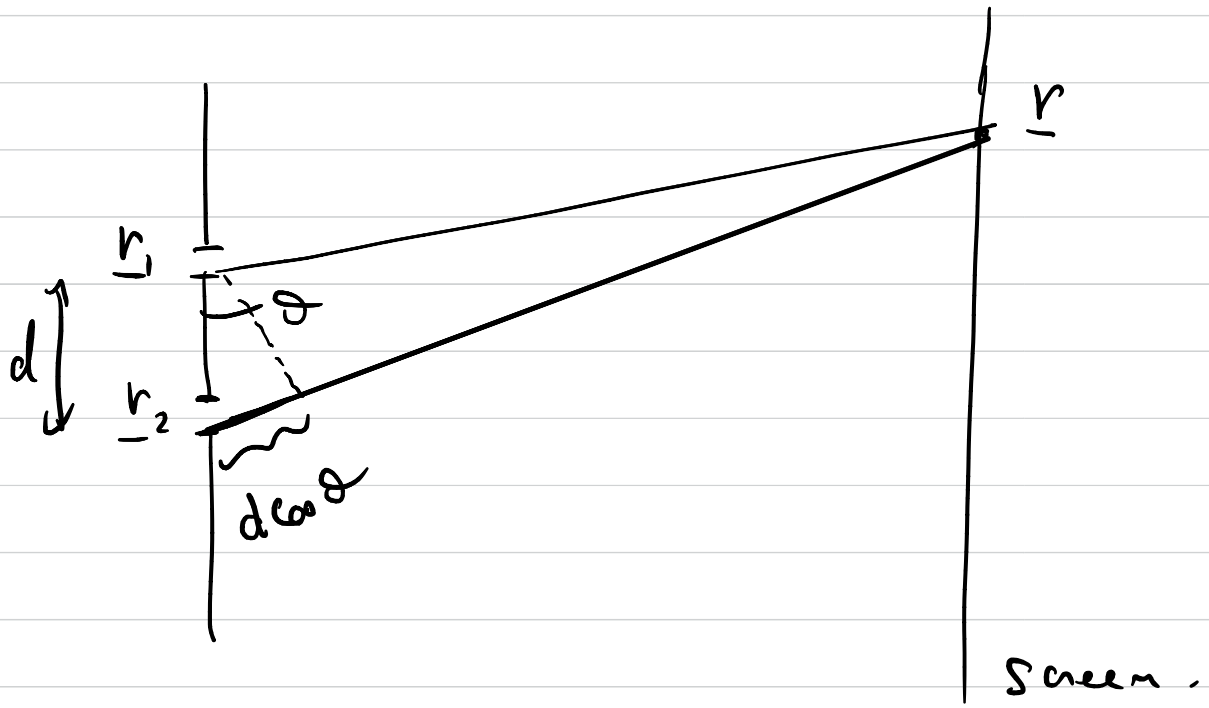
Appendix to week 1 (not mandatory).

Were messy calculation of prédiction for Double slit exp & Mach Zehnder interferometer

1) Young double slit experiment.



wave through slit 1:

$$\psi_1(\underline{r}) \simeq \frac{A}{D} e^{i\frac{2\pi}{\lambda}|\vec{r}-\vec{r}_1|}$$

wave through slit 2:

$$\psi_2(\underline{r}) \simeq \frac{A}{D} e^{i\frac{2\pi}{\lambda}|\vec{r}-\vec{r}_2|}$$

Here $\vec{r}$ is on the screen at distance D from slits and we have approximated the denominators $|\vec{r}-\vec{r}_1|$ & $|\vec{r}-\vec{r}_2|$ by D. However in the phase of the exponential we have to be more precise.

The total wave at $\vec{r}$ is

$$\psi(\underline{r}) = \psi_1(\underline{r}) + \psi_2(\underline{r})$$

$$= \frac{A}{D}\left\{e^{i\frac{2\pi}{\lambda}|\vec{r}-\vec{r}_1|} + e^{i\frac{2\pi}{\lambda}|\vec{r}-\vec{r}_2|}\right\}$$

