Renewable Energy: Geothermal Solution

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Problem 1

(a) The mass of the air to be heated is:

$$m_{\mathrm{air}} = V \cdot \rho_{\mathrm{air}} = 8000\,\mathrm{m}^3 \cdot 1.29\,\mathrm{kg/m}^3 = 10320\,\mathrm{kg}$$

The required energy change:

$$\Delta\tilde{E} = m_{\mathrm{air}} \cdot c_{\mathrm{air}} \cdot \frac{\Delta T}{\Delta t} = 10320\,\mathrm{kg} \cdot 1000\,\frac{\mathrm{J}}{\mathrm{kg}\cdot\mathrm{K}} \cdot 7.3 \cdot 10^{-3}\,\mathrm{K/s} = 75336\,\mathrm{J/s}$$

Installed heating capacity:

$$P_n = 75.3\,\mathrm{kW}$$

(b) COP of heat pump = 4.2
E = Electrical energy (for pump)
Q_n = Useful heat
Q_u = Ambient heat

$$Q_n = Q_u + E$$

Coefficient of power for heat pump is defined as:

$$\mathrm{COP} = \frac{Q_n}{E} = \frac{P_n}{P_{\mathrm{el}}}$$

This implies electrical power:

$$P_{\mathrm{el}} = \frac{75.3\,\mathrm{kW}}{4.2} = 17.9\,\mathrm{kW}$$

Heating power of the probe corresponds to Q_n, hence:

$$P_n = P_u + P_{\mathrm{el}} \implies P_u = 75.3\,\mathrm{kW} - 17.9\,\mathrm{kW} = 57.4\,\mathrm{kW}$$

(c) l is the length of the probe. Using P_u from part (b):

$$l \cdot 52\,\frac{\mathrm{W}}{\mathrm{m}} = 57.4\,\mathrm{kW} \implies l = 1104\,\mathrm{m}$$

We also have:

$$\frac{Q_u}{l} = \frac{Q_n - E}{l} = \frac{Q_n - \frac{Q_n}{\mathrm{COP}}}{l}$$

Insertion of annual heating demand Q_n = 135000 kWh gives:

$$\frac{Q_u}{l} = 93.5\,\frac{\mathrm{kWh}}{\mathrm{m}} < 110\,\frac{\mathrm{kWh}}{\mathrm{m}}$$

(d)

$$E = \frac{Q_n}{\mathrm{COP}} = 32143\,\mathrm{kWh}$$

The electricity cost:

$$32143\,\mathrm{kWh} \cdot 0.13\,\mathrm{CHF/kWh} = 4178.59\,\mathrm{CHF}$$

(e) Oil heating: Annual heating demand in MJ:

$$135000\,\text{kWh} = 486000\,\text{MJ}$$

Volume of oil required:

$$V_{\text{oil}} = \frac{486000\,\text{MJ}}{42.6\,\text{MJ/kg} \cdot 0.86\,\text{kg/L}} = 13266\,\text{litres}$$

The price of the oil:

$$13266\,\text{litres} \cdot 0.86\,\text{CHF/litre} = 11408\,\text{CHF}$$

Problem 2

(a) Available heat in the geothermal source:

$$\dot{Q} = \dot{m}c_p(T_{\text{in}} - T_{\text{out}}) = 50 \cdot 4180 \cdot (190 - 85) = 21.945\,\text{MW}$$

Exergy available in the geothermal source:

$$T_{\text{logmean}} = \frac{T_{\text{h,in}} - T_{\text{h,out}}}{\ln \frac{T_{\text{h,in}}}{T_{\text{h,out}}}} = \frac{463 - 358}{\ln \frac{463}{358}} = \frac{105}{0.2572} = 408.25\,\text{K}$$

$$Ex_{\text{source}} = \dot{Q} \cdot \left(1 - \frac{T_a}{T_{\text{logmean}}}\right) = 21.945 \cdot \left(1 - \frac{287}{408.25}\right) = 21.945 \cdot (1 - 0.703) = 6.52\,\text{MW}$$

Exergy for district heating:

$$T_{\text{logmean}} = \frac{T_{\text{h,in}} - T_{\text{h,out}}}{\ln \frac{T_{\text{h,in}}}{T_{\text{h,out}}}} = \frac{333 - 313}{\ln \frac{333}{313}} = \frac{20}{0.06194} = 322.9\,\text{K}$$

$$Ex_{\text{heating}} = \dot{Q} \cdot \left(1 - \frac{T_a}{T_{\text{logmean}}}\right) = 12 \cdot \left(1 - \frac{287}{322.9}\right) = 12 \cdot (1 - 0.889) = 1.334\,\text{MW}$$

Energy Efficiency:

- Summer: $\frac{3.2\,\text{MW}}{21.945\,\text{MW}} = 14.6\%$
- Winter: $\frac{2.4\,\text{MW} + 12\,\text{MW}_{\text{th}}}{21.945\,\text{MW}} = 65.6\%$

Exergy Efficiency:

- Summer: $\epsilon = \frac{Ex_{\text{electrical}} + Ex_{\text{heating}}}{Ex_{\text{source}}} = \frac{3.2+0}{6.52} = 49.1\%$
- Winter: $\epsilon = \frac{Ex_{\text{electrical}} + Ex_{\text{heating}}}{Ex_{\text{source}}} = \frac{2.4+1.334}{6.52} = 57.3\%$

(b) The marginal electrical efficiency in winter is the electrical production from the residual heat ($21.945\,\text{MW} - 12\,\text{MW}_{\text{DH}} = 9.945\,\text{MW}$), thus:

$$\frac{2.4}{9.945} = 24.1\%$$