Geothermal heat for buildings

Buildings energy use

- There are essentially 2 heat demands in buildings
  - hot sanitary **water**
  - **space** heating
- Estimate for sanitary hot water:
  - 50 L/day.person (1 shower = 30 L, 1 bath = 200 L)
  - 50°C hot water delivered, 10°C cold source

→ per person:

$$\dot{Q} = \dot{m}C_P\Delta T = \frac{50L}{day}.\frac{4184J}{K.mol}.(50-10) = 8.37\frac{MJ}{day} = 2.32\frac{kWh}{day} \approx 100\ W$$

Space heating requirement

LOSS: compensate for ΔT ($= T_{in} - T_{out}$)

- conduction heat loss through walls, windows, floor, roof, doors
- convection heat loss by hot air escaping through any openings

GAINS: any internal heat sources

- people, cooking, lighting, sun input (=> $T_{building} > T_{out}$)

Energy_balance = LOSSES – GAINS (yearly basis, per m^2 floor)

$$= \eta_{system} * E_{heating\_system}$$

$$= power_{system} * time$$

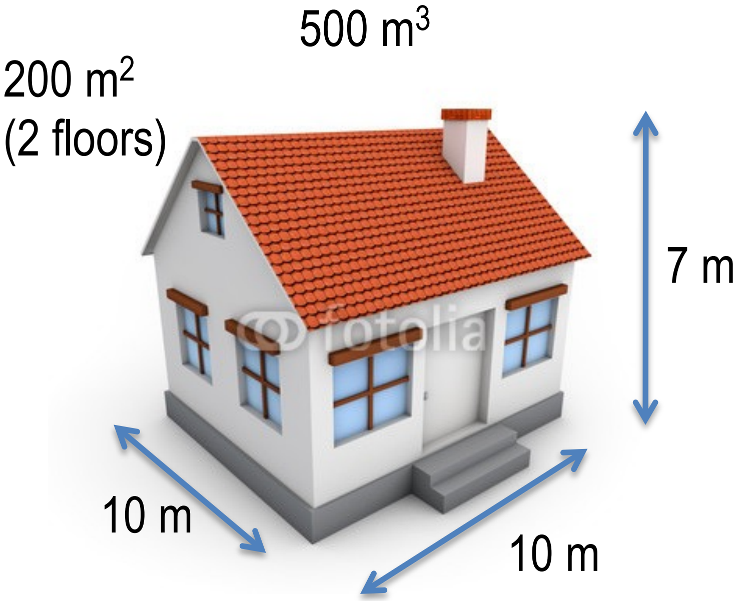
'heating degree-days'

Conduction losses

$$\dot{Q} = Area\,[m^2].\,U\left[\frac{W}{m^2K}\right].\,\Delta T_{out}^{in}[K]$$

- Area : walls, windows, roof, doors, floor against soil
- U = thermal conduction coefficient
- R = 1/U = thermal resistance coefficient
- for multiple layers of material i: $R_{tot} = \Sigma R_i$

Material	Area (m^2)	x U ($W/m^2.K$)	= $Q/\Delta T$ (W/K)
Floor against soil	100	x 0.3	= 30
Roof	106	0.2	21
Walls	180	0.3	54
Windows	40	1.3	52
Doors	4	2	8
			165 W/K



Convection losses

$$\dot{Q} = vol\,[m^3].N\,\lfloor s^{-1} \rfloor.C_{P,air}.\Delta T_{out}^{in}[K]$$

- C_P(air) = 1200 J/m^3.K (1 kJ/kg.K, density 1.2 kg/m^3)
- N = *air replacement number* via leaks, openings, ventilation (natural or forced)
- 1 person requires 30 m^3/h air replacement to evacuate his H_2O, CO_2, heat and odor; for the average 4-people family, this is 120 m^3/h or 0.25/h (for a 500 m^3 house): i.e. every 6 h the air is totally replaced

$$\Rightarrow \dot{Q} = 500\,[m^3].\,6{,}95.\,10^{-5}\lfloor s^{-1} \rfloor.1200.\,\Delta T_{out}^{in}[K] = \boxed{42\left[\frac{W}{K}\right]}$$

