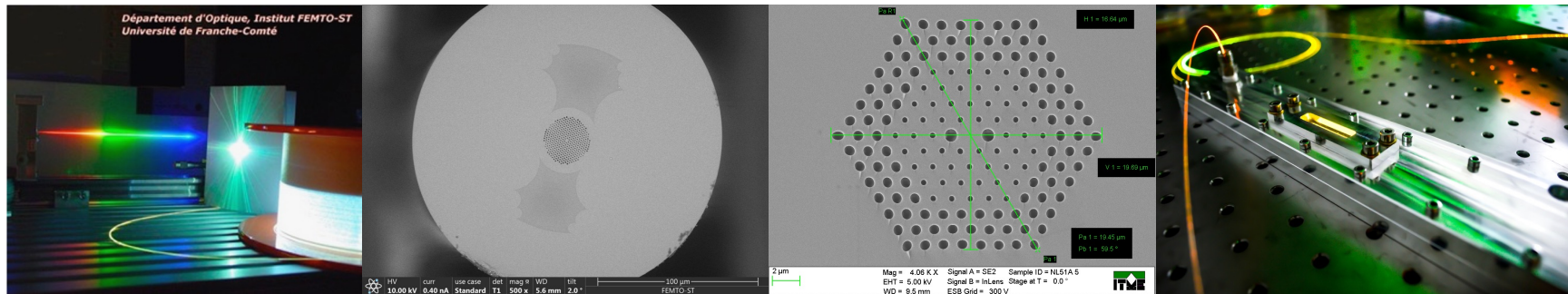




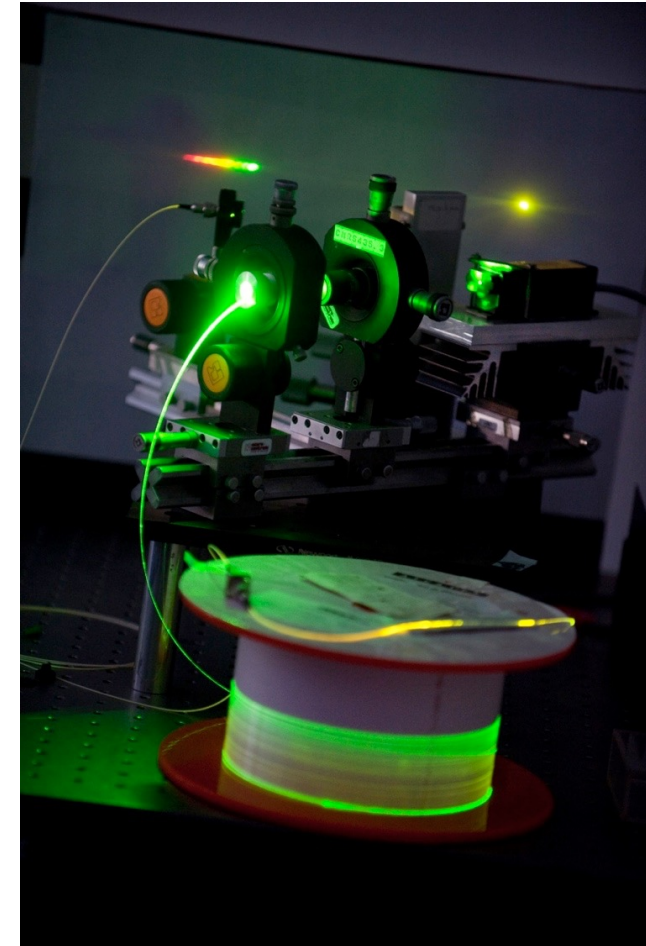
Nonlinear Effects in Optical Fibers : From Fundamentals to Applications



Thibaut Sylvestre, Research Director
Institut FEMTO-ST, CNRS, Université Marie et Louis Pasteur, Besançon, France

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- 2- The third-order nonlinear polarization
- 3- Basics of Optical Fibers
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 - 4-4 Applications : Amplifiers, lasers, sensors
- 5- The Optical Kerr Effect
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 - 5-2 Cross-phase modulation (XPM)
 - 5-3 Optical solitons (OS)
 - 5-4 Modulation instability (MI)
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 - 6-2 Dispersive wave generation
- 7- Stimulated Brillouin Scattering (SBS)
 - 7-1 Principles and basics & Applications
- 8- Conclusions and outlooks



« Nonlinear effects in an optical fiber »

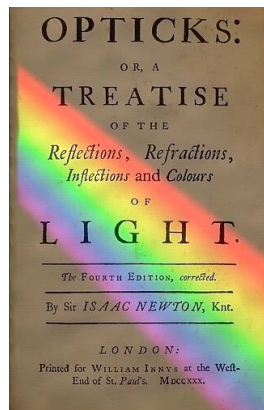
Introduction to Nonlinear Optics (NLO)

1672 : The Newton's Experiment

White light is the superposition of coloured rays !

Then he succeeded in reconstructing white light using a second prism.

He then tried to alter the color of pure rays, and he wrote in his books:



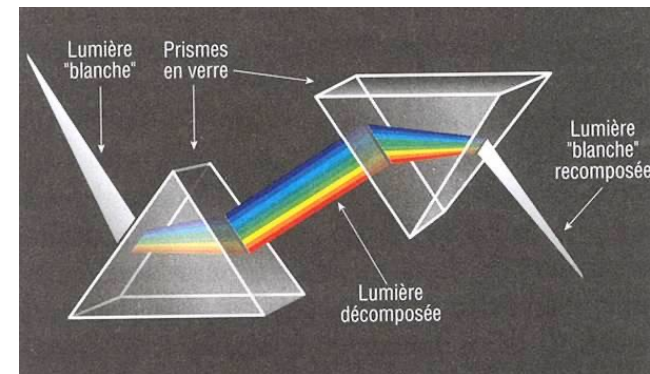
“They obstinately retained their colours, notwithstanding the utmost endeavours to change it....”

Isaac Newton (1643-1727)

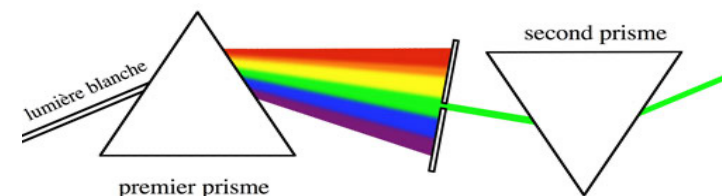
« Newton's Experimentum Crucis »



« White light reconstruction using a second prism »



«Newton' quest : change the color of a pure ray »

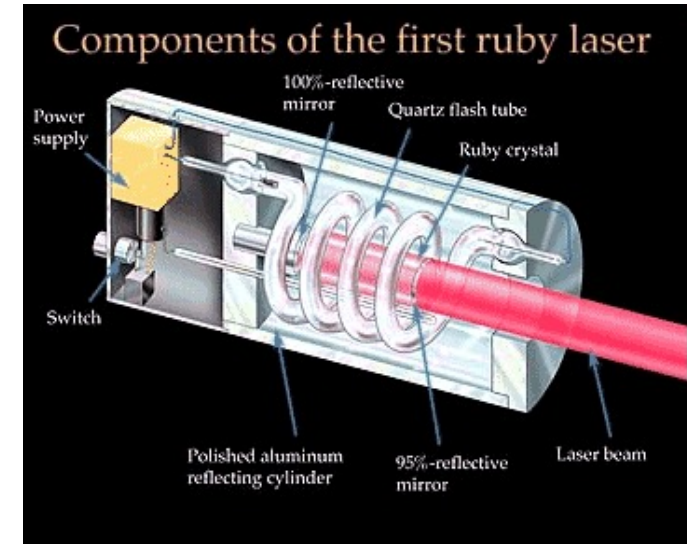


Introduction to Nonlinear Optics (NLO)

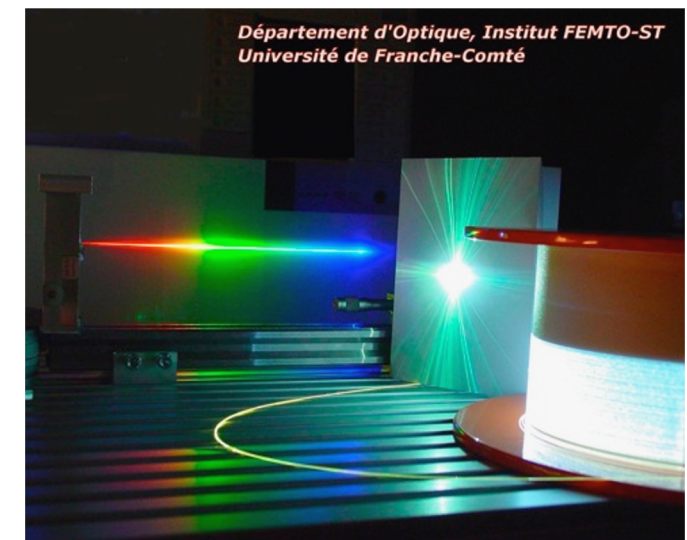
- ▶ 1960 : **Discovery of the LASER**,
high- brightness, coherence, monochromaticity,
enabling strong light-matter interactions
- ▶ 1961 : **Modern Nonlinear Optics (NLO)**,
Second harmonic generation (SHG)
- ▶ 1965-1990: Stimulated Raman scattering (SRS)
Third harmonic generation (THG), Difference and
Sum frequency generation (DFG, SFG), Parametric
amplification (OPA), Parametric oscillation (OPO),
Four-wave mixing (FWM), Phase conjugation (PC),
self-Focusing, Modulation instability, Solitons,
Supercontinuum generation
- ▶ 2000 : **Supercontinuum generation in photonic crystal
fiber = The « Rainbow » laser:**
Generation of a spatially-coherent white light from a
photonic crystal fiber

Definition: « **In nonlinear optics, we are concerned with
the effects that the light induces on itself as it
propagates through the medium »**

« The first Ruby laser » by Maiman

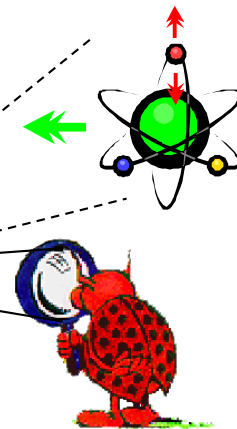
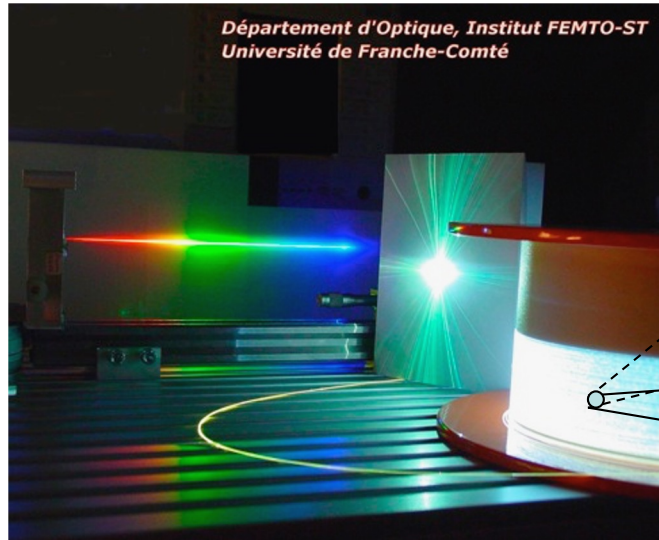


« The white-light laser » in 2000



Third-Order Nonlinear Effects

Nonlinear optical effects are remarkable phenomena that arise when an intense optical beam propagates through a transparent material (optical fiber)



Optical Kerr Effect
(electronic response)

$$n = n_0 + n_2 I(z, t)$$

Stimulated Raman Scattering
(Delayed Molecular response)

Third-Order Nonlinear Polarization

$$P_{NL}(t) = \epsilon_0 \chi_K^{(3)} : E(t)E(t)E(t) + \epsilon_0 E(t) \int_{-\infty}^t \chi_R^{(3)}(t-t') E(t') E(t') dt'$$

Optical Kerr effect:
Instantaneous Elastic effect
(No energy exchange with matter)

Raman effect :
Inelastic light scattering:
(Molecular vibrational states)

Linear and nonlinear propagation in optical fibers

➤ Linear effects : Dispersion & Absorption

➤ Nonlinear effects :

Self-phase modulation (SPM)

Optical Solitons (OS),

Stimulated Raman Scattering (SRS)

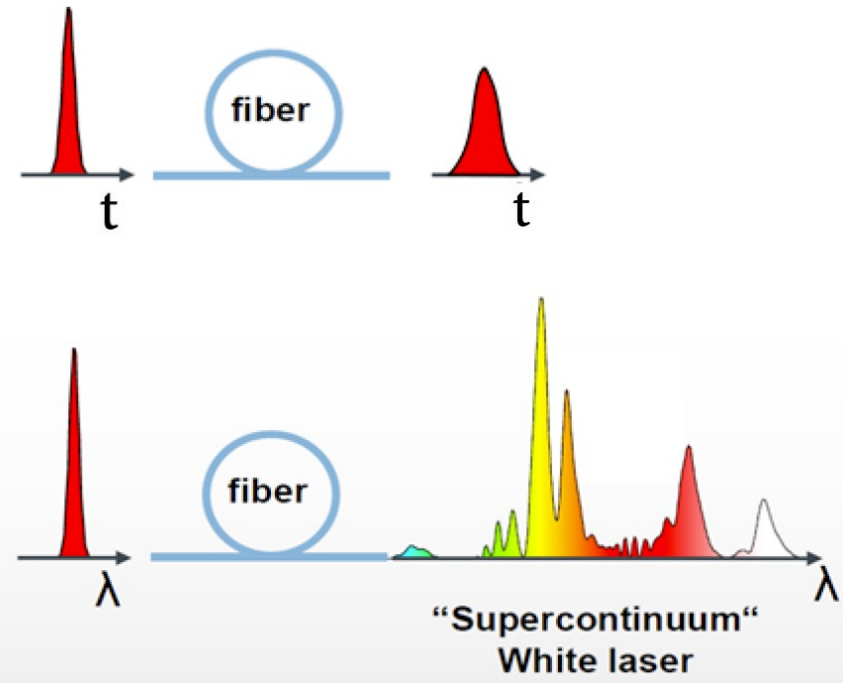
Dispersive Waves (DW)

Optical Wave Breaking (OWB)

Four-Wave Mixing (FWM)

Modulation Instability (MI)

All intensity dependent



➤ SC is broad as the sun (from UV to IR) and bright as a laser > 20k the sun

➤ Spatially coherent - single-mode beam output - Fiber delivery

➤ Supercontinuum light sources have many applications: OCT, Absorption Spectroscopy, Microscopy, Biomedical Imaging, OFC Metrology, etc...

R. R. Alfano, The Supercontinuum Laser Source (Springer, New York, 2016).

Linear and nonlinear propagation in optical fibers

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Self-phase modulation (SPM)

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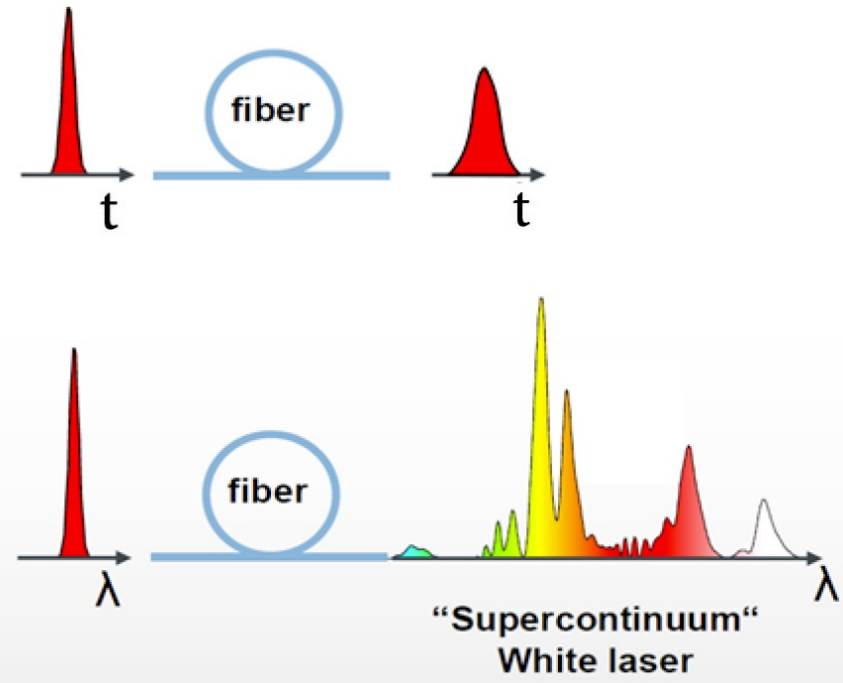
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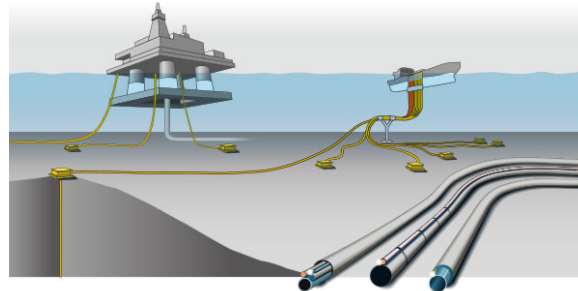
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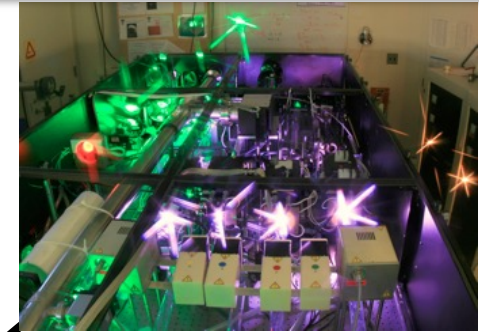
Key Applications of Nonlinear Fiber Optics



