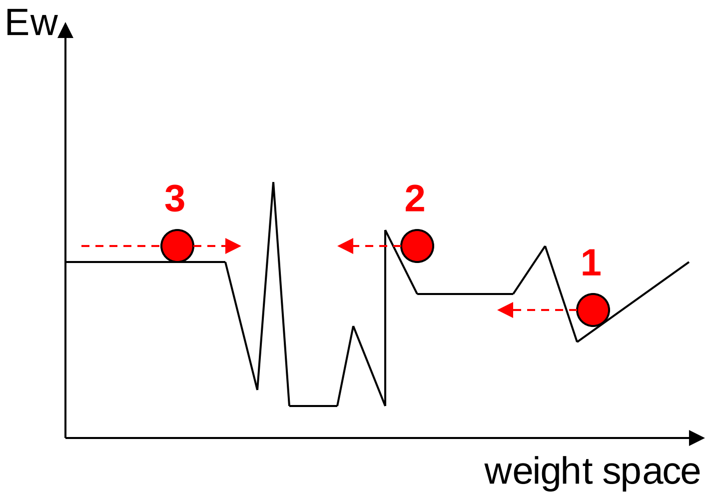
Algorithm



1. Initialize weights (random, around 0)
2. Present pattern $x_k^\mu = s_k^n$
3. Compute hidden $h_j^\mu = \Phi\left(\sum_k v_{jk} x_k^\mu\right)$
4. Compute output $y_i^\mu = \Phi\left(\sum_j w_{ij} h_j^\mu\right)$
5. Compute delta output $\delta_i^\mu = \dot{\Phi}\left(\sum_j w_{ij} h_j^\mu\right)\left(t_i^\mu - y_i^\mu\right)$ $\quad \delta_i^\mu = y_i^\mu\left(1 - y_i^\mu\right)\left(t_i^\mu - y_i^\mu\right)$
6. Compute delta hidden $\delta_j^\mu = h_j^\mu\left(1 - h_j^\mu\right)\sum_i w_{ij} \delta_i^\mu$
7. Compute weight change $\Delta w_{ij}^\mu = \delta_i^\mu h_j^\mu, \quad \Delta v_{jk}^\mu = \delta_j^\mu x_k^\mu$
8. Update weights $w_{ij}^t = w_{ij}^{t-1} + \eta \Delta w_{ij}^\mu, \quad v_{jk}^t = v_{jk}^{t-1} + \eta \Delta v_{jk}^\mu$