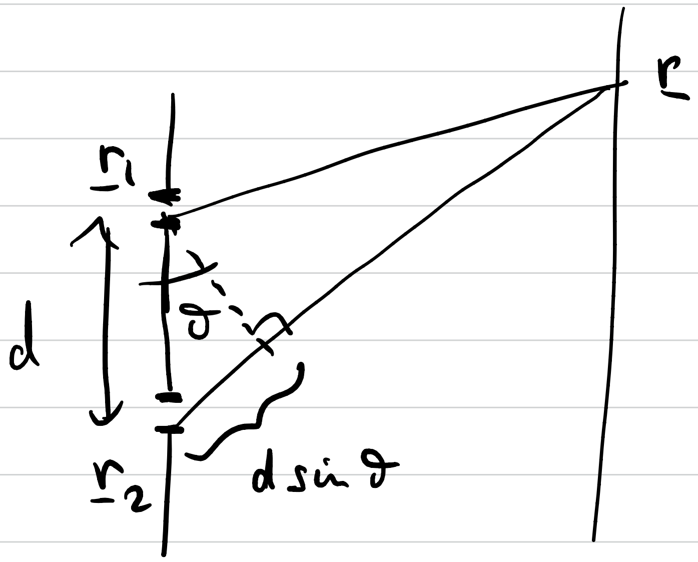
$$= \frac{A}{D}e^{i\frac{2\pi}{\lambda}|\vec{r}-\vec{r}_1|}\left\{1 + e^{i\frac{2\pi}{\lambda}\left(|\vec{r}-\vec{r}_2|-|\vec{r}-\vec{r}_1|\right)}\right\}$$

The intensity on the screen is:

$$|\psi(\vec{r})|^2 = \frac{A^2}{D^2}\left|1 + e^{i\frac{2\pi}{\lambda}\left(|\vec{r}-\vec{r}_2|-|\vec{r}-\vec{r}_1|\right)}\right|^2$$

Now we have to approximate the relative phase between the two "paths" correctly.

$$|\vec{r}-\vec{r}_2| - |\vec{r}-\vec{r}_1| \approx d \sin\vartheta \simeq d\frac{\rho}{D}$$



$$\Rightarrow |\psi(\vec{r})|^2 = \frac{A^2}{D^2}\left|1 + e^{i\frac{2\pi}{\lambda}\frac{d\rho}{D}}\right|^2$$

$$= \frac{A^2}{D^2}\left[\left(1+\cos\frac{2\pi}{\lambda}\frac{d\rho}{D}\right)^2 + \sin^2\frac{2\pi}{\lambda}\frac{d\rho}{D}\right]$$

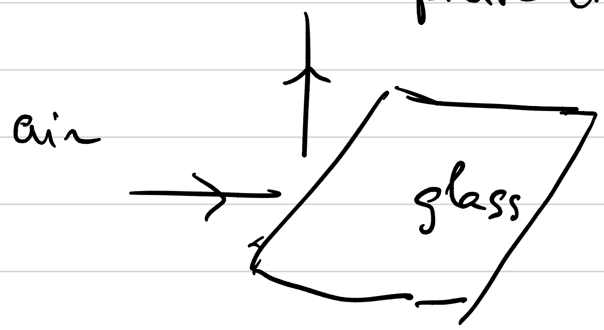
$$= \frac{2A^2}{D^2}\left[1+\cos\frac{2\pi}{\lambda}\frac{d\rho}{D}\right] = \frac{4A^2}{D^2}\left(\cos\frac{\pi}{\lambda}\frac{d\rho}{D}\right)^2$$

(using $1+\cos 2\alpha = 2\cos^2\alpha$).

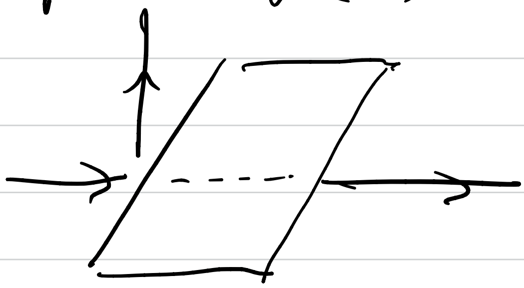
2. Mach-Zehnder interferometer.

Laws of reflection with perfect mirrors and semi-transparent mirrors.

phase change (−1) when beam reflect from air on glass.

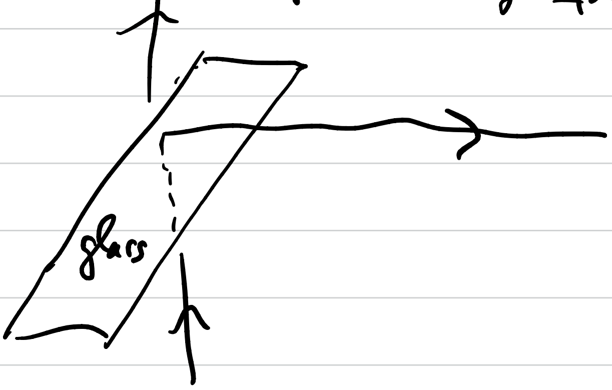


phase change (−1)