Telecommunications
(all-optical processing)



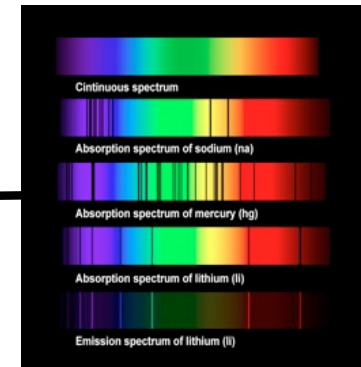
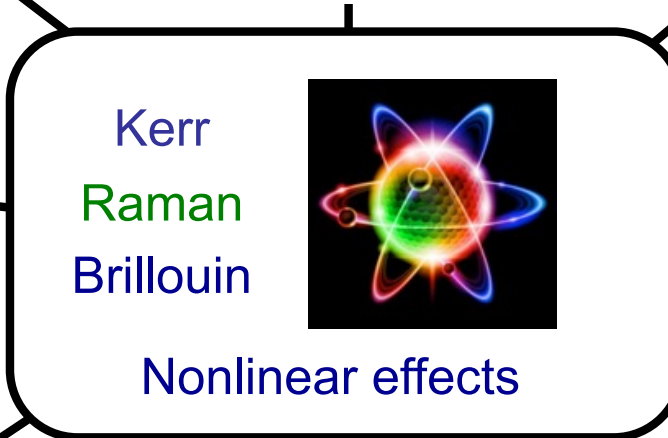
Fiber Optical Sensors



Fiber Lasers & Amplifiers



Extreme & Quantum physics



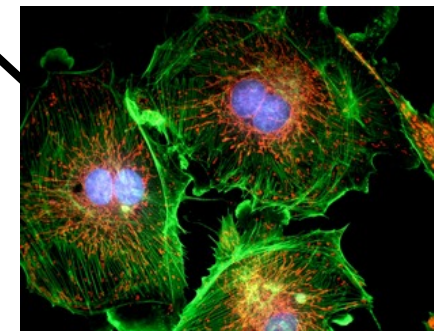
Spectroscopy



Supercontinuum sources



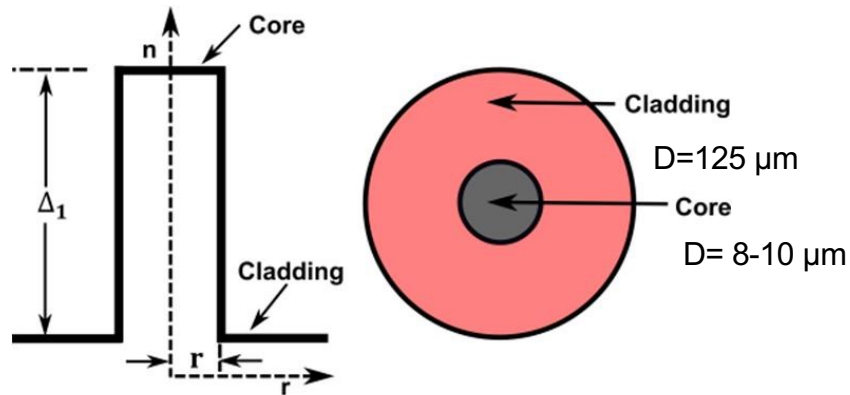
Time-Frequency metrology
(Frequency combs)



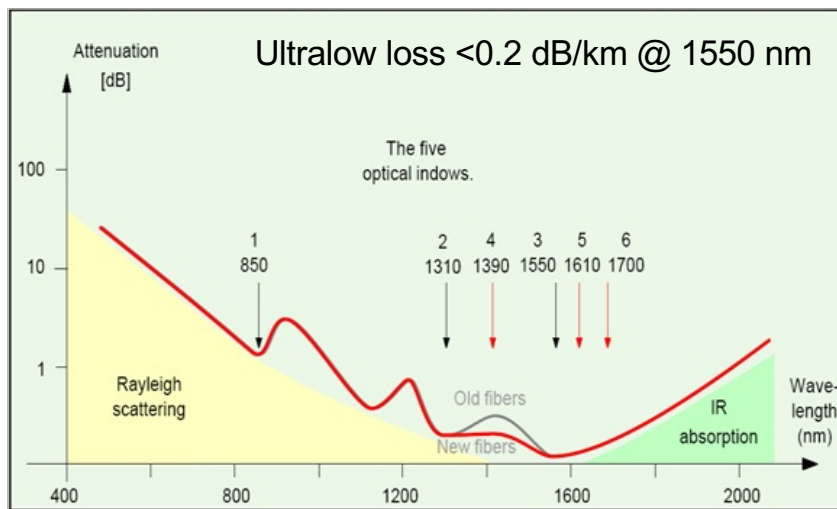
Imaging, Biophotonics

Basics of Fiber Optics

Typical refractive index profile and cross-section of a step-index fused silica (SiO_2) fiber

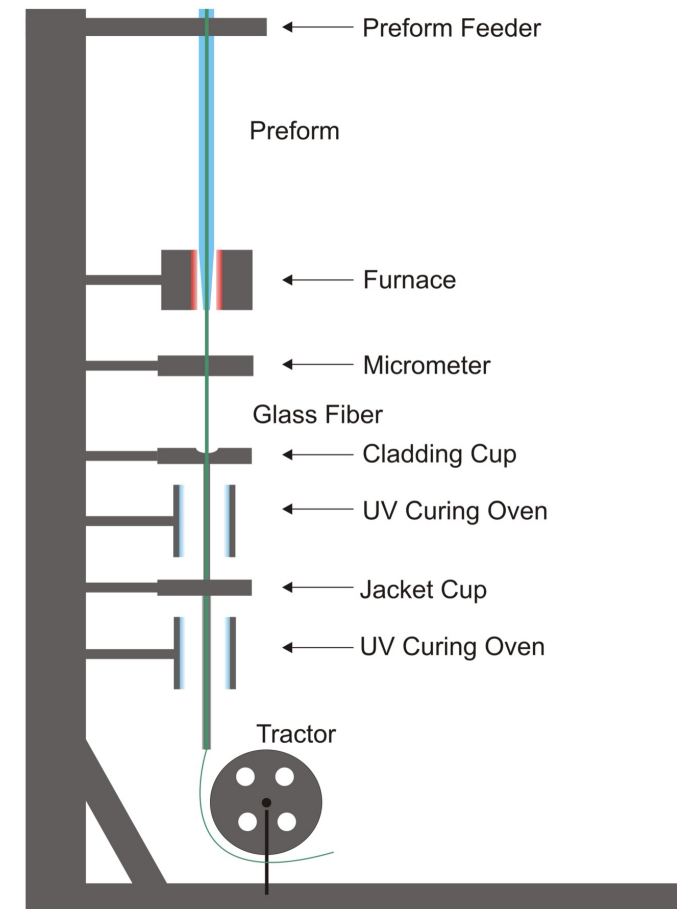


Attenuation spectrum of silica fibers



Manufacturing

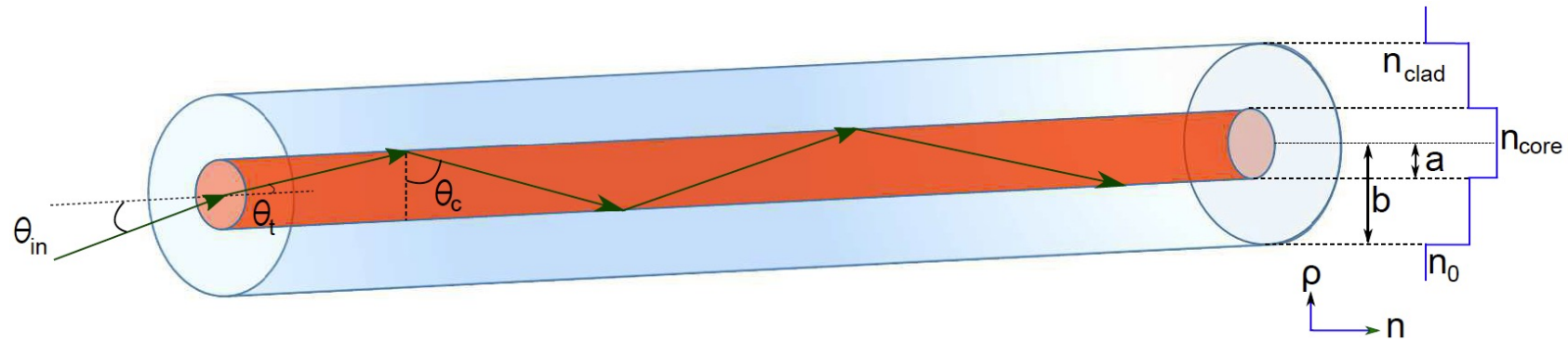
Fiber Drawing Tower



➤ More than 2 billion kilometers installed fibers on our planet for communications !

Basics of Fiber Optics

Light guiding mechanism in standard step-index fibers by total internal reflection (TIR)



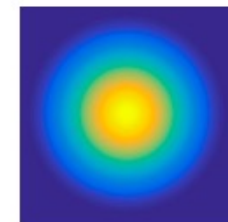
➤ Numerical Aperture (NA) $\sin(\theta_{in}) = \sqrt{n_{core}^2 - n_{clad}^2} = NA$

➤ Normalized frequency $V = p_c a = a k_0 (n_{core}^2 - n_{clad}^2)^{1/2} = \frac{2\pi a}{\lambda} NA$

✓ When V is lower than $V < 2.405$, the fiber can support only the fundamental mode LP01 which has the minimum loss of 0.2 dB/km at 1550 nm

✓ If $V > 2.405$, the waveguiding fiber is multimode

LP01 spatial mode

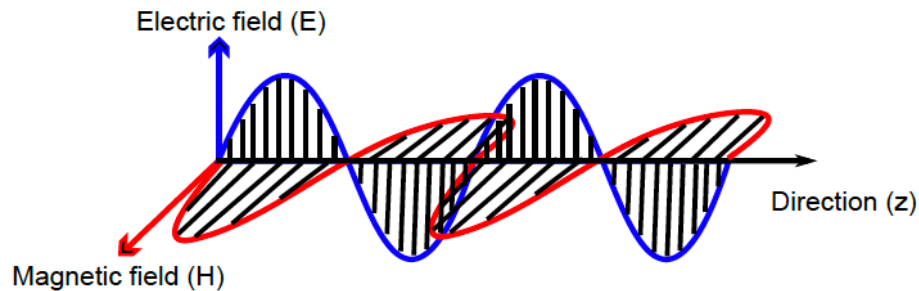


Basics of Fiber Optics

Light guiding is governed by the Helmholtz equations

- Electric field (FT)

$$\tilde{E}(\vec{r}, \omega - \omega_0) = F(x, y)\tilde{A}(z, \omega - \omega_0)\exp(i\beta_0 z).$$



$$\nabla_{\perp}^2 E + (k^2 - \beta^2)E = 0$$

$$\nabla_{\perp}^2 H + (k^2 - \beta^2)H = 0$$

$$k^2 = \frac{\omega^2}{c^2}n^2 = k_o^2 n^2$$

Wavevector

Propagation constant

$$\beta = \frac{2\pi}{\lambda_0}n_{\text{eff}}$$

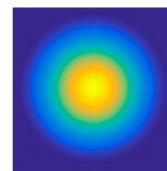
- Modal distribution (Bessel functions)

$$\frac{d^2 F}{d\rho^2} + \frac{1}{\rho} \frac{dF}{d\rho} + \left(n^2 k_0^2 - \beta^2 - \frac{m^2}{\rho^2} \right) F = 0$$

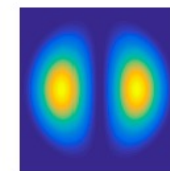
- Effective mode area (EMA)

$$A_{\text{eff}} = \frac{\left(\iint_{-\infty}^{\infty} |F(x, y)|^2 dx dy \right)^2}{\iint_{-\infty}^{\infty} |F(x, y)|^4 dx dy}$$

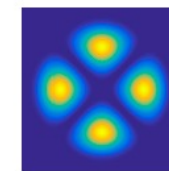
- Linearly-polarized (LP) fiber modes



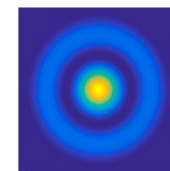
LP01



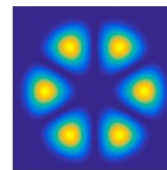
LP11



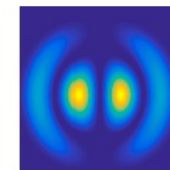
LP21



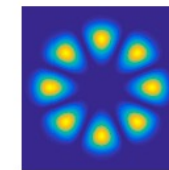
LP02



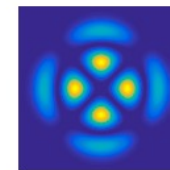
LP31



LP12

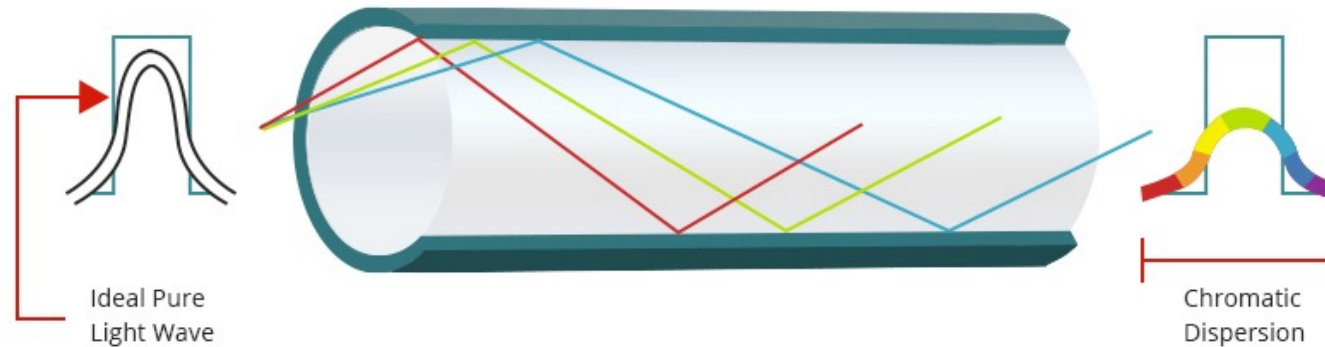


LP41



LP22

Chromatic dispersion



- Signal or pulse spreading over time resulting from the different speeds of light rays
- A combination of the material and waveguide dispersion effects
- **Material dispersion** : wavelength dependence of the refractive index on the core material
- **Waveguide dispersion** : dependence of the propagation constant β on the fiber parameters
- Sellmeier equation for the refractive index

$$n^2(\lambda) = 1 + \sum_{j=1}^k \frac{A_j \lambda^2}{\lambda^2 - B_j^2}$$

$$n_{\text{eff}} = \frac{\beta}{k_0}$$

Where A_j and B_j are the Sellmeier coefficients and n_{eff} the effective index

Chromatic dispersion

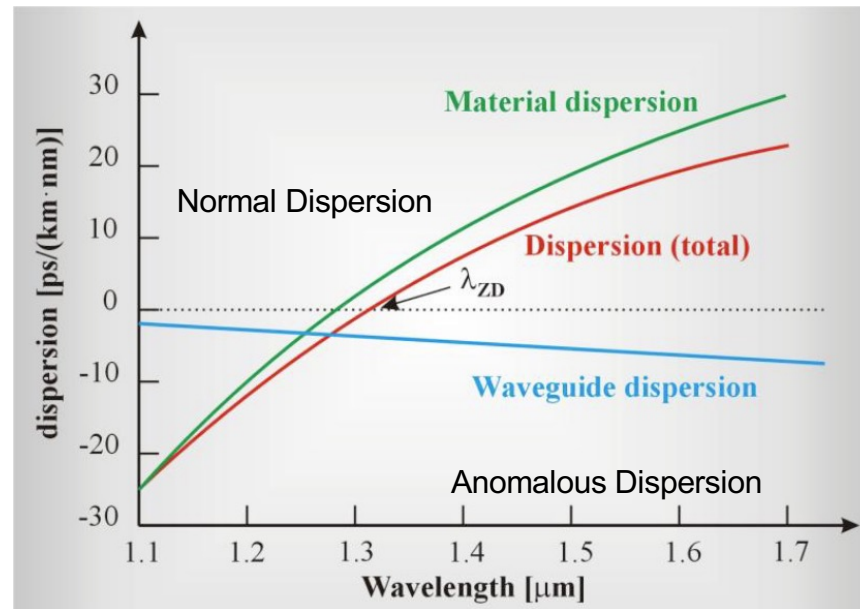
- Expanding the propagation constant as a Taylor series around the central frequency

$$\beta(\omega) = \beta_0 + \beta_1(\omega - \omega_0) + \frac{1}{2}\beta_2(\omega - \omega_0)^2 + \frac{1}{6}\beta_3(\omega - \omega_0)^3 + \dots + \frac{1}{m!}\beta_m(\omega - \omega_0)^m$$

$$\beta_2 = \frac{1}{c} \left(2 \frac{dn_{\text{eff}}}{d\omega} + \omega \frac{d^2 n_{\text{eff}}}{d\omega^2} \right) \text{ in s}^2/\text{m}$$

$$D = \frac{d\beta_1}{d\lambda} = -\frac{2\pi c}{\lambda^2} \beta_2 = -\frac{\lambda}{c} \frac{d^2 n_{\text{eff}}}{d\lambda^2}$$

