Fundamentals of electrical circuits and systems I
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Problem Set 4

Problem 1: Laplace transform basics

(a) Determine the Laplace transform and the associated region of convergence for the following functions of time:

i. $x(t) = e^{-2t}u(t) - e^{3t}u(-t)$, *ii.* $x(t) = \delta(t - t_0) - u(-t)$, *iii.* $x(t) = te^{-3|t|}$.

(b) Determine the inverse Laplace transform of

$$X(s) = \frac{3s + 5}{s^2 + s - 12}, \quad Re\{s\} > 3.$$

Problem 2: The Region of Convergence

The Laplace transform for an absolutely integrable signal $x(t)$ is known to have a pole at $s = 2$. Answer the following questions and briefly justify your answer.

(a) Could $x(t)$ be of finite duration?

(b) Could $x(t)$ be left-sided?

(c) Could $x(t)$ be right-sided?

(d) Could $x(t)$ be two-sided?

Problem 3: The Region of Convergence II

In this problem, we study the pole-zero plot of a signal $x(t)$.

(a) Figure 1 shows three different pole-zero plots. For *each* case separately, determine all possible ROCs.

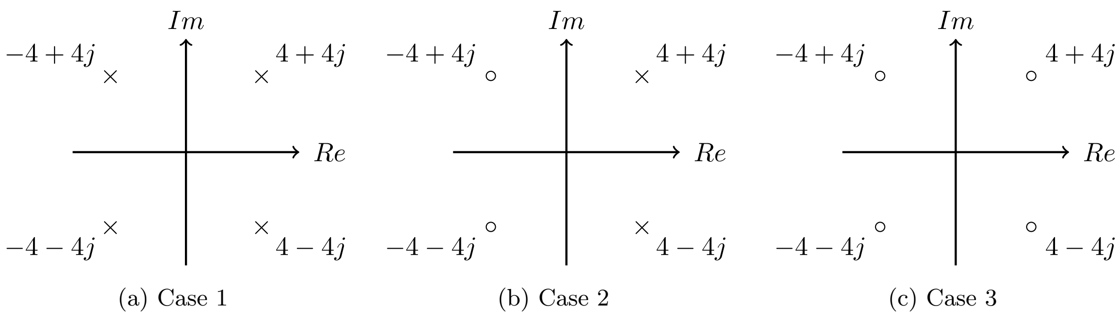


Figure 1: Three possible pole-zero plots

(b) Suppose we know that $x(t)e^{5t}$ is absolutely integrable. Using this information about the signal, you can now identify which of the possible ROCs that you have found in Part *(a)* is the right one. Do this separately for each of the three cases.

(c) Suppose we know that $x(t) * (e^{2t}u(-t))$ is absolutely integrable, where $u(t)$ is the step function. Again, we can leverage this information about the signal to find the right ROC. Do this separately for each of the three cases.

Problem 4: Transfer Function

Consider the system with impulse response

$$h(t) = \begin{cases} e^{-at}\cos(\omega_0 t) & t \geq 0, \\ -e^{bt}\sin(\omega_0 t), & t < 0, \end{cases}$$

where a, b and ω_0 are positive real numbers.

(a) Is this system stable?

(b) Find the transfer function $H(s)$ and the ROC.

(c) Draw the pole-plot for this transfer function. How many other systems do there exist that have exactly the same transfer function as the system considered here (but a different ROC)? Indicate the corresponding ROCs.

Problem 5: LTI Systems and Laplace transform

For a linear time-invariant system, it is known that the transfer function is given by

$$H(s) = \frac{2(s-2)}{(s+1)(s^2-2s+2)}.$$

As in lecture, we now study the pole-zero plot of this transfer function. That is, we study a transfer function that is a rational equation of two polynomials: $H(s) = \frac{P(s)}{Q(s)}$. We draw the roots of $Q(s)$ with a $\times$-symbol in the s-plane and we call them the *poles* of our system. Then, we draw the roots of $P(s)$ with an $\circ$-symbol and we call them the *zeros* of our system.

(a) Draw the pole/zero diagram for $H(s)$.

(b) Suppose that apart from $H(s)$, you are also told that the system is *causal.* Is the resulting system also stable?

(c) Determine the differential equation that describes this system.

Problem 6: Laplace transform and LTI systems

A causal LTI system $\mathcal{H}$ with inmpulse response $h(t)$ has its input $x(t)$ and output $y(t)$ related through a linear constant-coefficient differential equation of the form

$$\frac{d^3}{dt^3}y(t) + (1+\alpha)\frac{d^2}{dt^2}y(t) + \alpha(\alpha+1)\frac{d}{dt}y(t) + \alpha^2 y(t) = x(t).$$

(a) If

$$g(t) = \frac{d}{dt}h(t) + h(t),$$

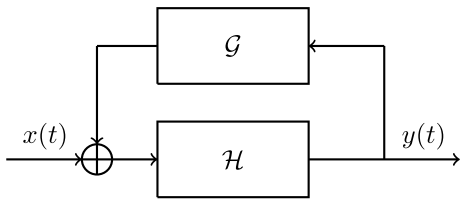
how many poles does $G(s)$ have? What are they?

(b) For what real values of the parameter α is $\mathcal{H}$ guaranteed to be stable?

(c) For what real values of the parameter α is the $ROC(G(s))$ strictly bigger than the $ROC(H(s))$?

Problem 7: Feedback system

In class we studied the feedback composition of two continuous-time systems and derived an equivalent transfer function in terms of $H(s)$ and $G(s)$.



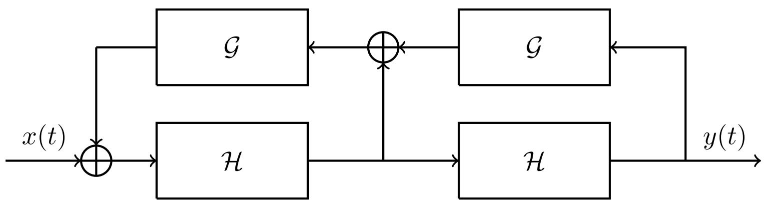
Suppose

$$H(s) = \frac{1}{s-1} \quad \text{and} \quad G(s) = b - s,$$

with the the overall feedback system assumed to be causal. For what real values of b is the feedback system also stable?

Problem 8: Transfer function of a composite system

(a) Consider the continuous-time system interconnect shown as follows, where the component systems $\mathcal{G}$ and $\mathcal{H}$ are LTI systems with transfer functions $G(s)$ and $H(s)$, respectively. Derive the overall transfer function of the system as a function of $G(s)$ and $H(s)$.



(b)(Optional) The interconnected system from Part *(a)* could be thought of as having $K = 2$ stages. We can extend this to an arbitrary length — the following figure shows the case of $K = 3$ stages. For arbitrary K, find the formula for the overall transfer function of the system as a function of $G(s)$ and $H(s)$.