Total losses are 165 W/K (transmission) and 42 W/K (ventilation) (80%:20%)

= 207 W/K **≈ 1 W /m^2.K** (for the 200 m^2 floor house)

= 5 kWh / day.K

total_loss_coefficient

Yearly energy demand – ‘Degree-days’ (DD)

Energy_loss (J) = power (W) x time (s)

$$= \text{loss-coefficient (W/K)} \times \underbrace{\Delta T\ \text{(K)} \times \text{time (s)}}_{\textit{“Degree-days” (temperature demand)}}$$

Example : T_{in} = 20°C, T_{out} = 10°C for 1 week => 70 degree-days

A typical year number for Switzerland is 3000 DD, i.e. on average a ΔT of 8°C between inside and outside has to be compensated for year-round.

Energy_loss (J) =
= 207 (W/K) x 3000 DD x 24 (h/d) x 3600 (s/h) ≈ 54 GJ ≈ **14’900 kWh**
= (for 200 m^2 heated floor area) = 270 $MJ/m^2.yr$

‘old house’ : > 400 $MJ/m^2.yr$
new house, highly insulated : 100 $MJ/m^2.yr$

Gains and net demand

- Gains: (2 adults, 2 children)
  - 100 W per adult (presence = 50%), 60 W per child (presence = 60%)
    => 2 * 50 W + 2 * 30 W = 160 W
  - Household apparatus : ca. 2 W/m^2 (empirical value)
    => 200 m^2 * 2 W/m^2 = 400 W

⇒ *total gains = 560 W = 4900 kWh/yr*

- Net demand = losses – gains = 14’900 – 4’900 = **10’000 kWh / yr**
  => (200 m^2 house) : 50 kWh/m^2.yr, or **180 MJ/m^2.yr**
  => (÷ 365 days) : 27.4 kWh/day = 1.14 kW (for the house, not per m^2)
- for 90% efficient heating, a 1.26 kW (on average!) heating system would be needed (for winter-peak with -10°C outside : 207 W/K * 30 (ΔT) = 6 kW)
- The 4-people family thus requires 400 W for hot water and 1.2 kW for space heating (≈ a 1:3 ratio between both services), or a total of 1.6 kW
- Rem: the **Minergie**-standard for housing is 142 MJ/m^2.yr (40 kWh/m^2.yr)

Smart building – smart heating

1. Thick insulation everywhere
    - reduce conduction loss
2. Sealed envelope (active ventilation)
    - reduce convection loss
3. Passive sun heating
    - increase gains

A reduction by a ***factor 4*** between current consumption and future construction standards is realistic.