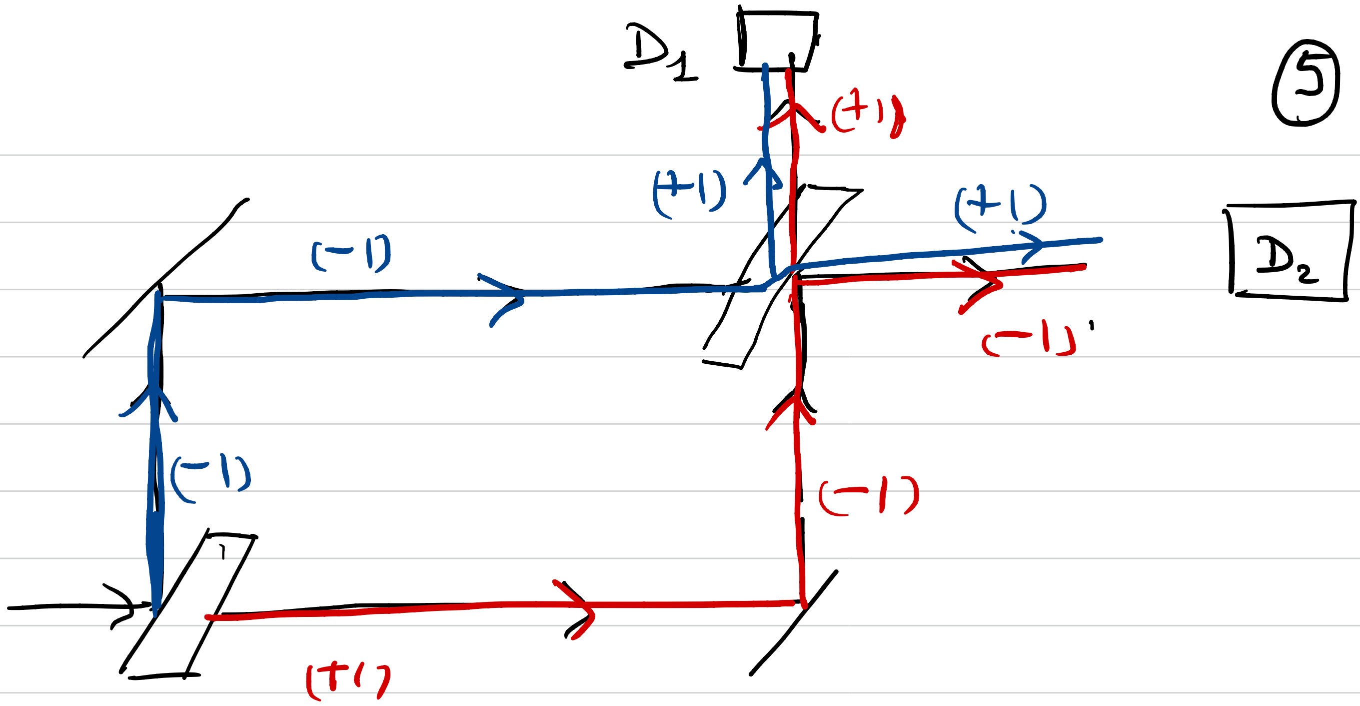


no phase change for transmission

No phase change for transmission. or (+1)



No phase change because reflection is from glass on air or (+1)



- incident wave $\quad A e^{i\varphi}$. (intensity A^2).
- phase change through lower path ending in D_2 (red):

$$\frac{A}{2} e^{i\varphi} (+1)(-1)(-1)\, e^{\frac{2\pi i}{\lambda} L}$$

L = length of path.

- phase change through upper path ending in D_2 (blue)

$$\frac{A}{2} e^{i\varphi} (-1)(-1)(+1)\, e^{\frac{2\pi i}{\lambda} L}$$

Total wave entering D_2:

$$\frac{A}{2} e^{i\varphi} e^{\frac{2\pi i L}{\lambda}} \Big[\underbrace{(+1)(-1)(-1) + (-1)(-1)(+1)}_{+2} \Big]$$

$\Rightarrow$ $\boxed{\text{Intensity} = A^2 \text{ in } D_2}$.

- phase change through upper path in D_1:

$$\frac{A}{2} e^{i\varphi} (-1)(-1)(+1)\, e^{2\pi i \frac{L}{d}}.$$

- phase change through lower path in D_1:

$$\frac{A}{2} e^{i\varphi} (+1)(-1)(+1)\, e^{2\pi i \frac{L}{d}}.$$

- Total wave in D_1:

$$\frac{A^2}{4} \left[(-1)(-1)(+1) + (+1)(-1)(+1) \right] = 0$$

$\boxed{\text{No intensity in } D_1.}$

$\boxed{\text{In conclusion all the intensity enters in } D_2.}$