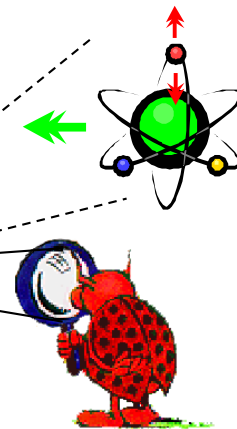
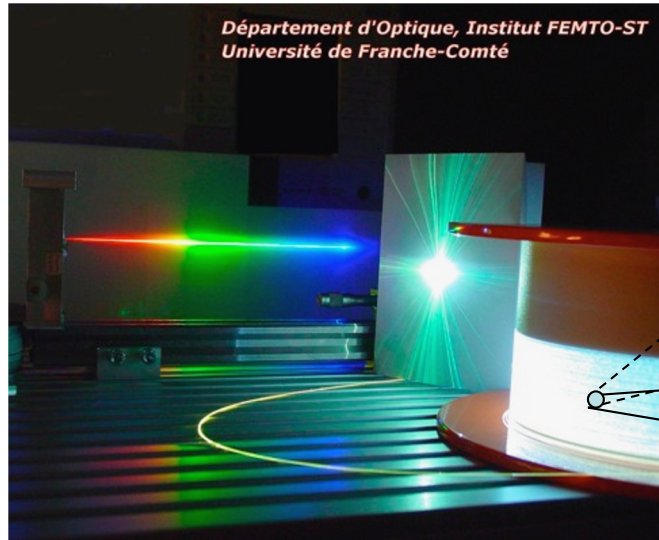
D is in ps/nm/km



- When D is negative, $D < 0 \Rightarrow \beta_2 > 0$: Normal dispersion regime (red light faster than blue)
- When D is positive, $D > 0 \Rightarrow \beta_2 < 0$: Anomalous dispersion (blue light faster than red)
- When D is zero, $D = 0 \Rightarrow$ the zero-dispersion wavelength (ZDW)
- β_3 and β_4 are the dispersion slope and curvature

Raman Light Scattering (RLS)

Nonlinear optical effects are remarkable phenomena that arise when an intense optical beam propagates through a transparent material (optical fiber)



Optical Kerr Effect
(electronic response)

$$n = n_0 + n_2 I(z, t)$$

Stimulated Raman Scattering
(Delayed Molecular response)

Third-Order Nonlinear Polarization

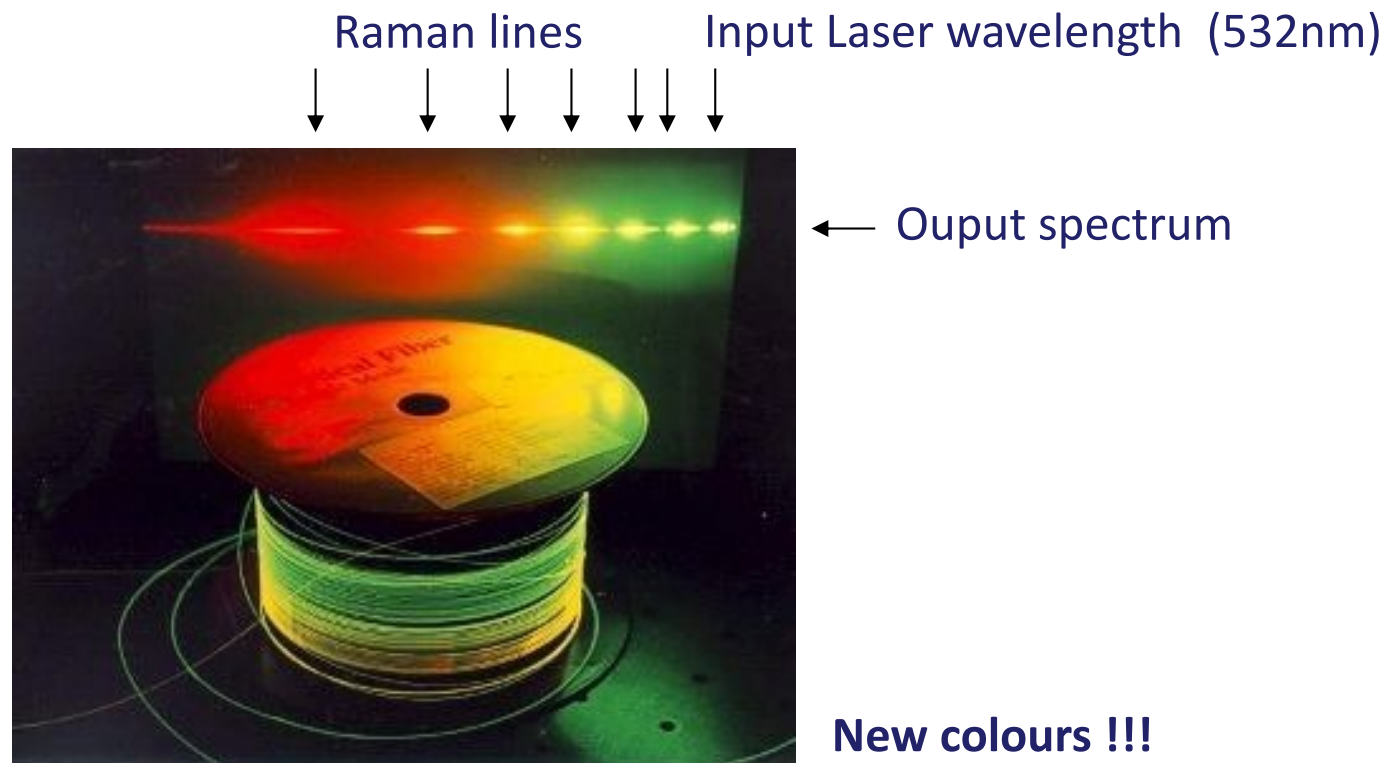
$$P_{NL}(t) = \epsilon_0 \chi_K^{(3)} : E(t)E(t)E(t) + \epsilon_0 E(t) \int_{-\infty}^t \chi_R^{(3)}(t-t') E(t') E(t') dt'$$

Optical Kerr effect:
Instantaneous Elastic effect
(No energy exchange with matter)

Raman effect :
Inelastic light scattering:
(Molecular vibrational states)

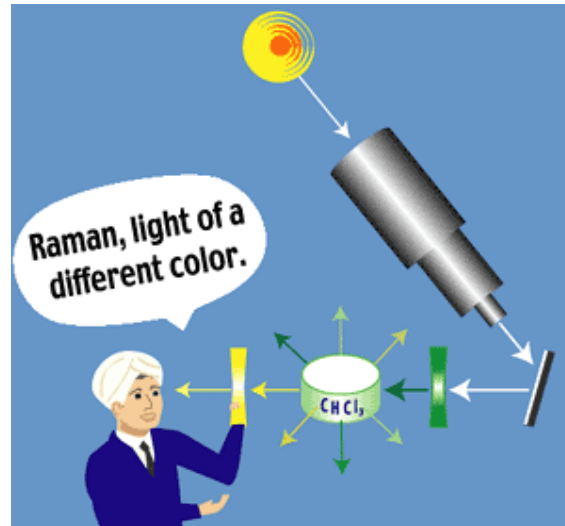
Raman Light Scattering (RLS)

- Appearance of additional lines in the spectrum of monochromatic light that has been scattered by a transparent material medium, e.g. an optical fiber



Raman Light Scattering (RLS)

The molecular scattering of light was discovered by C. V. Raman in 1928 in **liquids** and simultaneously by G. Landsberg and L. Mandelstan in **crystals**.



First Report of Raman Observation

Nature 121, 501-502 (31 March 1928)

A New Type of Secondary Radiation

C. V. RAMAN & K. S. KRISHNAN

Continue

The new type of light scattering discovered by us naturally requires very powerful illumination for its observation. In our experiments, a beam of sunlight was converged successively by a telescope objective of 18 cm. aperture and 230 cm. focal length, and by a second lens was placed the scattering material, which is either a liquid (carefully purified by repeated distillation *in vacuo*) or its dust-free vapour. To detect the presence of a modified scattered radiation, the method of complementary light-filters was used. A blue-violet filter, when coupled with a yellow-green filter and placed in the incident light, completely extinguished the track of the light through the liquid or vapour. The reappearance of the track when the yellow filter is transferred to a place between it and the observer's eye is proof of the existence of a modified scattered radiation. Spectroscopic confirmation is also available.

**C.V. Raman (1888 –1970) was an Indian physicist
Nobel Laureate recognised for his work on
the molecular scattering of light (spontaneous regime)**

C. V. Raman remains the only Indian to receive a Nobel Prize in science.

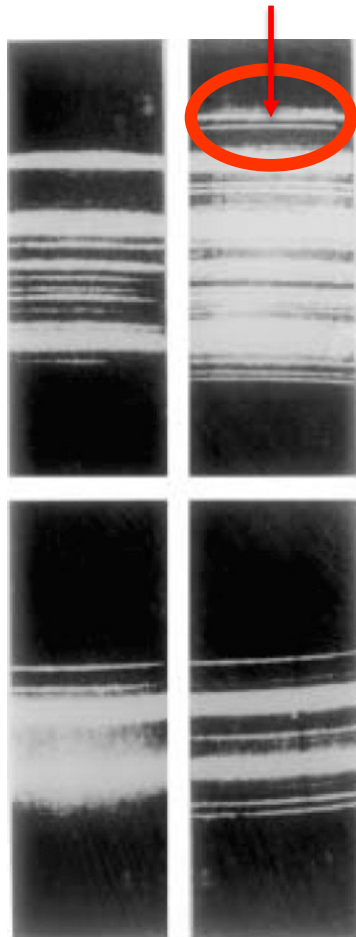
Raman Light Scattering (RLS)

Raman's article in *Nature* did not display any spectra.

They first appeared later in the *Indian Journal of Physics*.

Raman sent reprints of that article reporting his discovery to 2000 scientists in France, Germany, Russia, Canada, and the United States.

Raman lines



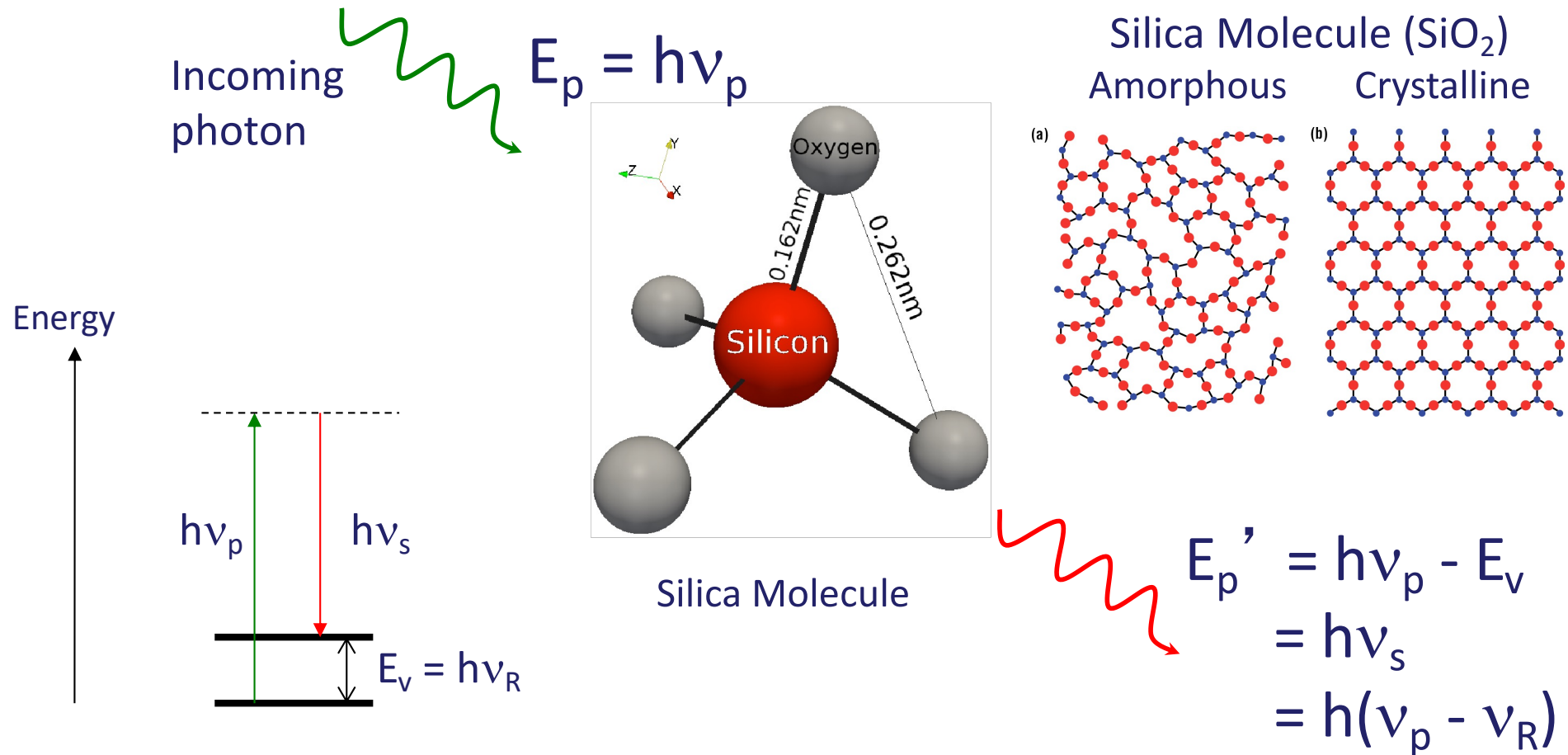
The first spectra taken by C. V. Raman and K. S. Krishnan

The upper-left photograph shows the incident light consisting of the spectrum of a quartz mercury arc lamp after passing through a blue filter that cuts out all wavelengths greater than the indigo line at 435.8 nm.

The upper-right photograph shows the same spectrum when scattered by liquid benzene and taken with a small spectroscope.

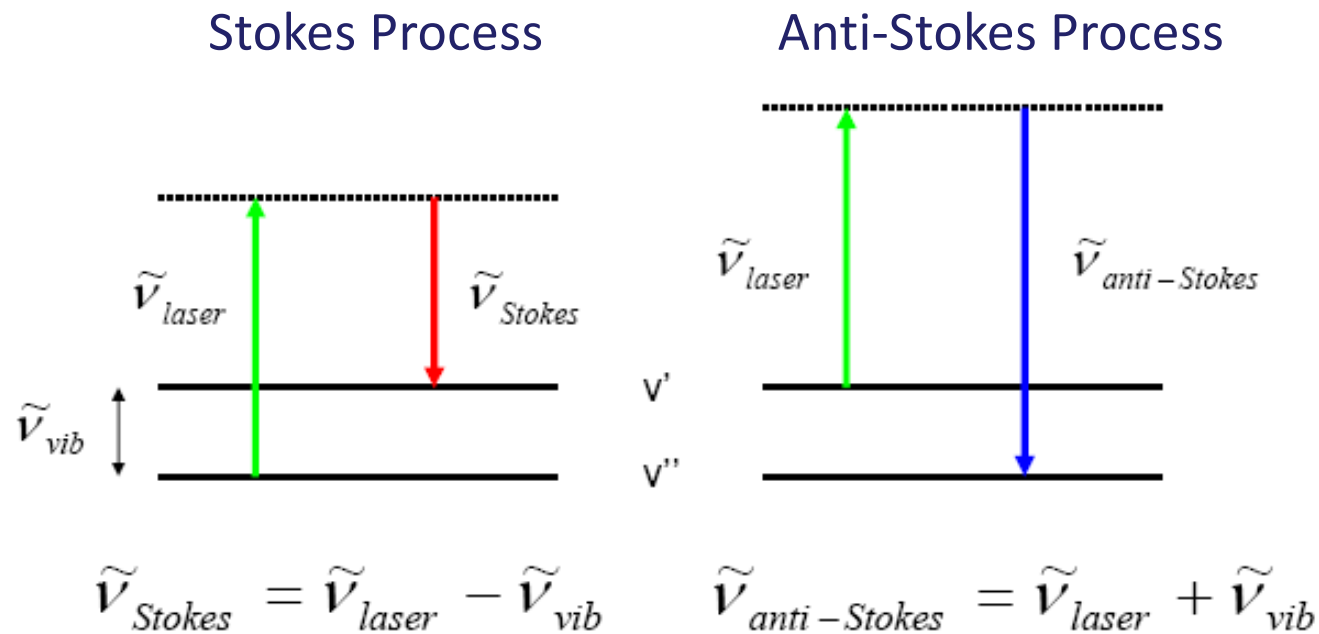
Note the appearance of modified lines owing to the Raman effect.