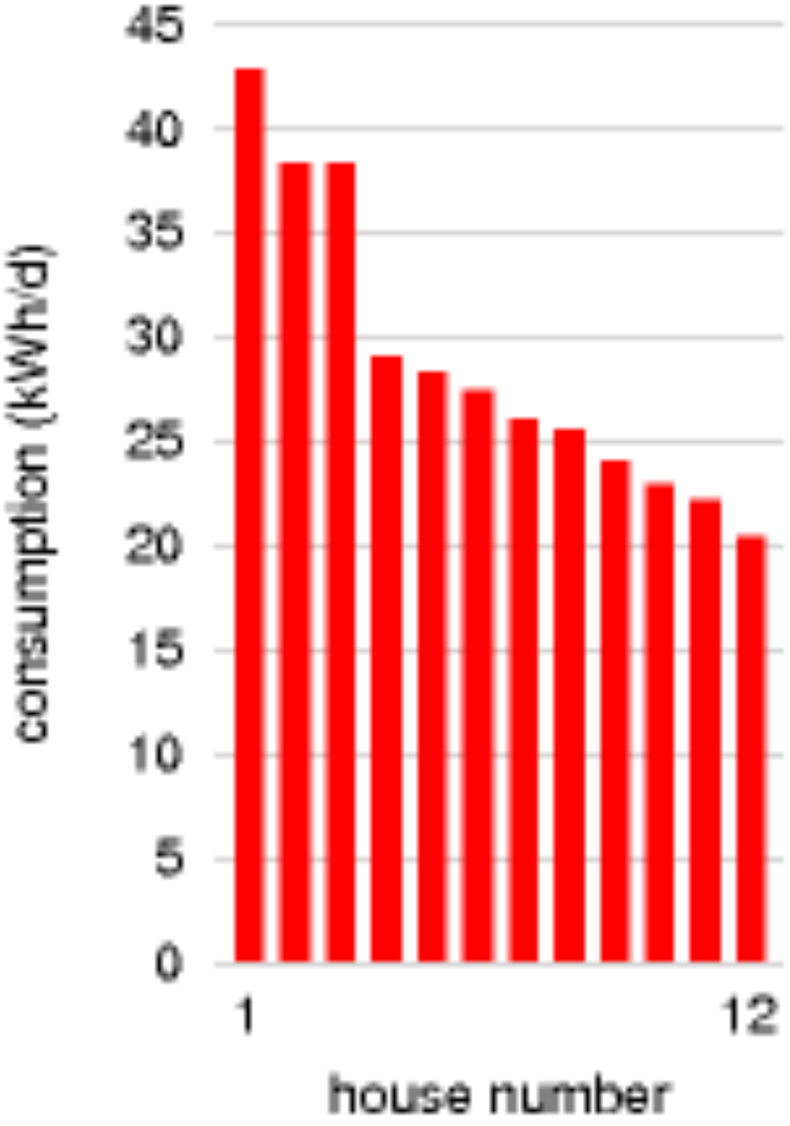
Occupant behaviour!

An actual test :

- 12 identical, low consumption houses
- 86 m^2
- loss_coefficient : 112.5 W/K
  (=> **1.3 W/m^2.K**)
- identical heating systems

The effective final consumption
still varies by a factor 2 !
= due to the occupants' lifestyle

D MacKay, 'Sustainable Energy'



Dynamic behaviour – heat diffusion 1st law

1st diffusion law (**Fourier**):

$$Q = -\lambda \frac{dT}{dx} \qquad \lambda \left[\frac{W}{mK}\right] = \varrho \left[\frac{kg}{m^3}\right] . C_P \left[\frac{J}{kg.K}\right] . D \left[\frac{m^2}{s}\right] = C_V \left[\frac{J}{m^3.K}\right] . D \left[\frac{m^2}{s}\right]$$

λ : thermal conductivity

U : thermal conduction coefficient

d : thickness of the material

$$\lambda \left[\frac{W}{mK}\right] = U \left[\frac{W}{m^2K}\right] . d[m]$$

C : heat capacity of the material (per <u>kg</u>)

C_V : heat capacity of the material (per <u>m</u>^{3}) = ρC

D : heat diffusion coefficient

$$D = \frac{\lambda}{C_V}$$

Materials thermal properties

Material	Density ρ (kg / m^3)	Heat capacity C (J / kg K)	Thermal conductivity λ (W / m K)	Thermal diffusion coefficient D (m^2 / s) = $\lambda / \rho C$
Concrete	2400	1000	1.8	0.75 E-6
Brick	1100	940	0.44	0.42 E-6
Wood	500	2500	0.15	0.12 E-6
Glass	2500	720	0.8	0.44 E-6
Steel	8000	830	60	9 E-6
Aluminium	2750	830	200	88 E-6
Rockwool	20	1000	0.036	1.8 E-6
Polystyrene	20	1370	0.036	1.3 E-6
Air	1.2	1000	0.024	20 E-6
Water H_2O	1000	4180	0.59	0.14 E-6
Clay soil, dry	2000	800	0.25	0.16 E-6
Clay soil, 50% sat.	2000	1150	1.18	0.5 E-6
Clay soil, 100% sat.	2000	1500	1.58	0.53 E-6

Dynamic behaviour – heat diffusion 2nd law

We are interested in the dynamic heat profile in materials (of buildings, of soil (for ground heat pumps)) with the variation of seasons and day/night.