Using Back-Propagation

Error space can be complex in multilayer networks: local minima and flat areas

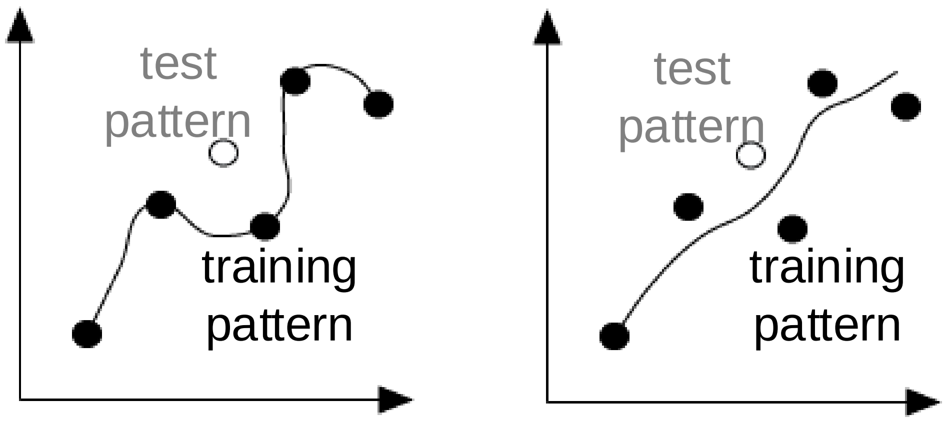


1. Large learning rate: take large steps in the direction of the gradient descent

2. Momentum: add direction component from last update $\Delta w_{ij}^{t} = \eta \delta_i + \alpha \Delta w_{ij}^{t-1}$

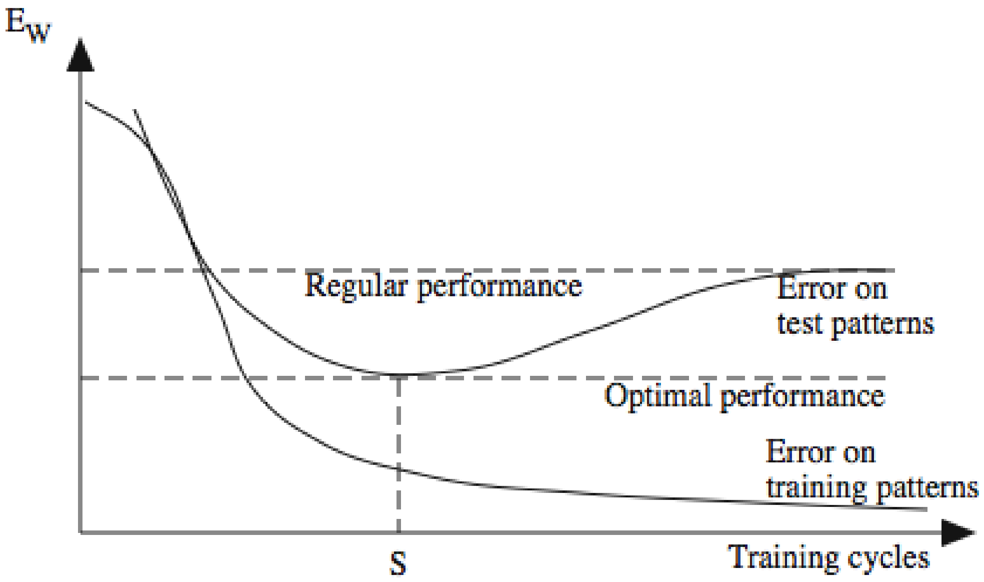
3. Additive constant: keep moving when no gradient $d_i^m = (\dot{F} + k)(t_i^m - y_i^m)$

Over-fitting



Overfitting training data leads to poor generalisation

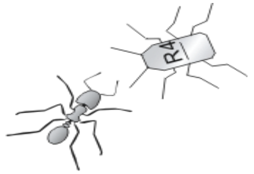
Overfitting can derive from too many weights and/or too long learning of training patterns



Solution: Use a Validation Set

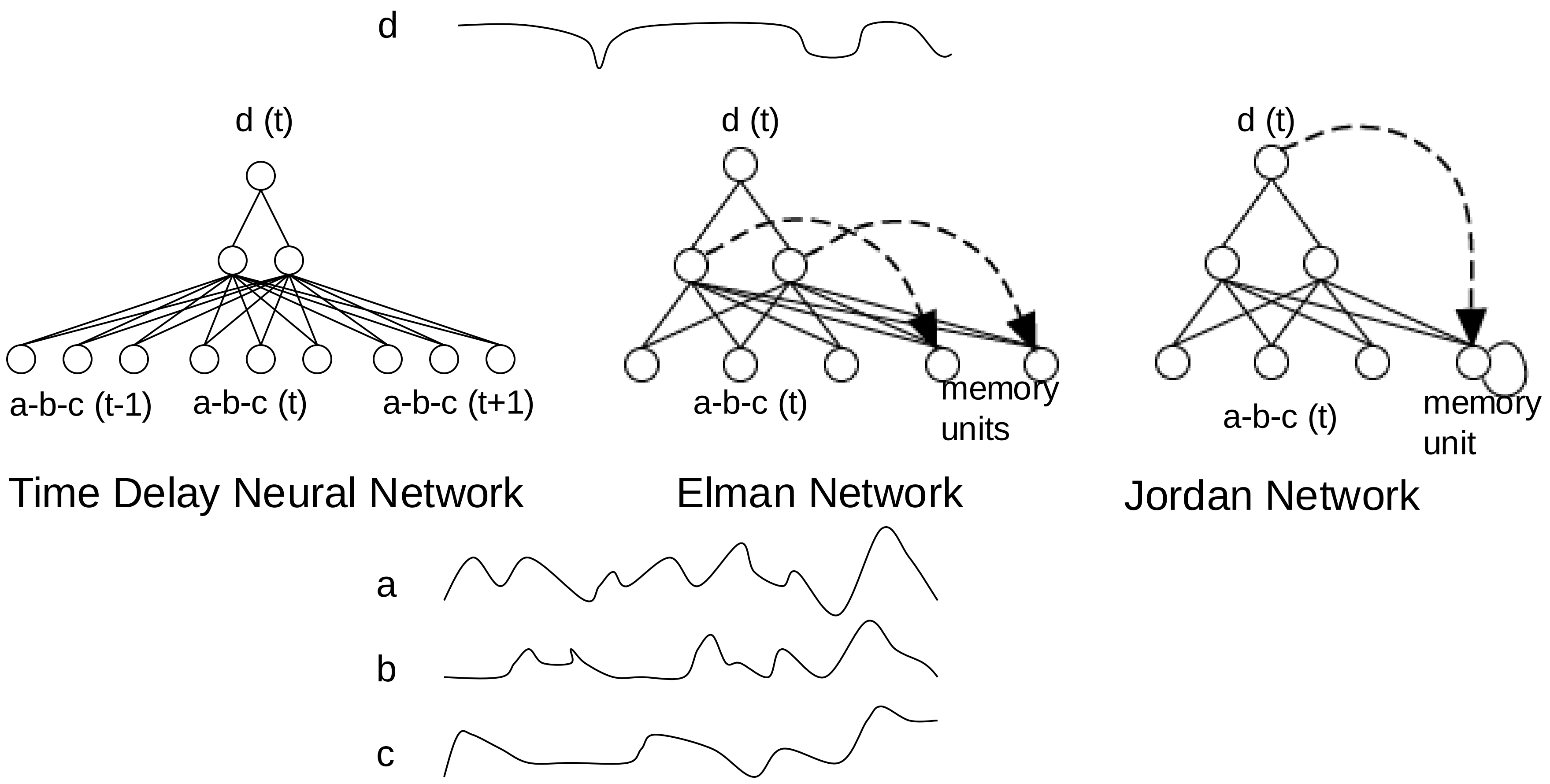
Divide available data into:
- training set (for weight update)
- validation set (for error monitoring)