Spontaneous Raman scattering: Quantum view



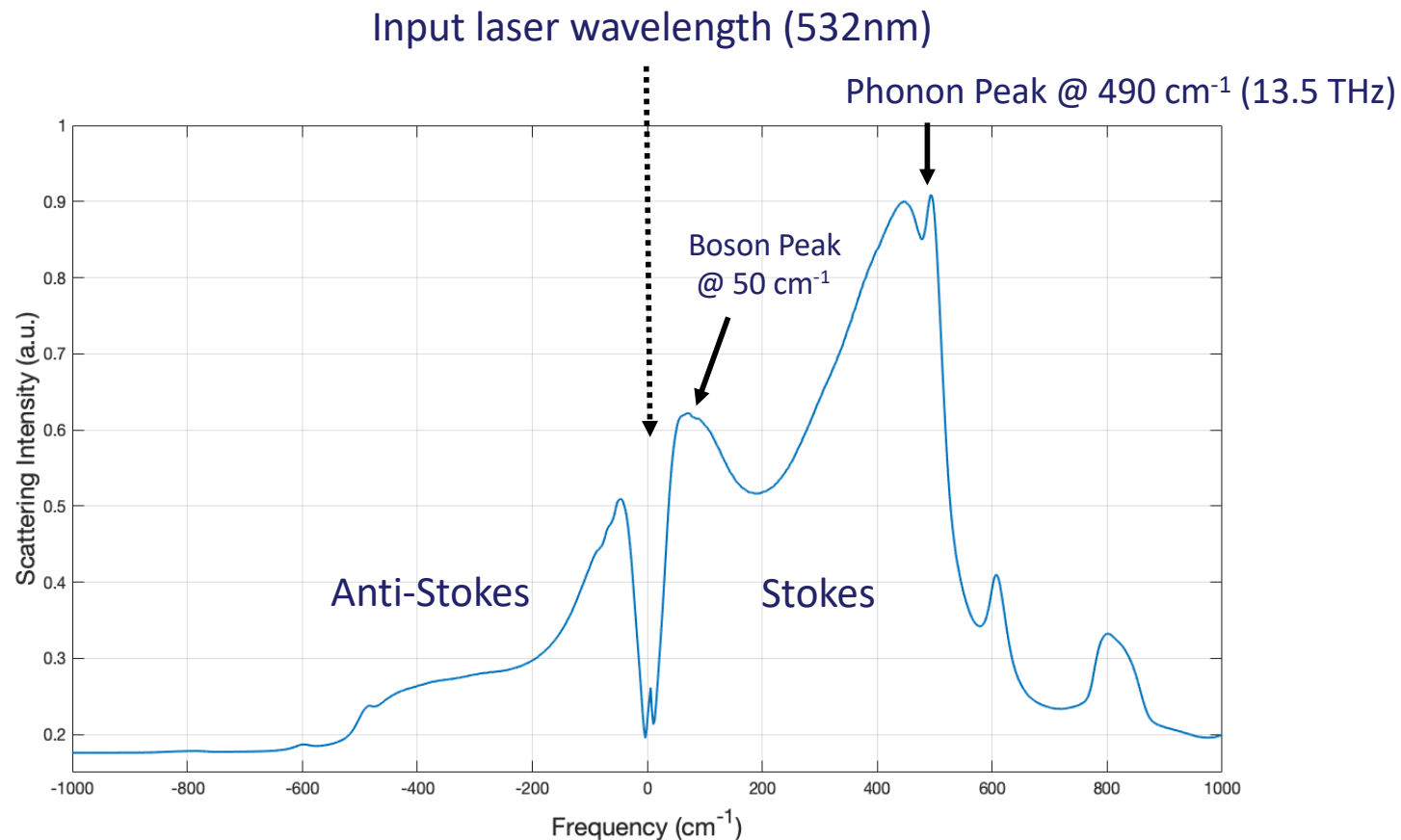
Raman scattering is an *inelastic scattering process*, where an incoming photon interacts with a coherently excited state of the system (e.g. the vibrational modes of a silica SiO₂ molecule). As a result of this interaction, a frequency down-converted (Stokes) photon is emitted.

Spontaneous Raman scattering: Stokes vs anti-Stokes



An up-converted (anti-Stokes) photon may also be emitted if the vibrational states are sufficiently populated. However this is rarely observed in optical fibers due to the ultrafast relaxation time (fs).

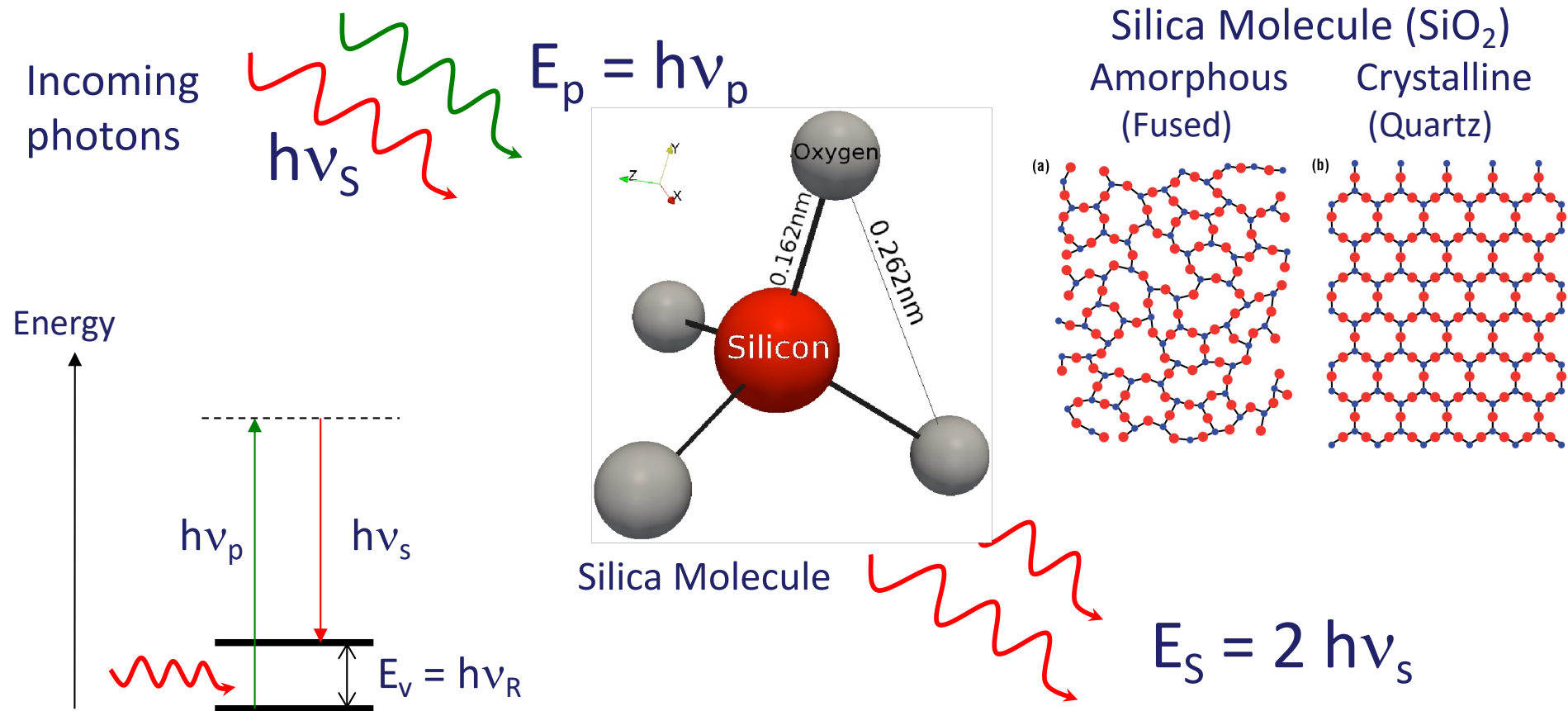
Spontaneous Raman scattering: Spectrum



The spontaneous Raman scattering efficiency is very weak and extends over a broad frequency range (1500 cm⁻¹). The ratio of the anti-Stokes to Stokes intensities follows the Maxwell-Boltzmann's distribution as:

$$I_S / I_{AS} \approx \exp\left(-\hbar\Omega_R / k_B T\right)$$

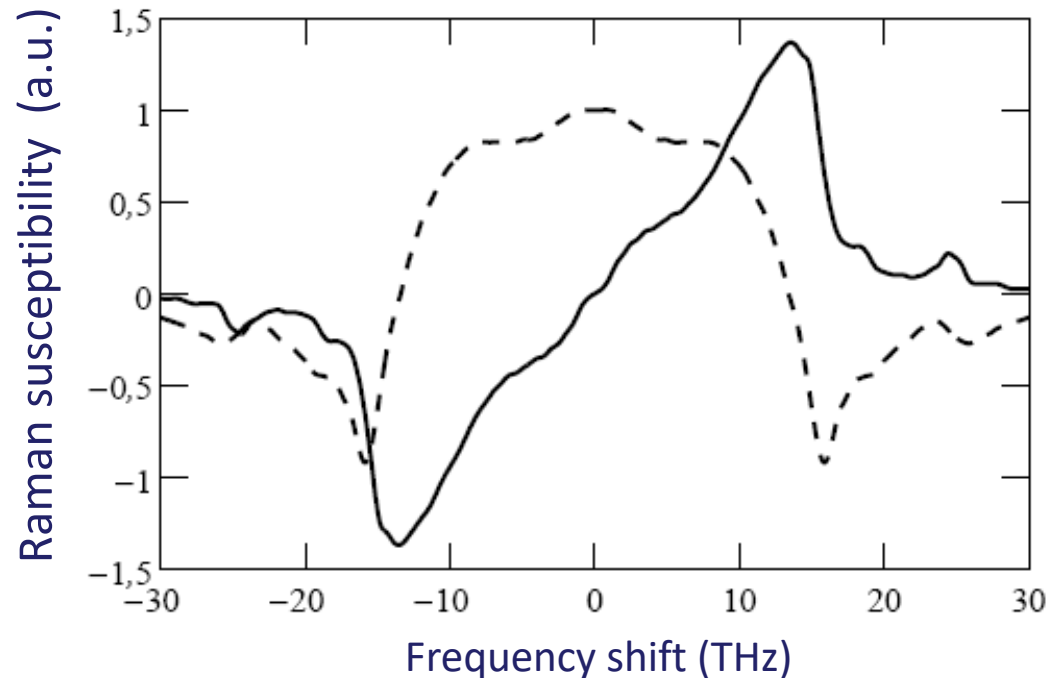
Stimulated Raman scattering (SRS)



If the incident laser beam is sufficiently intense, the photon-phonon scattering process becomes *self-stimulated*. The temporal beating between the pump and the Stokes waves stimulates the vibrational states at the Raman frequency, and the wave grows rapidly such that most of *the pump energy is transferred to it*.

Stimulated Raman scattering (SRS)

The Stimulated Raman gain spectrum in fused silica

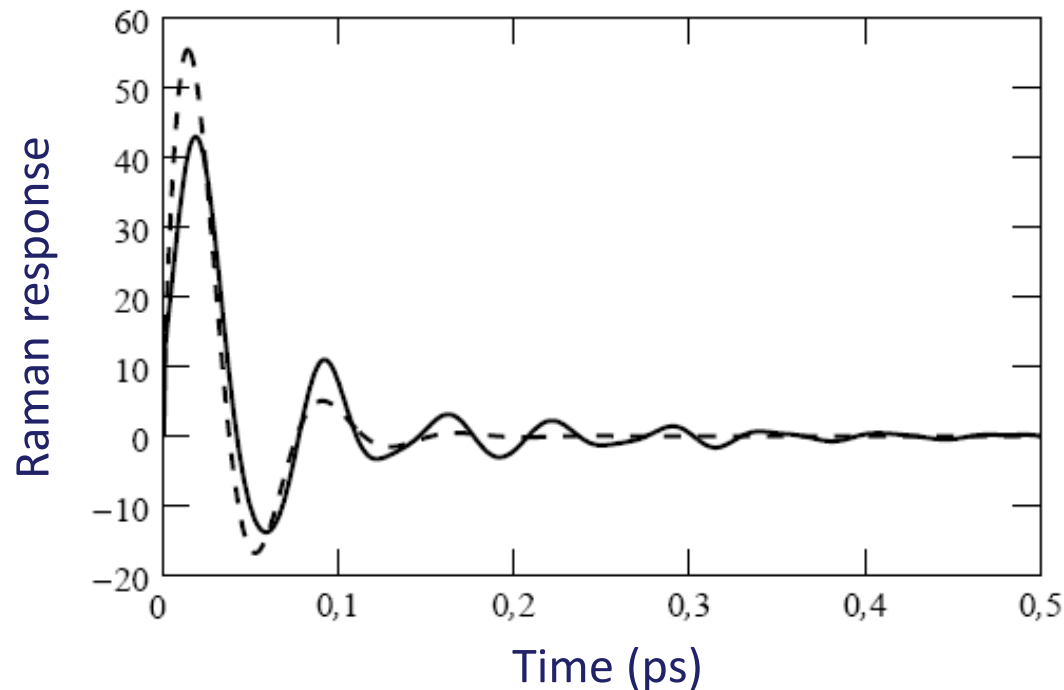


$$\chi_R^{(3)} = \chi_R'(\omega) + i\chi_R''(\omega) = FT(h_R(t))$$

The real part (symmetric dashed curve, nonlinear index) and imaginary part (antisymmetric solid curve, Raman gain) of the Raman susceptibility are related by the Kramers-Kronig relations.

Stimulated Raman scattering (SRS)

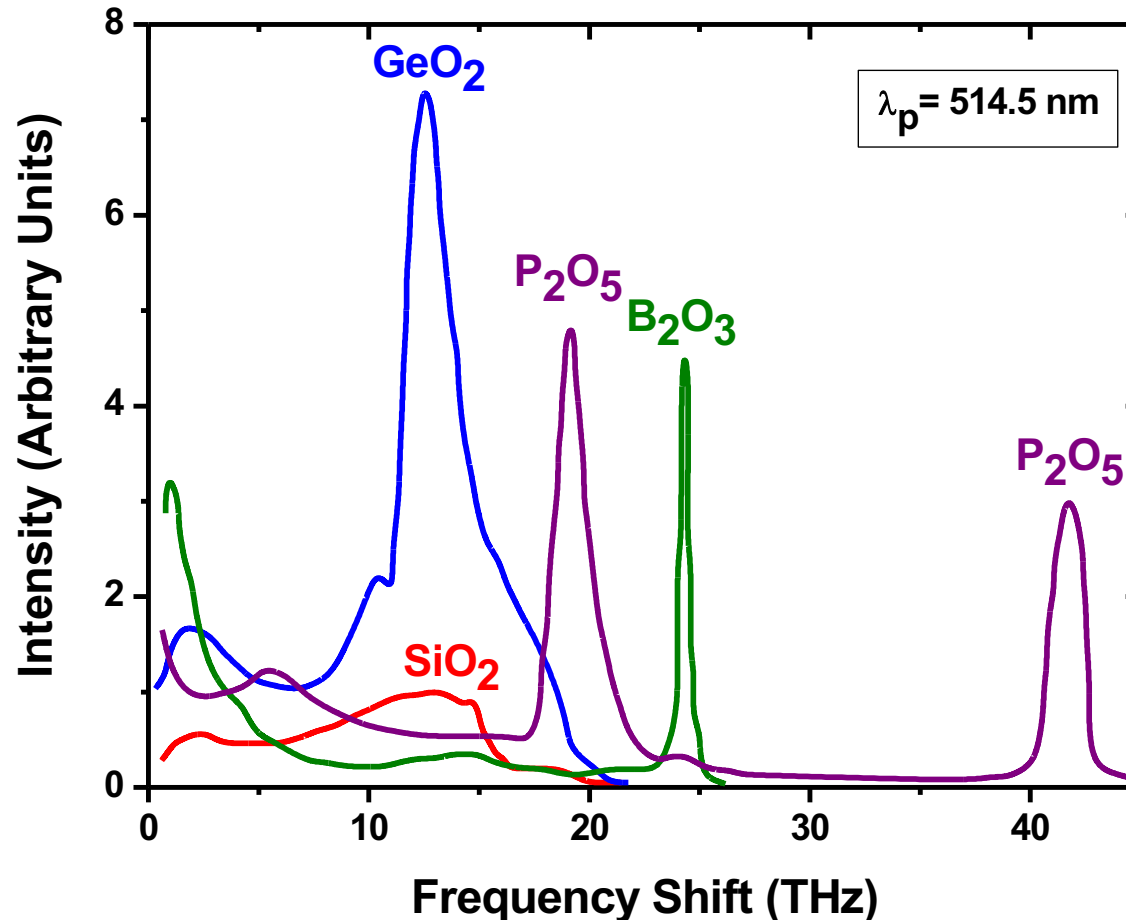
The Raman response function in the time domain is basically a decaying sinusoidal oscillation. The **oscillation period** corresponds to the **peak of the Raman gain** and the **decay rate** corresponds to the **width** of the gain spectrum.



$$h_R(t) = \frac{t_1^2 + t_2^2}{t_1 t_2} \sin\left(\frac{t}{t_1}\right) \exp\left(-\frac{t}{t_2}\right)$$

$t_1 = 12.2 \text{ fs} = 2\pi/\Omega_R$ (13.2 THz)
 $t_2 = 32 \text{ fs} = 2\pi/\Delta\Omega_R$ (5 THz)

Stimulated Raman scattering (SRS)



Galeener et al., Appl.
Phys. Lett. 32, 34 (1978)

The **Raman gain** can be **enhanced and frequency shifted** by use of dopants inside the silica core or using other nonlinear glasses (Chalcogenide, Tellurite, ZBLAN, fluoride)

Stimulated Raman scattering (SRS)