Which is the temperature T at a distance x (1-D case) in a material of properties λ, ρ, C, at a time t after receiving a quantity of heat (Q) at the surface at time zero ? The distribution is given by the diffusion equation :

$$\frac{dT(x,t)}{dt} = D\,\frac{d^2T(x,t)}{dx^2}$$

and the solution of this differential equation is: $T(x,t) = \frac{K}{\sqrt{t}}\,exp\left[-\frac{x^2}{4Dt}\right]$

The integration constant K is determined by considering the injected amount of heat:

$$Q = \int mCdT = \int_{vol} \rho C.T.dvol = \int_{-\infty}^{\infty} \rho CT dx = \rho C\frac{K}{\sqrt{t}}\int_{-\infty}^{\infty} exp\left[-\frac{x^2}{4Dt}\right]dx$$

Solution of the diffusion equation

Integral tables: $\int_{-\infty}^{+\infty} exp[-ax^2] = \sqrt{\frac{\pi}{a}} \xrightarrow[a = \frac{1}{4Dt}]{} \int_{-\infty}^{+\infty} exp\left[-\frac{x^2}{4Dt}\right] = \sqrt{4\pi Dt} = 2\sqrt{\pi Dt}$

$$Q = \rho C \frac{K}{\sqrt{t}} \int_{-\infty}^{\infty} exp\left[-\frac{x^2}{4Dt}\right] dx = \rho C \frac{K}{\sqrt{t}} . 2\sqrt{\pi Dt} = 2\rho C K \sqrt{\pi D} \Rightarrow$$

$$K = \frac{Q}{2\rho C\sqrt{\pi D}}$$

$$K = \frac{Q}{2\sqrt{\pi}\sqrt{\rho^2 C^2 D}}$$

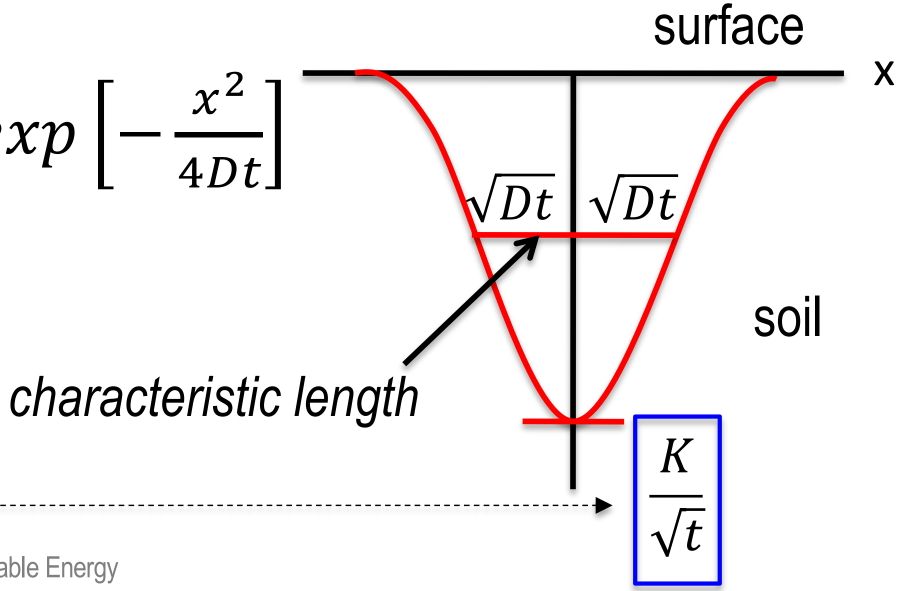
$$K = \frac{Q}{2\sqrt{\pi}\sqrt{\rho C\lambda}}$$