Stop training when error for validation set starts growing



How to learn time-dependent features

Learning of time-dependent features is necessary in production of language, behavior, predictions



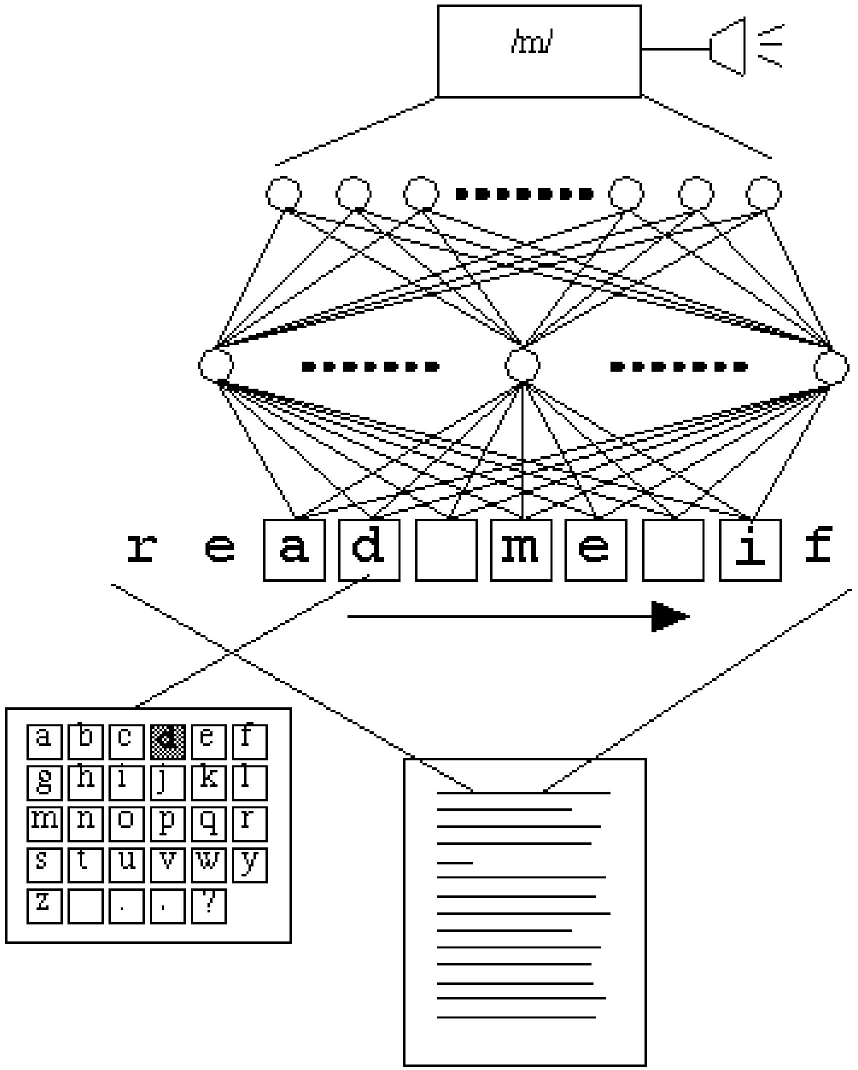
NETtalk

A neural network that learns to read aloud written text:

• 7 x 29 input units encode characters within a 7-position window(TDNN)
• 26 output units encode english phonemes
• approx. 80 hidden units

Training on 1000-word text, reads any text with 95% accuracy

Learns like humans: segmentation, bla-bla, short words, long words



[Sejnowski & Rosenberg, 1987]