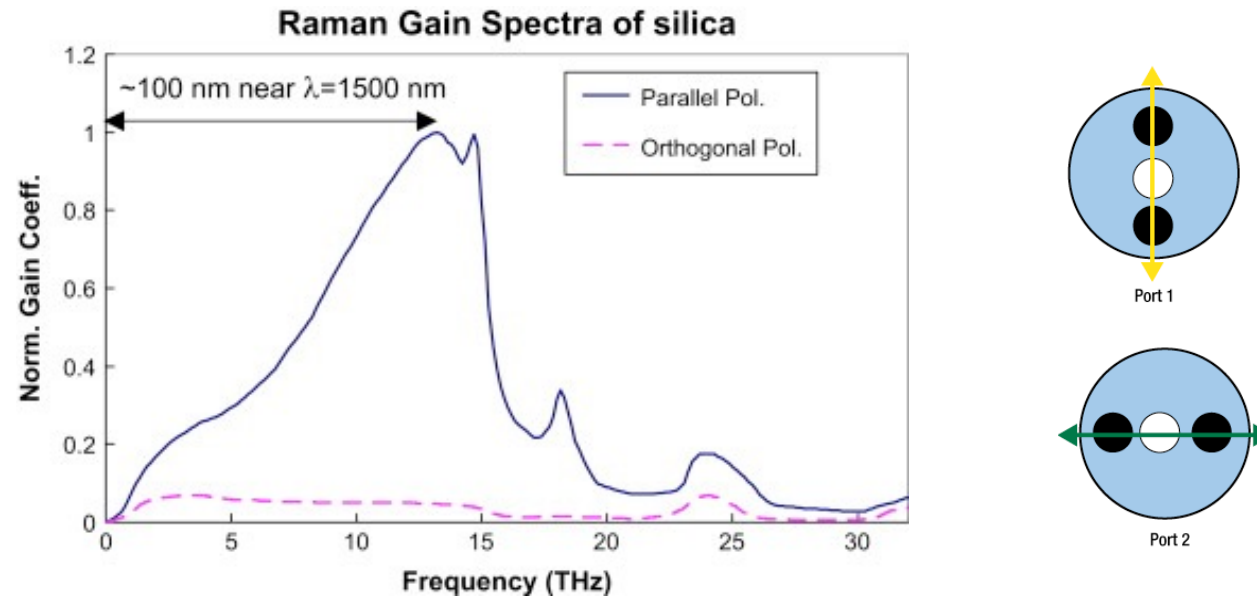
The Raman threshold is defined as the **input pump power** at which the **Stokes power** becomes equal to the **pump power** at the fiber output (Saturation regime) :

$$P_{Th} (W) = \frac{16A_{eff}}{g_R L_{eff}}$$

- ✓ Where A_{eff} is the effective area of the HE11 mode :
$$A_{eff} = \frac{\left(\int \int_{-\infty}^{+\infty} |F(x,y)|^2 dx dy \right)^2}{\int \int_{-\infty}^{+\infty} |F(x,y)|^4 dx dy}$$
- ✓ g_R is the Raman gain, typically 10^{-13}m.W^{-1} for silica fibers
- ✓ L_{eff} is the effective length that accounts for loss:
$$L_{eff} = \int_0^L e^{-\alpha z} dz = \frac{1 - e^{-\alpha L}}{\alpha}$$

Stimulated Raman scattering (SRS)

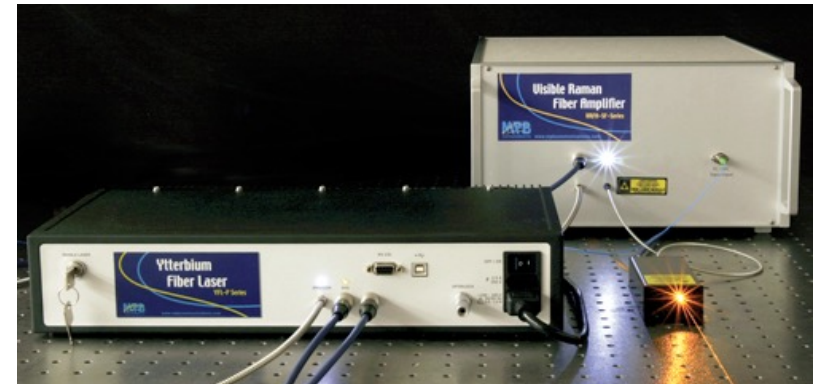
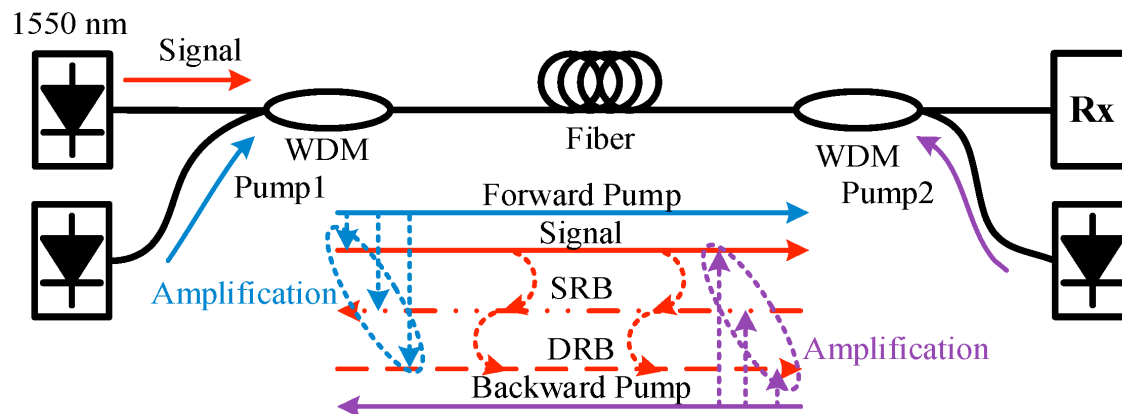
The effect of polarization state of pump light



- Raman scattering is strongly polarization dependent
- The Raman gain coefficient for a signal perpendicular to the pump is **30 times** lower than the parallel Raman gain
- However, by using cross-polarized pumps or pump polarization diversity, the Raman gain can be made independent of the state of polarization of the incident signal.

Key Applications : Fiber Raman Amplifiers

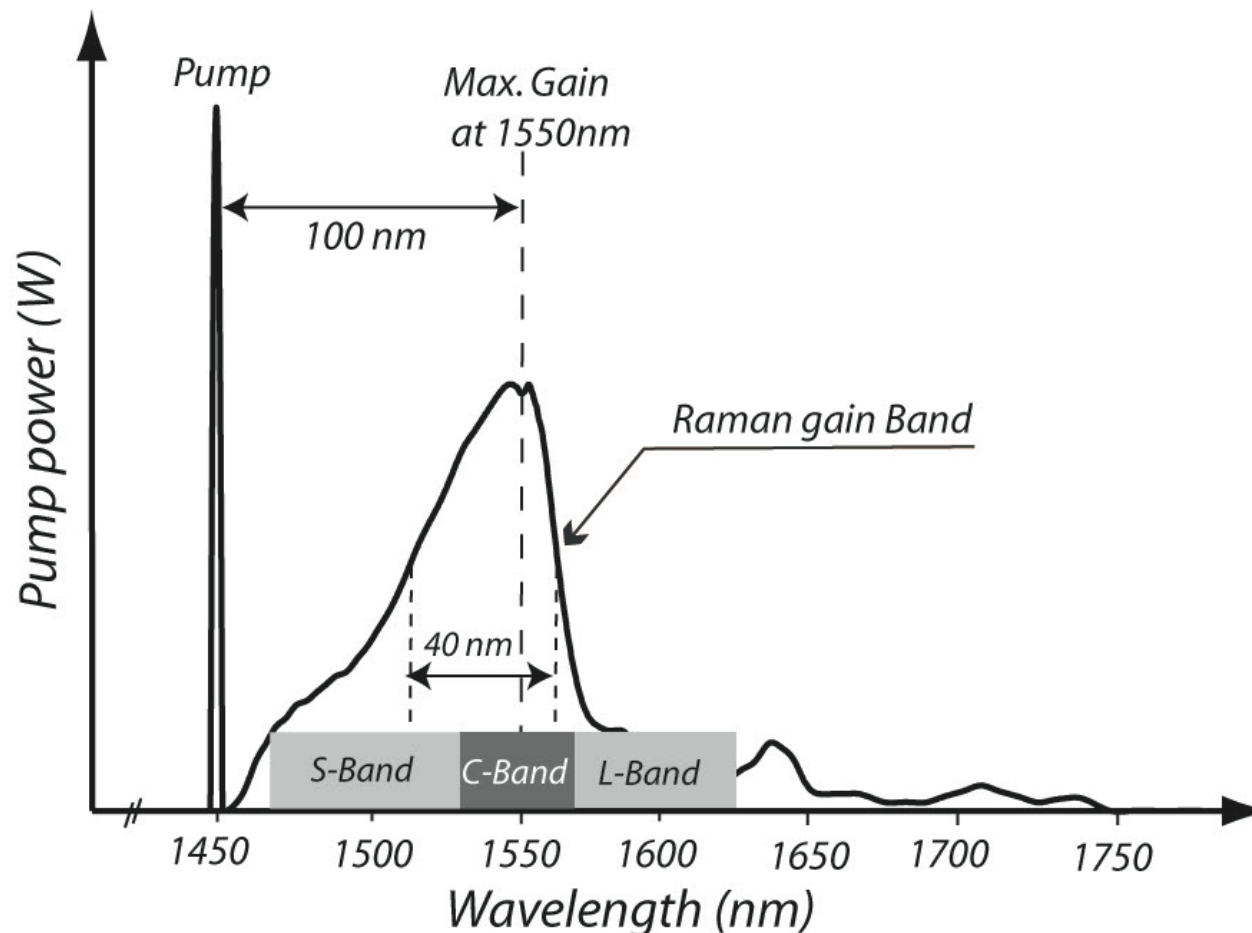
Stimulated Raman scattering can serve as an **optical amplifier** for applications in telecommunications (typically 50nm bandwidth @1.55 μm).



Raman amplifiers can operated in very different wavelength regions
Gain spectrum can be tailored by using different pump wavelengths
Distributed forward and backward pumping
Low noise (Noise figure < 3 dB)

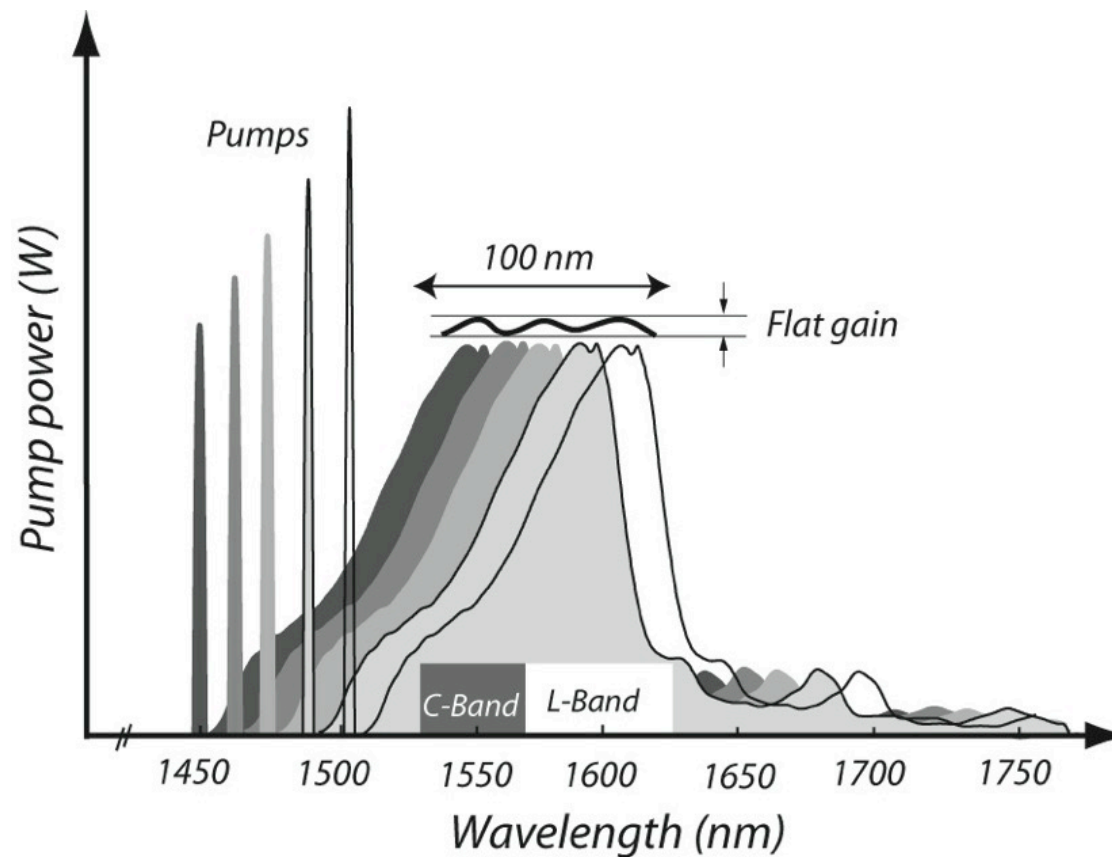
Key Applications : Fiber Raman Amplifiers

In optical fibers, the Raman effect is a broadband gain process. This wide gain bandwidth is very advantageous for making Raman fiber amplifiers (RFA) and Raman fiber lasers (RFL). For a pump wavelength at 1450 nm, the gain band provides maximum gain at 1550 nm over a range of 40 nm.



Key Applications : Fiber Raman Amplifiers

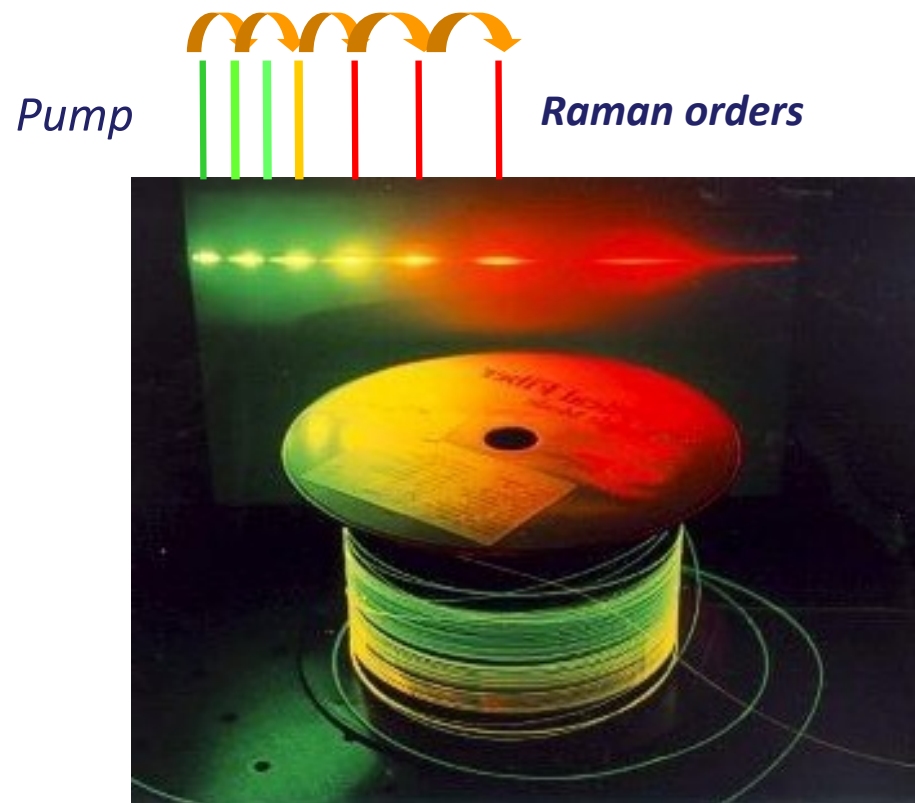
It is possible to combine in the same optical fiber several pump lasers such as to generate a flat and wide gain bandwidth. Such a large gain bandwidth has recently enabled to transmit more than 20 Tbit/s data rate over more than 10000 km.



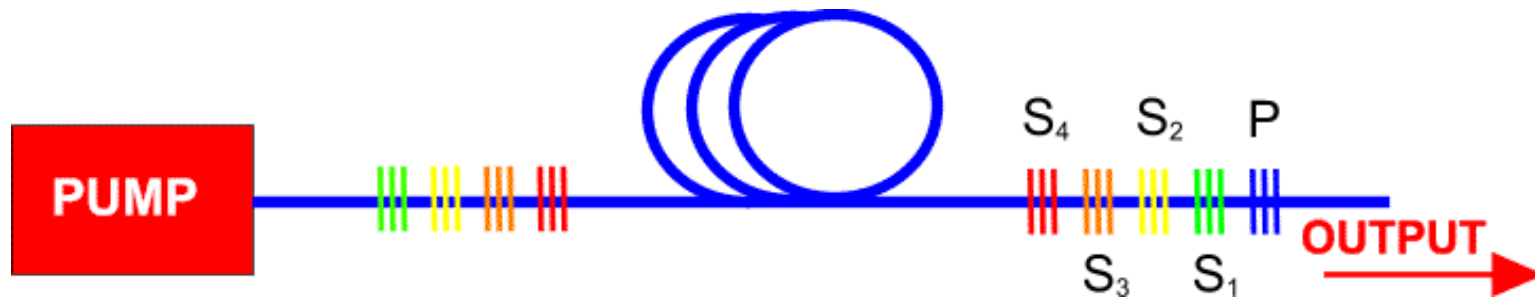
Principle of multi-wavelength pumping for flat and broadband Raman amplification at telecommunication wavelengths (C and L bands).