$\rho C\lambda$ = thermal effusivity ≅ a measure of heat storage

Inserting K in the solution, we finally find:

$$T(x,t) = \boxed{\frac{K}{\sqrt{t}}} exp\left[-\frac{x^2}{4Dt}\right] = \frac{Q}{2\sqrt{\pi}\rho C\sqrt{Dt}} . exp\left[-\frac{x^2}{4Dt}\right]$$

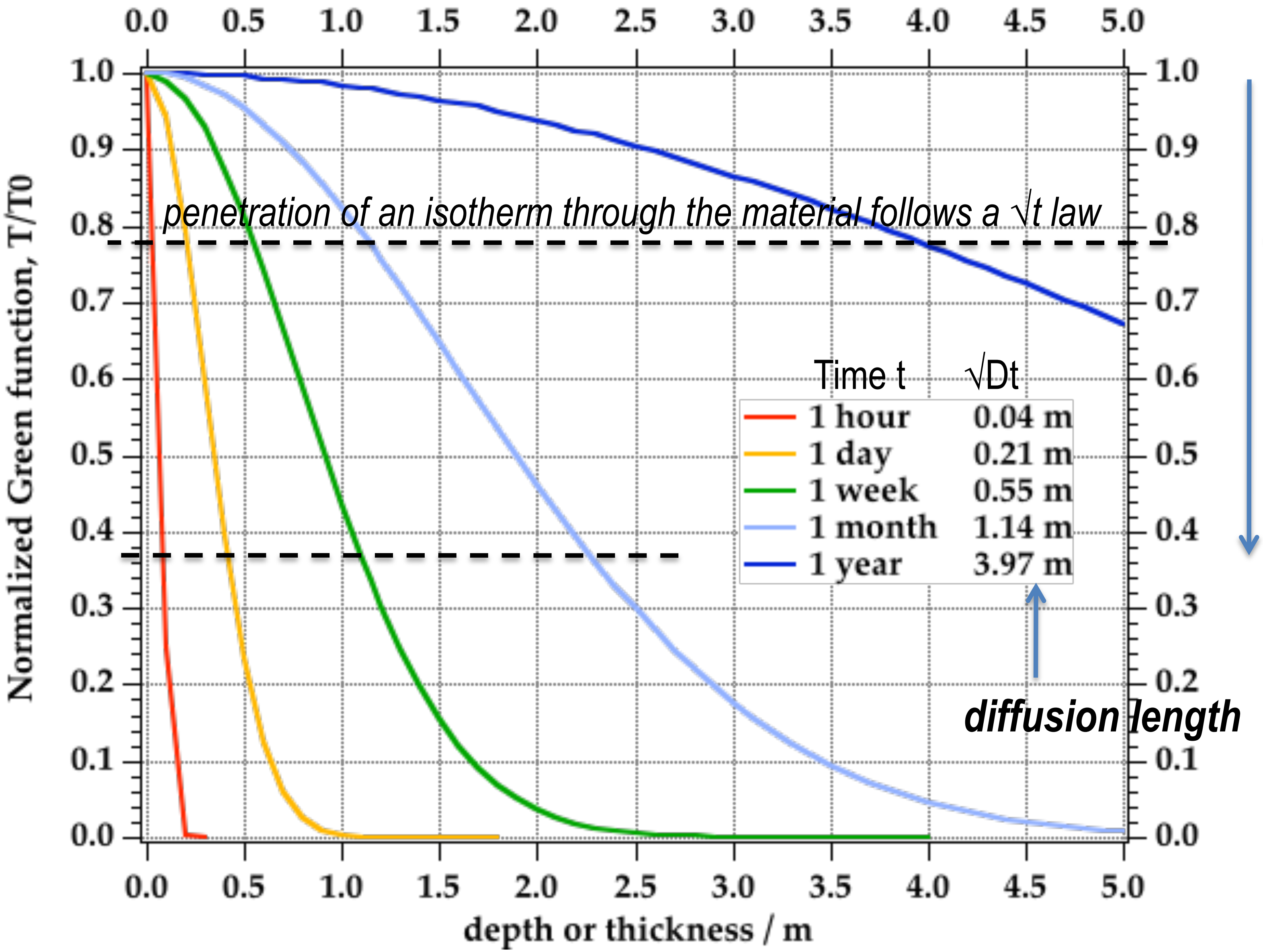
« Green Function »