Cascaded Raman Scattering (CRS)

- ▶ Cascaded Raman generation is an iteration of fundamental stimulated Raman scattering (SRS) processes in which each generated Stokes wave acts as a pump to produce an **additional order**



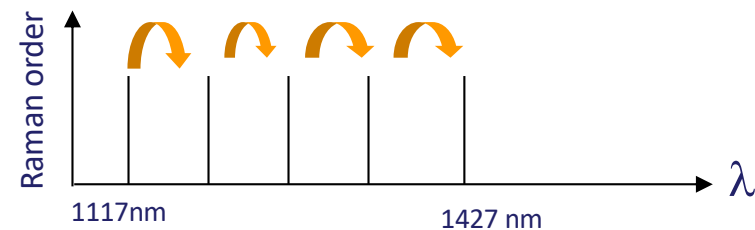
Cascaded Raman fiber lasers (RFL)



Cascaded Raman fiber lasers can be built with nested pairs of fiber Bragg gratings. Oscillation on one Raman order is used for pumping another order, so that larger frequency offsets can be bridged.

► Advantages

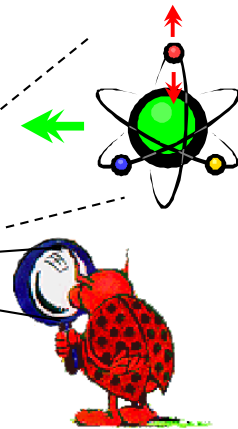
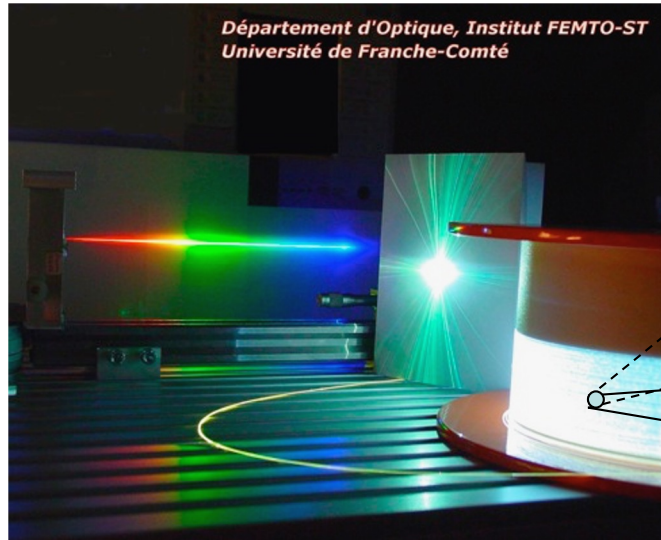
- ☺ Very high output power delivery
- ☺ Wavelength versatile
- ☺ Ideal for Raman amplification in WDM systems (low noise, wideband,...)



Principle of operation

The Optical Kerr Effect

Nonlinear optical effects are striking phenomena that arise when an intense optical beam propagates through an optical fiber



**Optical Kerr Effect
(electronic response)**

$$n = n_0 + n_2 I(z, t)$$

**Stimulated Raman Scattering
(Delayed Molecular response)**

Third-Order Nonlinear Polarization

$$P_{NL}(t) = \epsilon_0 \chi_K^{(3)} : E(t)E(t)E(t) + \epsilon_0 E(t) \int_{-\infty}^t \chi_R^{(3)}(t-t') E(t') E(t') dt'$$

Optical Kerr effect:
Instantaneous Elastic effect
(No energy exchange with matter)

Raman effect :
Inelastic light scattering:
(Molecular vibrational state)

The Optical Kerr Effect

The optical **Kerr effect** can be described as an instantaneous and local change in the refractive index, proportional to the optical intensity:

$$n(z,t) = n_0 + n_2 I(z,t)$$



John Kerr
(1824-1907)

with n_2 the **nonlinear Kerr index**

(typically $3 \times 10^{-16} \text{cm}^2/\text{W}$ for silica)

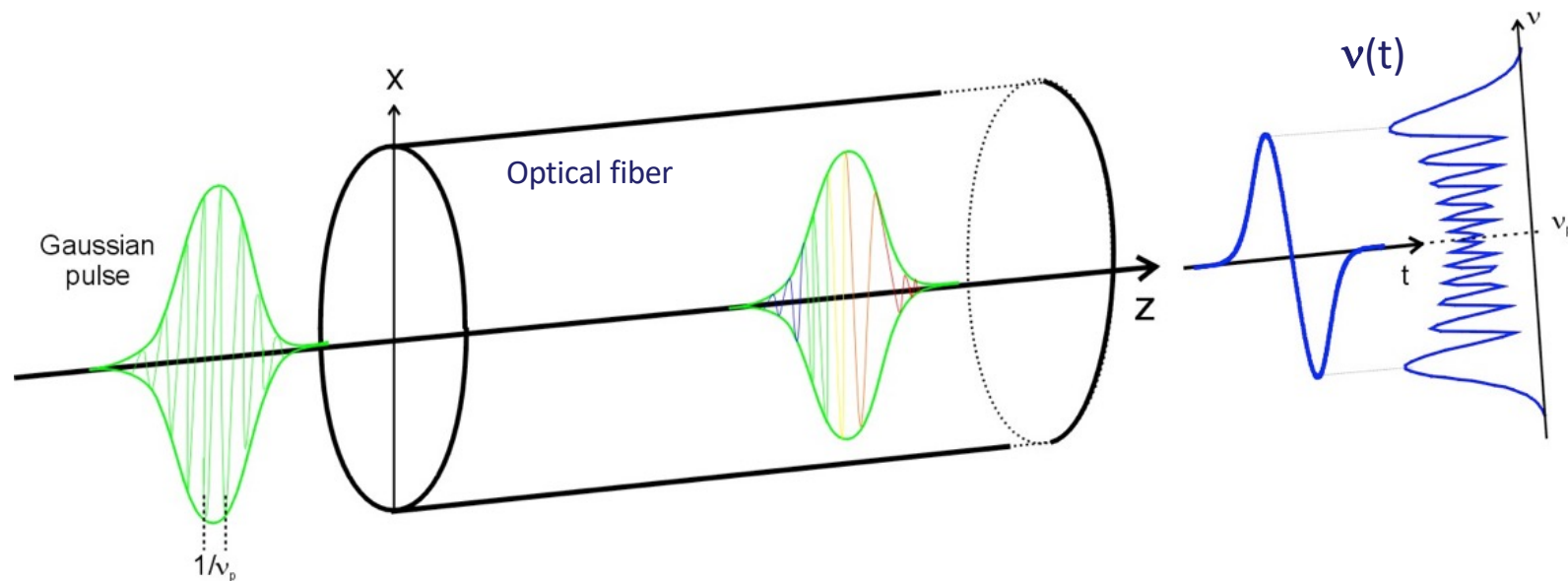
In optical fibers, we use the nonlinear coefficient : $\gamma = \frac{2\pi n_2}{\lambda A_{eff}} \text{ (W}^{-1} \text{ km}^{-1}\text{)}$

This is an elastic (non-resonant) scattering : the incident photons suffer phase and/or frequency shifts **but overall there is no energy exchange with the material**

- Kerr materials exhibit many quantum properties: noise squeezing, correlation, twin-photon, entanglement, EPR paradox, etc...

Self-phase modulation (SPM)

If an optical pulse is transmitted through an optical fiber, the Kerr effect causes a time-dependent phase shift according to the pulse profile. In this way, the pulse acquires a so-called chirp, i.e., a temporally varying instantaneous frequency, and therefore a spectral broadening.

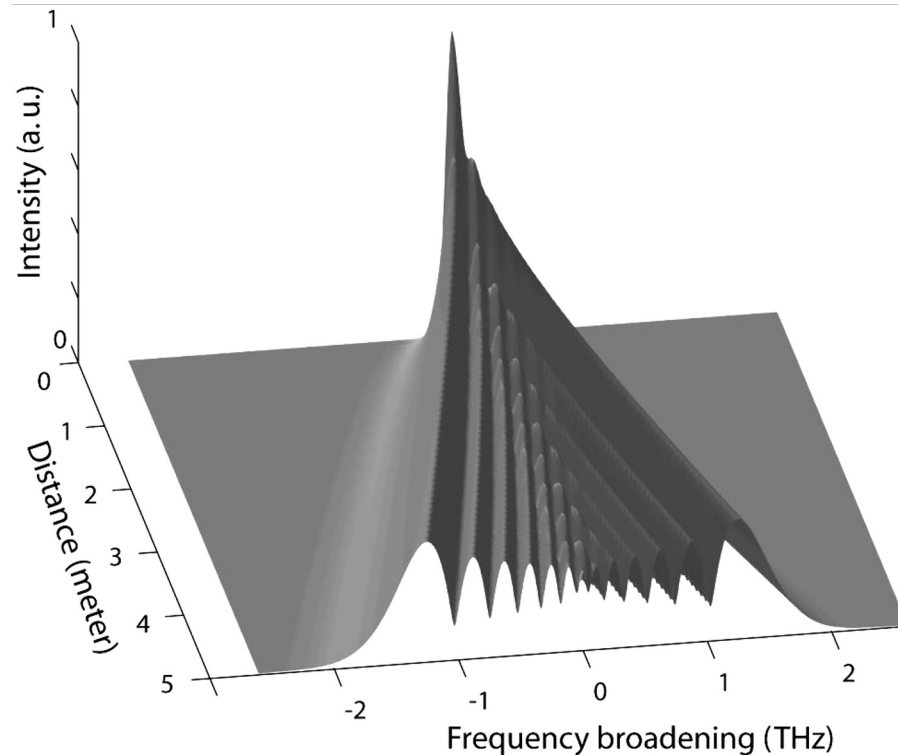


$$\text{Nonlinear phase shift: } \varphi_{NL}(t) = \gamma P(t)L$$

$$\text{Instantaneous frequency chirp: } \nu(t) = -\frac{1}{2\pi} \frac{\partial \varphi_{NL}}{\partial t} = -\frac{\gamma L}{2\pi} \frac{\partial P(t)}{\partial t}$$

Self-phase modulation (SPM)

The Fourier spectrum of the optical pulses exhibits strong oscillations due to constructive and destructive interferences



$$A(z, t) = A(0, \tau) \exp(j\gamma L |A|^2)$$

$$|A(\tau)|^2 = P_0 \exp(-\tau^2/\tau_0^2)$$

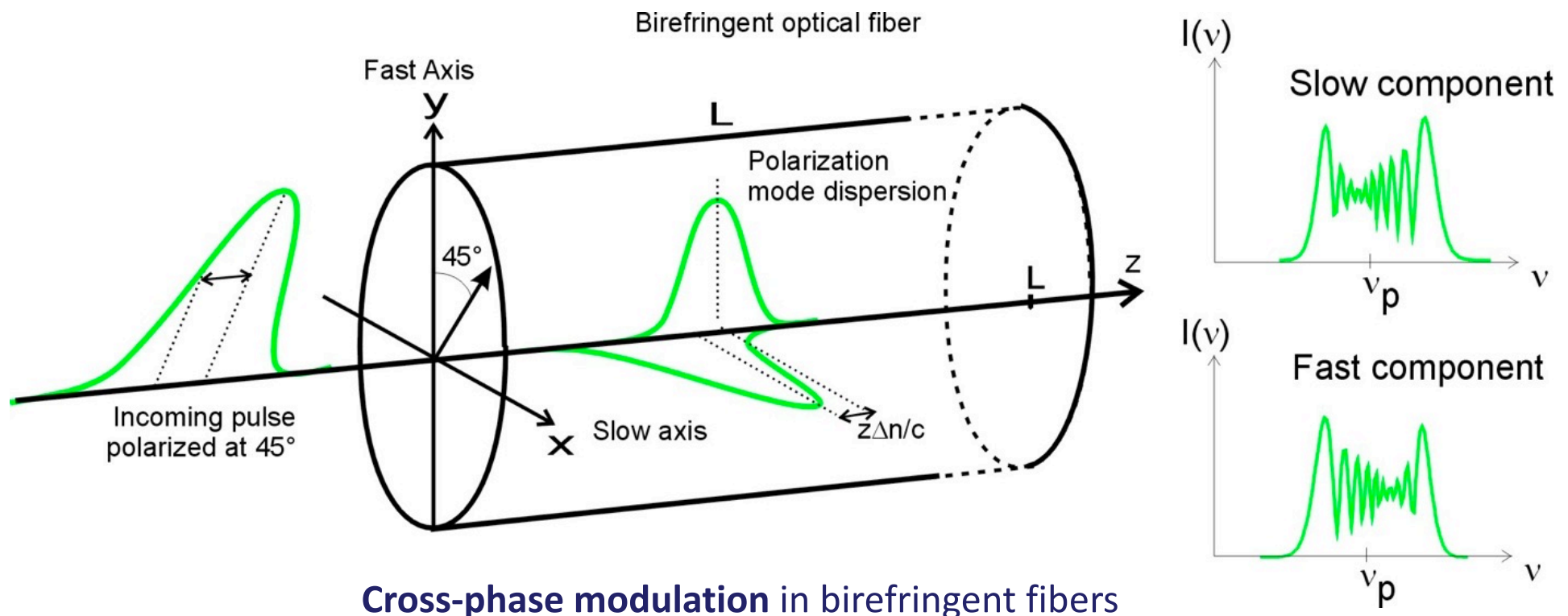
$$\delta\omega(t) = -\gamma L \frac{\partial}{\partial t} |A|^2 = 2\gamma P_0 L \tau \exp\left(-\frac{\tau^2}{\tau_0^2}\right)$$

The number of fringes N is proportional to the nonlinear phase shift by $N\pi$.

R.H. Stolen and C.H. Lin, Phys. Rev. A 17, 1448-1453 (1978).

Cross-phase modulation (XPM)

Cross-phase modulation (XPM) is the change in the optical phase of a light pulse caused by the interaction with another pulse of different color or polarization. Compared with self-phase modulation, there is an additional factor of 2 and for cross-polarized beams in birefringent fiber, it must be replaced with 2/3.

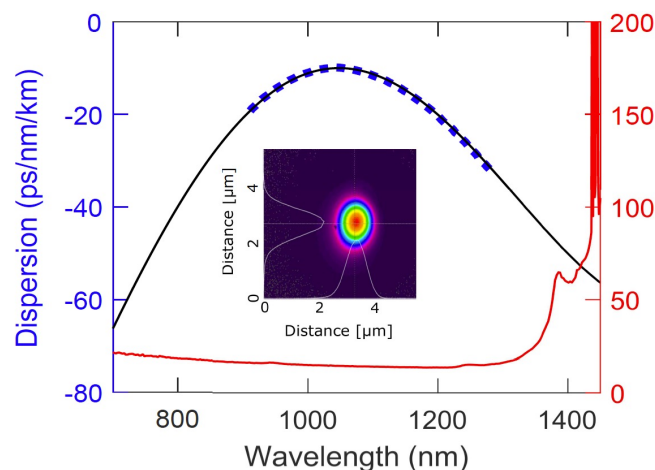


Optical Wave Breaking (OWB)

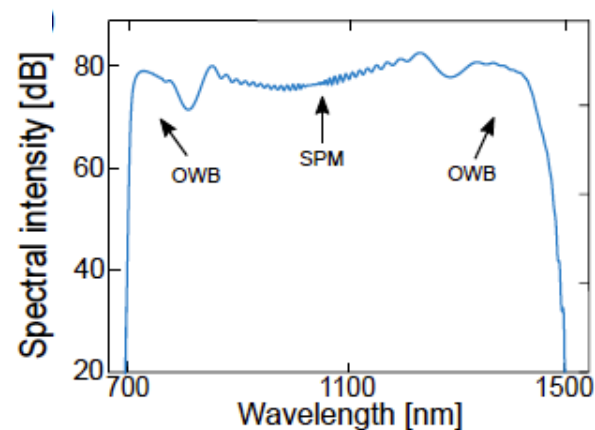
- Modelling with a generalized nonlinear Schrödinger equation (NLSE)

$$\frac{\partial A}{\partial z} = \underbrace{-\frac{\alpha(\omega)}{2}A}_{\text{loss}} + \underbrace{\sum_{k \geq 2} \frac{i^{k+1}}{k!} \beta_k \frac{\partial^k A}{\partial T^k}}_{\text{dispersion}} + \underbrace{i\gamma \left(1 + i\tau_0 \frac{\partial}{\partial T}\right)}_{\text{Self-steepening}} \left(A \int_{-\infty}^{+\infty} R(T') |A(z, T - T')|^2 dT' \right) \underbrace{\quad}_{\text{Nonlinear response (Kerr + Raman)}}$$

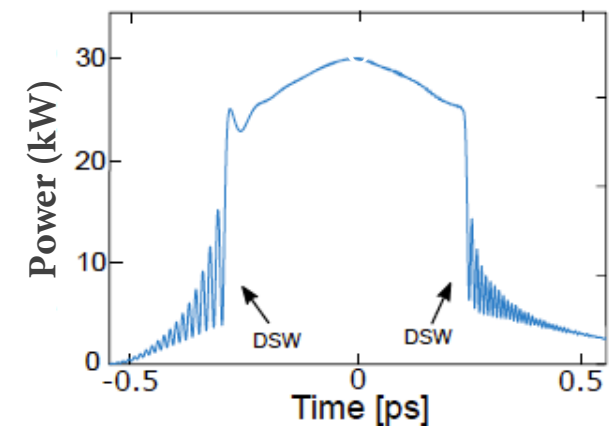
- Pumping in the normal dispersion regime
- Self-phase modulation (SPM) + Optical-Wave Breaking (OWB)
- OWB = Dispersive Shock Wave (DSW) => FWM between SPM and residual pump
- Smooth and flat SC spectrum – Pulse-preserved
- Gives low noise and high coherence level



A Heidt et al (2017). JOSAB, 34(4), 764.



C. Finot et al, JOSAB 25, 1938 (2008).



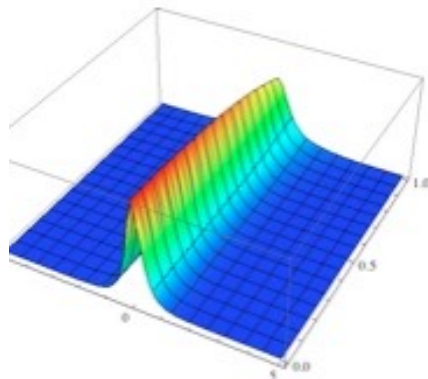
Temporal Optical Soliton (OS)

A fundamental optical soliton (N=1) is a pulse that preserves its shape during the propagation, being unaffected by dispersion and nonlinearities or collision.

Soliton envelope:
$$A(z,t) = \sqrt{P_0} \operatorname{sech}\left(\frac{\tau}{\sqrt{|\beta_2|/\gamma P_0}}\right) \exp\left(i\frac{\gamma P_0}{2}z\right)$$

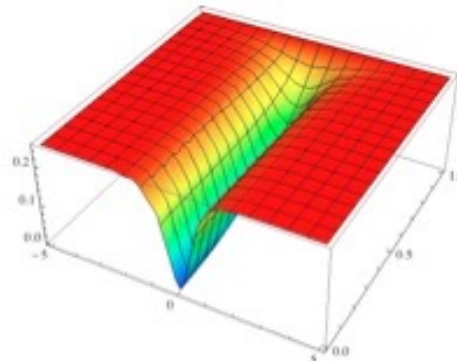
Soliton number:
$$N = \sqrt{\frac{\gamma P_0 T_0^2}{|\beta_2|}} = 1$$

Bright Soliton



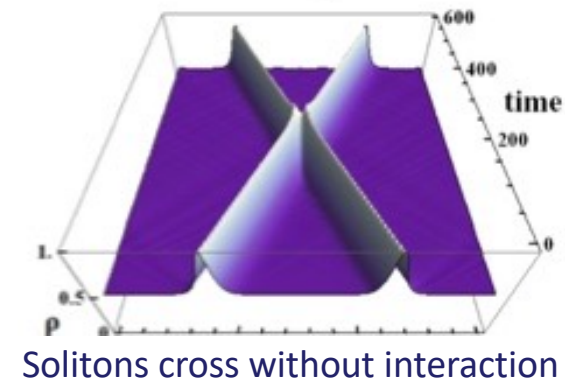
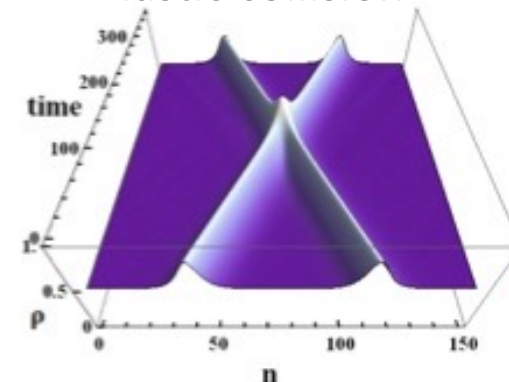
Anomalous dispersion ($\beta_2 < 0$)

Dark Soliton



Normal dispersion ($\beta_2 > 0$)

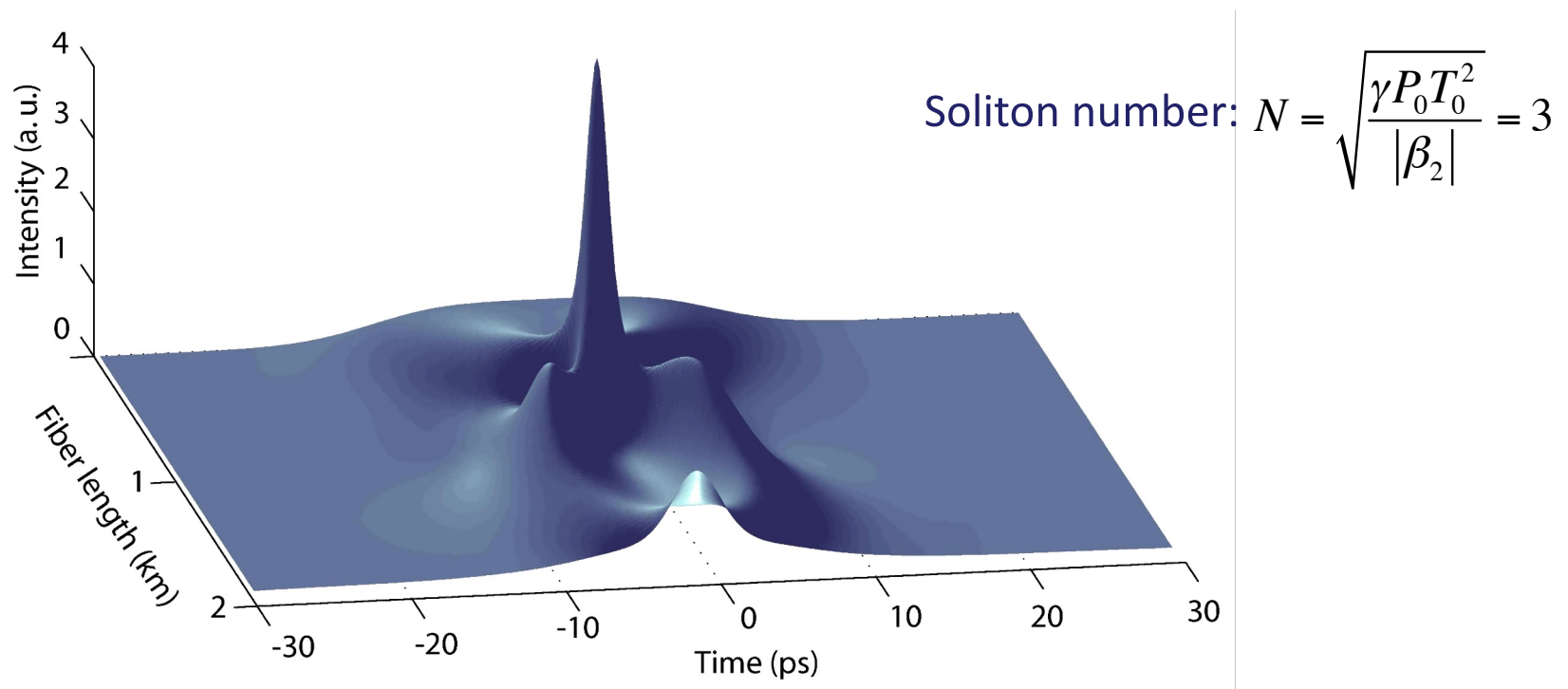
Elastic collision



Solitons cross without interaction

High-order Optical Soliton (HOS)

A high-order optical soliton ($N > 1$) is a pulse that periodically changes its shape during the propagation, always returning to its original shape.



Third-order soliton ($N=3$) propagating in a 2 km-long single-mode fiber

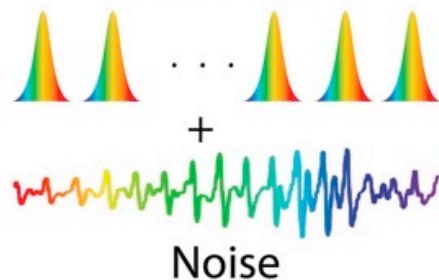
Modulation Instability (MI)

Modulation instability manifests as the break-up of a continuous field or long pulses into a train of ultra-short pulses (fs).

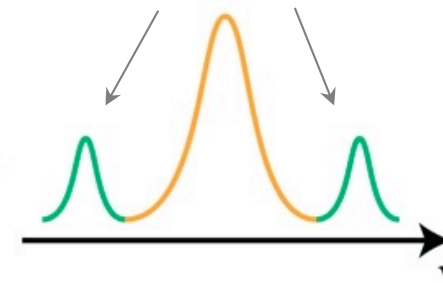
$$\text{MI gain: } g(\Omega) = |\beta_2 \Omega| \sqrt{\frac{4\gamma P_0}{|\beta_2|} - \Omega^2}$$

$$\text{MI modulation frequency : } \Omega_{\max} = \pm \sqrt{\frac{2\gamma P_0}{|\beta_2|}}$$

Long pulse train or CW



MI (FWM) Sidebands (0.1 to 10 THz)

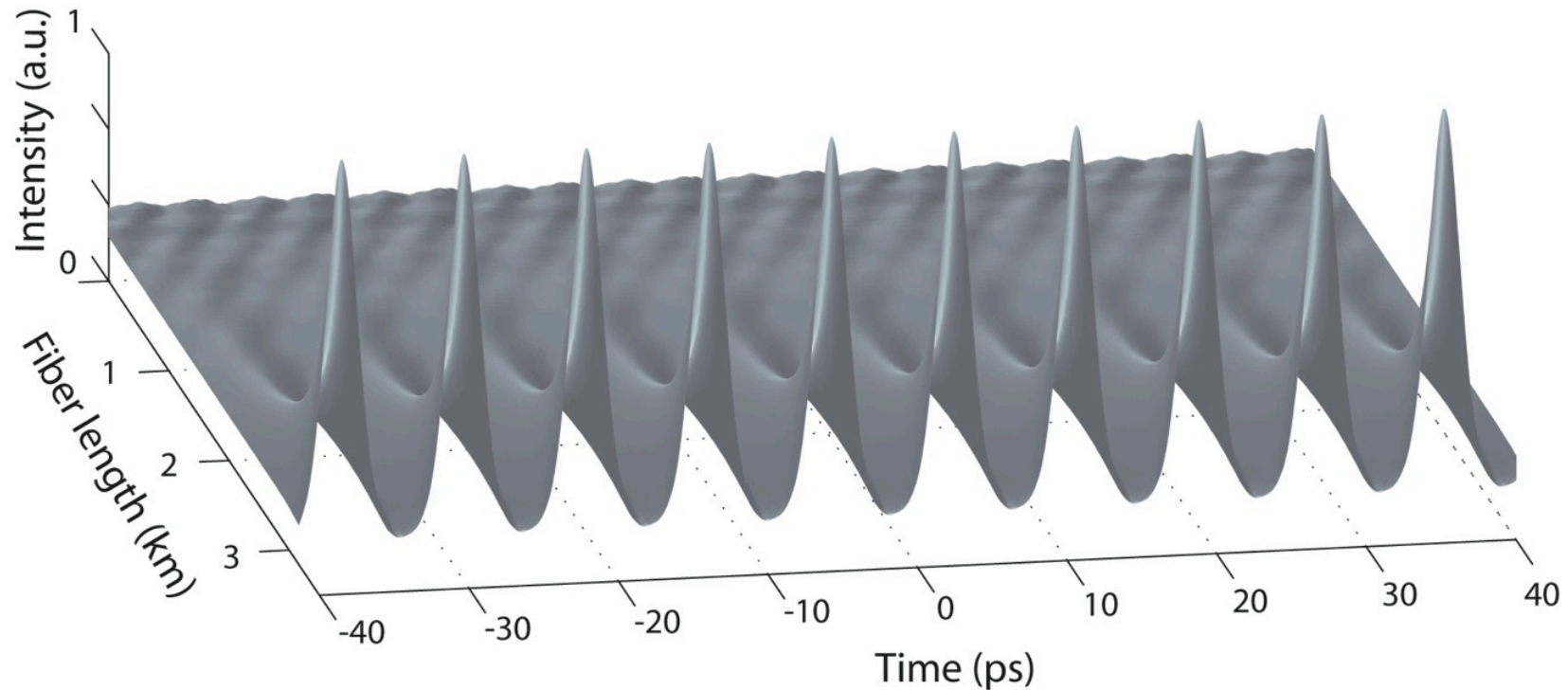


In the frequency domain, it gives rise to two symmetric FWM sidebands (Stokes and anti-Stokes).

Modulation instability plays a fundamental role in supercontinuum generation with cw or long pulse pumping.

Modulation Instability (MI)

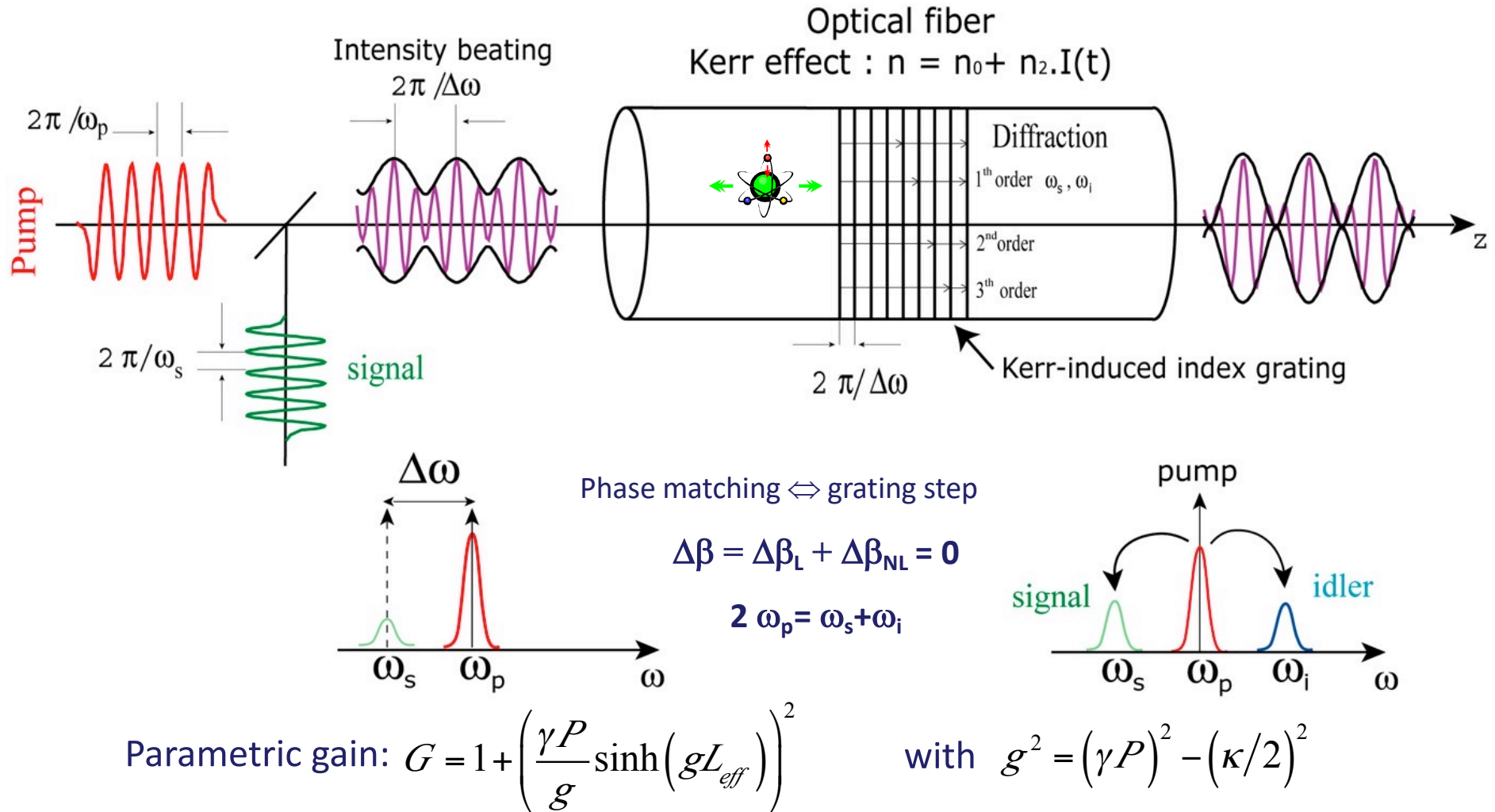
Modulation instability manifests as the break-up of a continuous field or long pulses into a train of ultra-short pulses (fs).