T-diffusion plot

$$\frac{1}{\sqrt{Dt}} . exp\left[-\frac{x^2}{4Dt}\right]$$ *normalised to 1 at x = 0*

D = 0.5 E-6 m^2/s (soil, brick)



x = √Dt

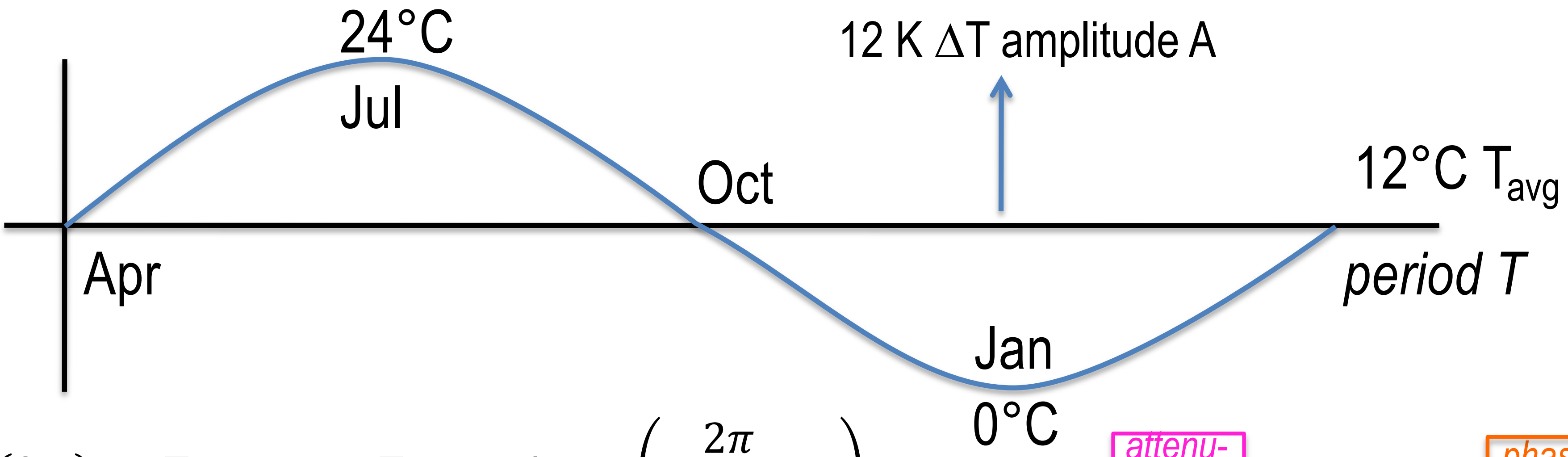
$e^{-1/4} = 0.78$

x = 2 √Dt

$e^{-1} = 0.37$

Application to soil heat pumps

Assume a seasonal sinusoidal variation of temperature T at the ground surface



$$T(0,t) = T_{surface} = T_{avg} + A.cos\left(\frac{2\pi}{T_{period}}.t\right)$$

The solution is described by: $$T(x,t) = T_{x\_soildepth} = T_{avg} + A.exp\overbrace{\left[-\frac{x}{x_L}\right]}^{\text{attenuation of the oscillation}}.cos\left(\frac{2\pi}{T_{period}}.t\overbrace{-\frac{x}{x_L}}^{\text{phase shift of the oscillation}}\right)$$

$$x_L = \sqrt{\frac{DT}{\pi}}$$

characteristic depth (m) *of both the decay (exp) and the oscillation (cos)*

Resulting heat flux

$$Q(x)\left[\frac{W}{m^2}\right] = -\lambda\frac{dT}{dx}\frac{A}{x_L}exp\left[-\frac{x}{x_L}\right]\sqrt{2}.sin\left(\frac{2\pi}{T_{period}}t-\frac{x}{x_L}-\frac{\pi}{4}\right)$$

Peak flux at the surface x = 0 (July, January, max amplitude):

$$Q(0) = \lambda\frac{A}{x_L}\sqrt{2} = \lambda\frac{A\sqrt{2}}{\sqrt{\frac{DT}{\pi}}} = A\sqrt{\frac{2\pi}{T}}\frac{\lambda}{\sqrt{D}} = A\sqrt{\frac{2\pi}{T}}\frac{\lambda}{\sqrt{\frac{\lambda}{\varrho C}}} = A\sqrt{\frac{2\pi}{T}}\sqrt{\lambda\varrho C}$$

Example:

clay soil, ρC = 2’300’000, λ =1.18 => D = 0.5 E-6 => **x_L (annual) = 2.24 m**

Peak Q(0) = 8.8 W/m^2 (A=12K), and corresponding flux at depth of e.g. 4 m = 1 W/m^2 (i.e. very much larger than the sustained geothermal flux of 0.05 W/m^2).

In other words, the heat is in fact coming from the warm surface air (and thus the sun) and not from the intrinsic geothermal flux !

If we pump more heat than the flux (at x=0), **the soil cools progressively down**.

This can be solved by pumping back heat into the soil during summertime.

Consequences

At **$x = 2\ x_L$**, the oscillation amplitude has decayed to exp(-1/2) = 0.135 = 1/7
At **$x = 2\ x_L$**, the oscillation phase shift is -2 (rad) = -115°

At **$x = 3\ x_L$**, amplitude has decayed to exp(-1/3) = 0.05 = 1/20
At **$x = 3\ x_L$**, the oscillation phase shift is -3 (rad) = -172° ≈ half a cycle

Example : amplitude A (ΔT) = 12 K
=> at 2 x_L, amplitude is only 12/7 = 1.6 K; at 3 x_L, it is only 0.6 K

Example : rock
λ = 2.1 W/mK, ρ = 2500 m^3/kg, C = 820 J/kgK => D = 1 E-6 m^2/s

Seasonal variation : T_{period} = 3600 * 24 * 365 = 31'536'000 s
=> **$x_L = 3.17$ m**

Daily variation : T_{period} = 3600 * 24 = 86'400 s
$\Rightarrow$ **$x_L = 0.166$ m** => with a 33 cm thick granite wall, one can practically absorb the temperature oscillation between night and day

Resulting soil temperature profile

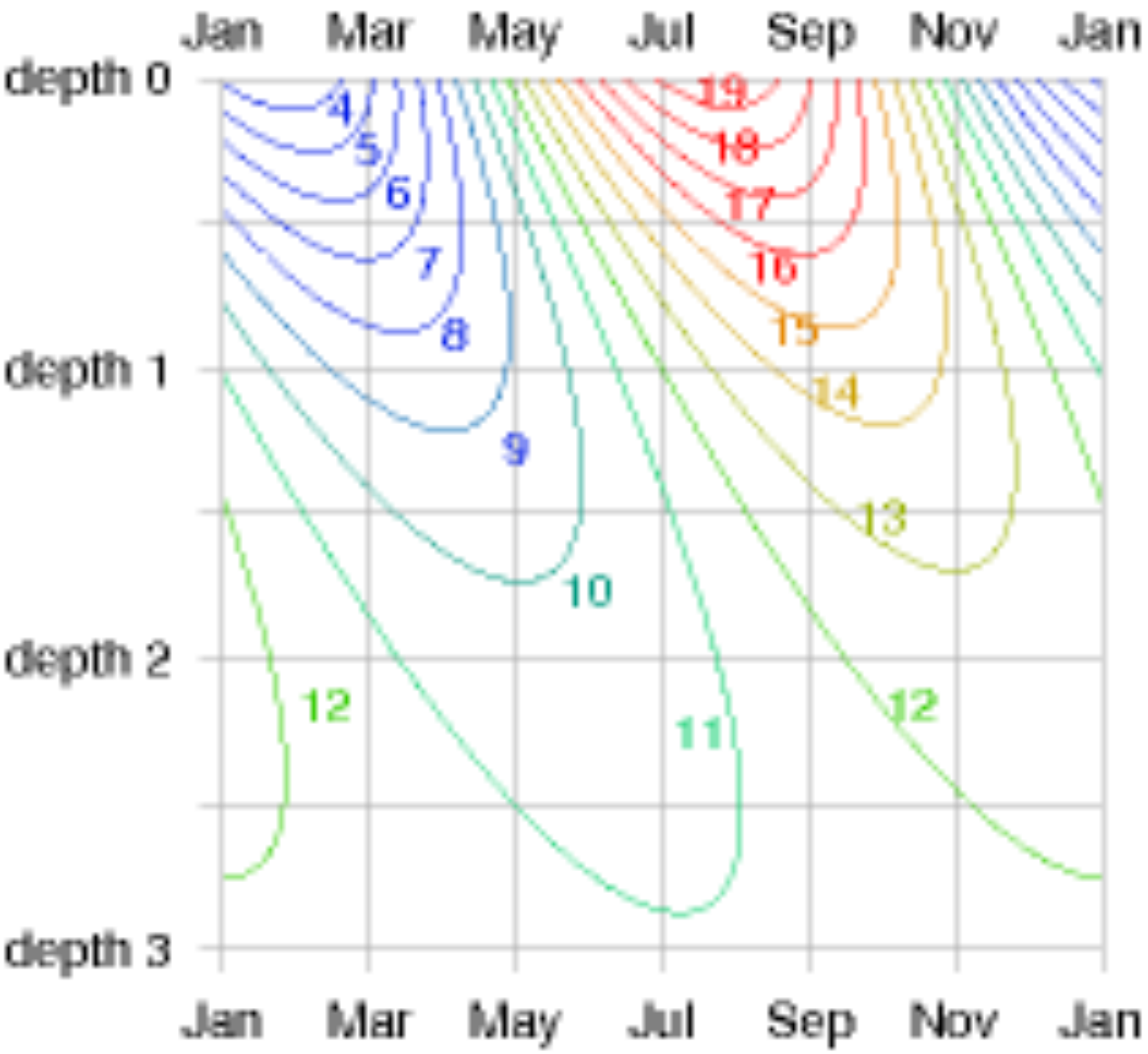
At x_L = 3, the soil temperature oscillation is delayed by half a cycle (phase shift).

At x_L = 1, the soil temperature oscillation amplitude is 5.5 K (= 15 K .exp(-1))

At x_L = 2, the soil temperature oscillation amplitude is < 2 K

At x_L = 3, the soil temperature oscillation amplitude is < 1 K

D MacKay, 'Sustainable Energy'



Application to shallow heat pumps

Suppose a serpentine heat collecting tube buried **2 m (=d) below** the surface.

Assume we can allow to cool the underground at 2 m to **ΔT = 5 K below** the ground surface temperature, which we assume constant.
Assume thermal conductivity of the soil as 1.18 W / m K (moist clay).

$\Rightarrow$The heat flux we can collect from the serpentine tube is then

$$Q = \lambda * \Delta T/d = 1.18 * 5/2 = \mathbf{3\ W/m^2}$$

From a 100 m^2 area under the house we could then pump 300 W, which is far too low for heating.
Remember we need at least 1.2 kW on average (hence 400 m^2).
This indicates there is a *limit to the density of soil heat pumps* in populated neighbourhoods.

Application to deep soil heat pumps

Suppose a 50 m deep borehole (≈2.5 kW heat extraction). This excludes any exchange with the atmosphere, and cooling of the soil is only caused by the heat extraction.

Distance from hole	Cooling of the soil (in K) at 50 m deep after x time of operation			
	2 yr	5 yr	11 yr	30 yr
1 m	1.1	1.2	1.5	1.7
5 m	0.8	1.0	1.2	1.56
10 m	0.44	0.65	0.87	1.2
20 m	0.1	0.2	0.4	0.64
40 m	0	0	0.1	0.25