Modulation instability of a noisy continuous field and a soliton-like (breathers) pulse train generated in a 3-km optical fiber, obtained from a numerical simulation of NLSE

Optical Parametric Amplification (OPA)

A phenomenological approach based on interference and diffraction



Parametric amplification \Leftrightarrow Four-wave mixing \Leftrightarrow Modulation Instability \Leftrightarrow optical solitons

Optical Parametric Amplification (OPA)

Phase-matching condition near the zero-dispersion wavelength

$$\kappa = \Delta\beta_L + \Delta\beta_{NL}$$

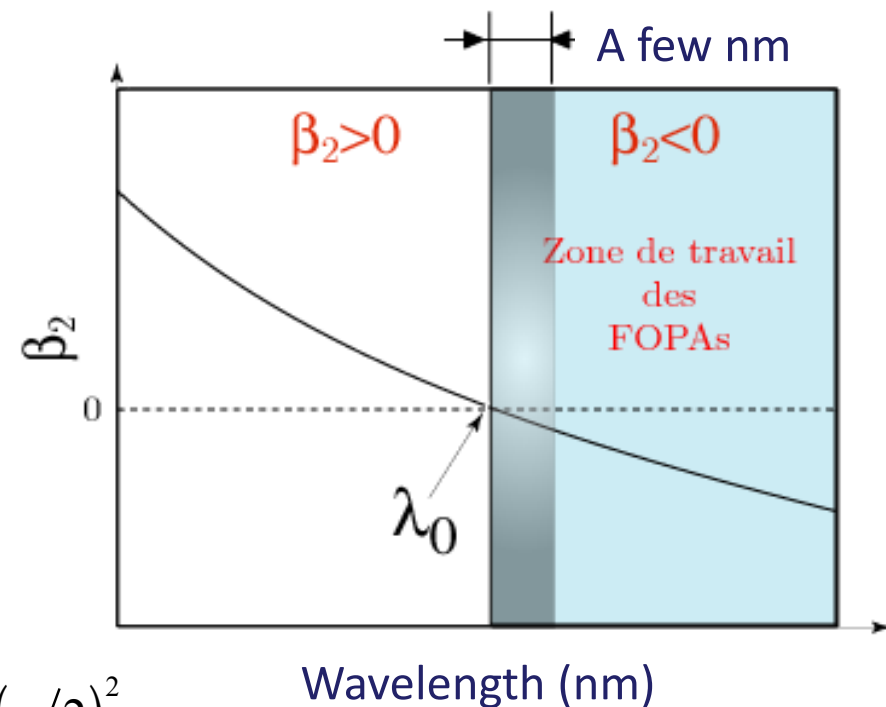
$$\Delta\beta_L = \beta_2 \Delta\omega^2 + \frac{\beta_4}{12} \Delta\omega^4$$

Linear phase shift

$$\Delta\beta_{NL} = 2\gamma P_0 > 0$$

Nonlinear phase shift

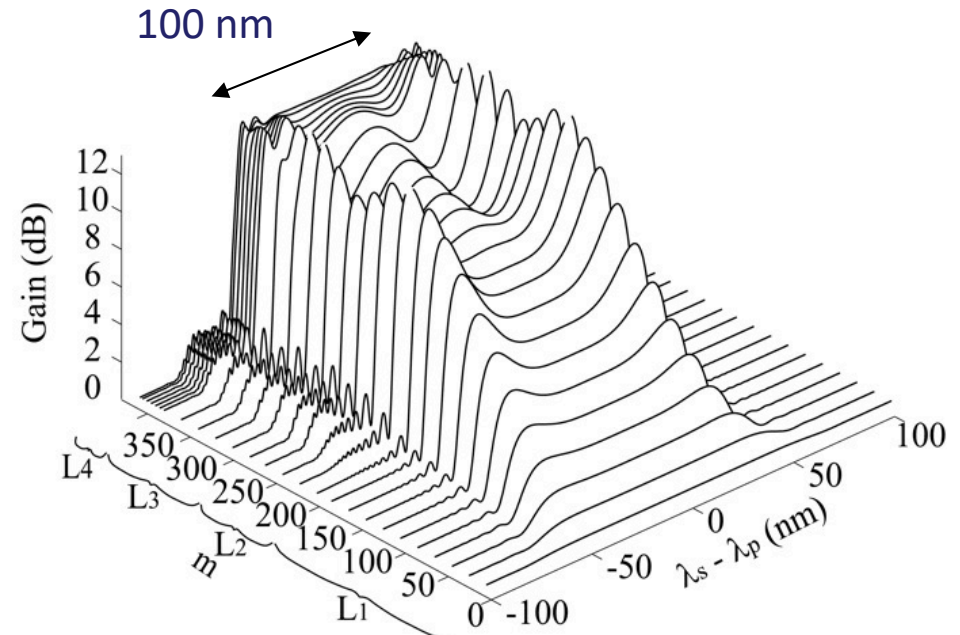
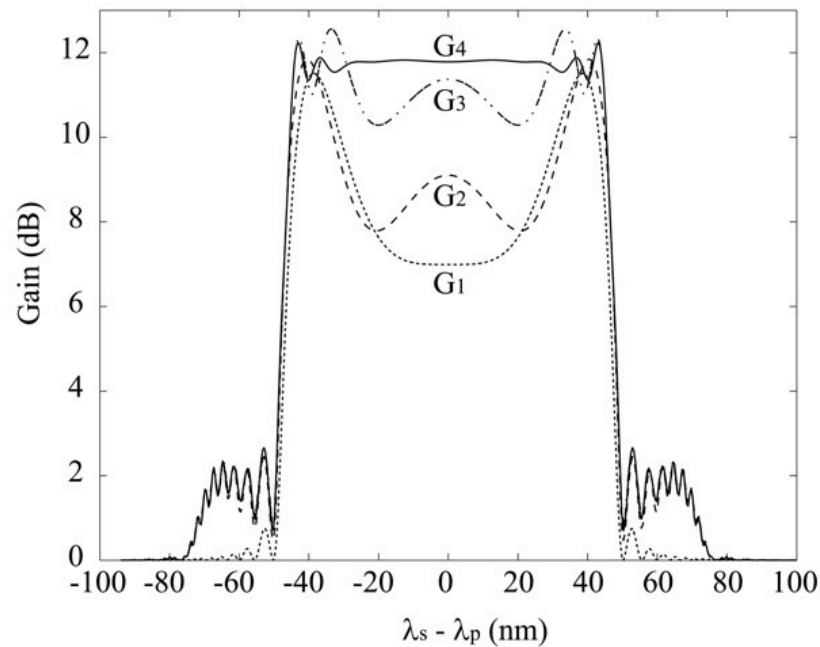
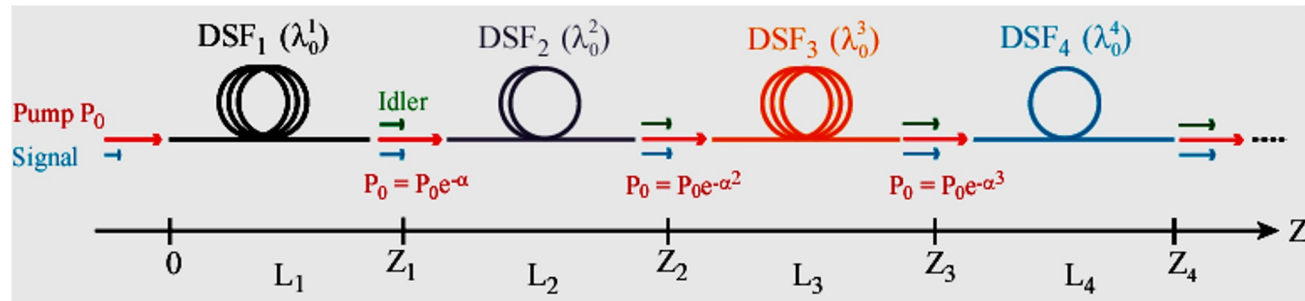
- ⇒ Dispersion and nonlinearity must cancel out
- ⇒ Amplification if β_2 and/or $\beta_4 < 0$
 - if λ_p close to λ_0 (1.55 μm)
- ⇒ Achievement of ultra-wide gain bandwidth



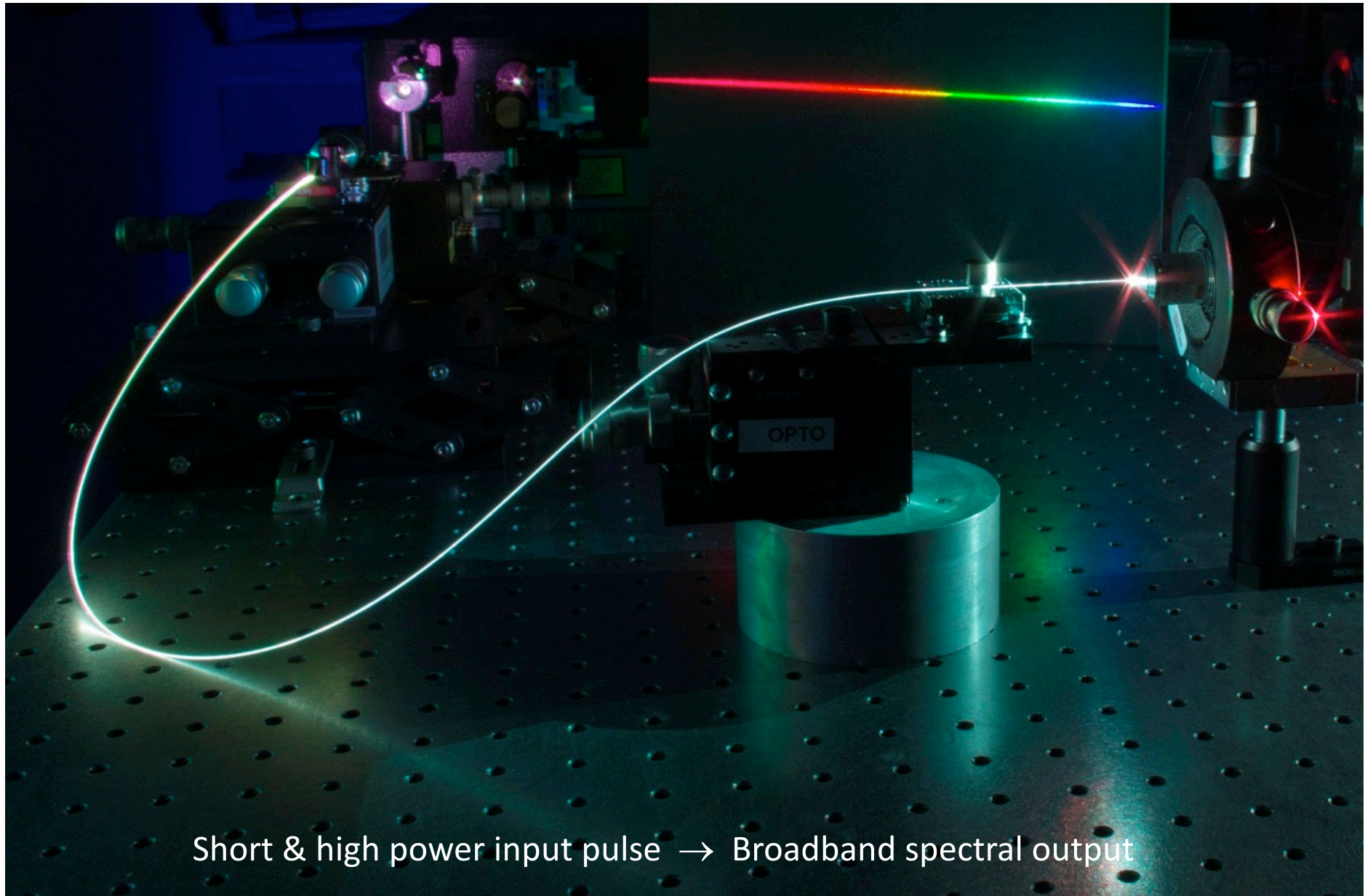
$$G = 1 + \left(\frac{\gamma P}{g} \sinh(gL_{eff}) \right)^2 \quad g^2 = (\gamma P)^2 - (\kappa/2)^2$$

Optical Parametric Amplification (OPA)

- ▶ OPA can provide flat and broad gain bandwidth by dispersion management



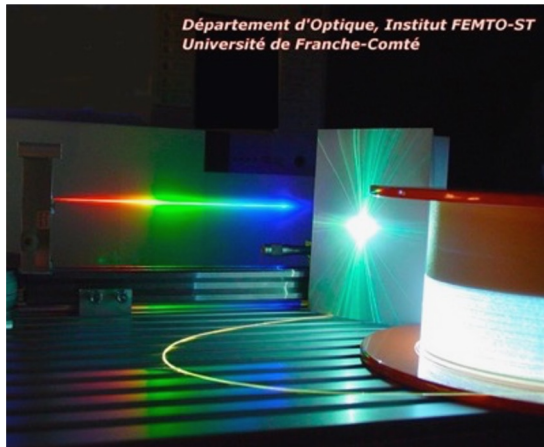
Supercontinuum generation (SC)



Short & high power input pulse → Broadband spectral output

Why Supercontinuum light ?

- **New hybrid broadband light source: The “white-light” laser**
- **Broad as a lamp or as the sun (from UV to infrared)**
- **Bright as a laser > 20000 times the sun**
- **Spatially coherent - single-mode beam output - Fiber delivery**
- **Temporally coherent to some extent (Soliton Number $N < 16$)**
- **Nothing except synchrotron can give wider bandwidth**



SC generation in a silica photonic crystal fiber



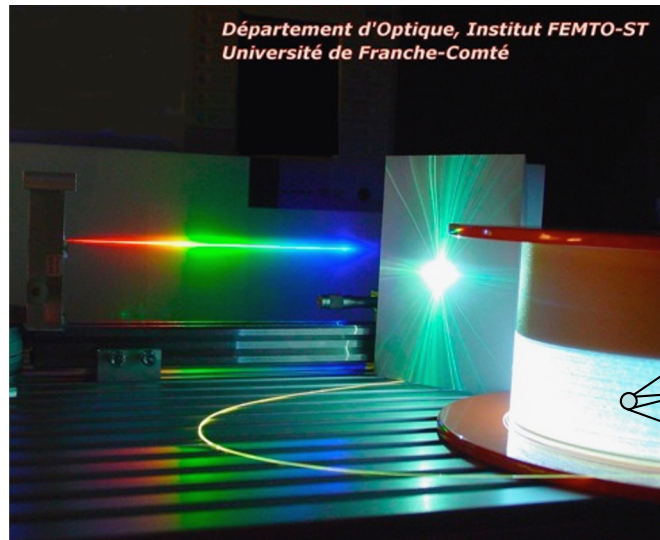
The SuperK SC Laser
NKT photonics
(400 nm-2400 nm)



European synchrotron

Supercontinuum Physics

- Beautiful fundamental physics !
- Exploit fiber nonlinearities to convert laser light to new wavelengths
- Based on nonlinear Kerr and Raman responses
- Interaction between linear dispersion effects and nonlinear effects



Optical Kerr Effect
(electronic response)

$$n = n_0 + n_2 I(z, t)$$

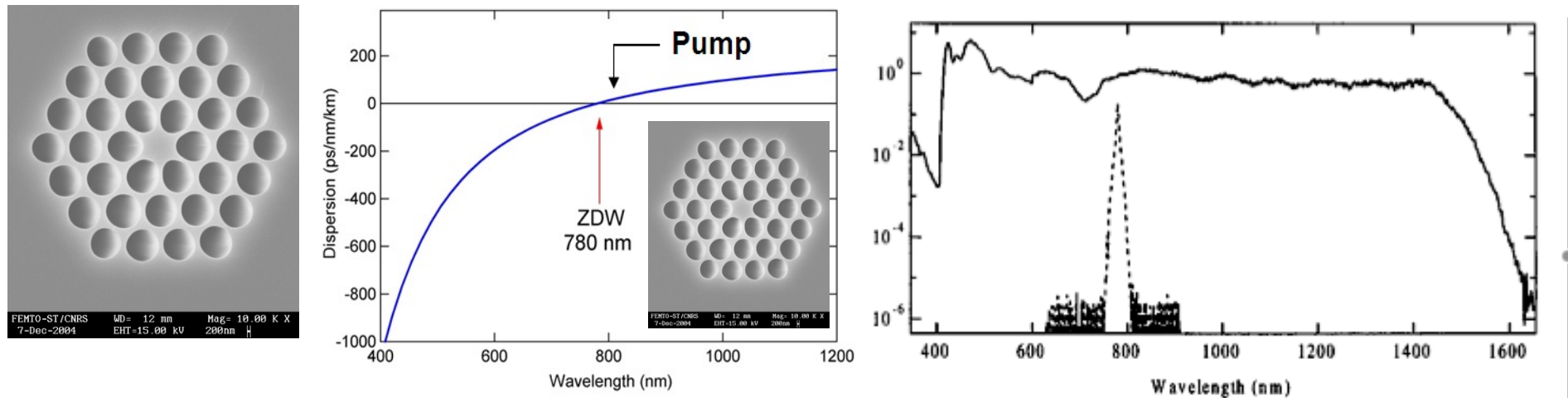
Stimulated Raman Scattering
(Delayed Molecular response)

$$P_{NL}(t) = \underbrace{\epsilon_0 \chi_K^{(3)} : E(t)E(t)E(t)}_{\text{Instantaneous Elastic Kerr effect}} + \underbrace{\epsilon_0 E(t) \int_{-\infty}^t \chi_R^{(3)}(t-t') E(t') E(t') dt'}_{\text{Inelastic Raman light scattering}}$$

Instantaneous Elastic Kerr effect
(No energy exchange with matter)

Inelastic Raman light scattering:
(Molecular vibrational state)

Supercontinuum Physics



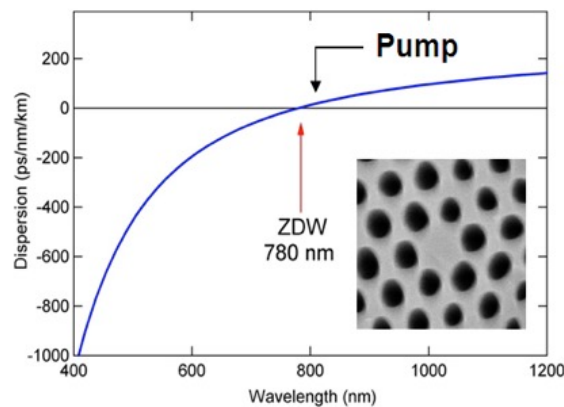
The optical supercontinuum is formed when a high power femtosecond pulse @ 800 nm is sent to a microstructured optical fiber with both small effective mode area and zero-dispersion wavelength close to the pump that cause severe spectral broadening.

Supercontinuum Physics

Modelling with a generalized nonlinear Schrodinger equation (GNLSE)

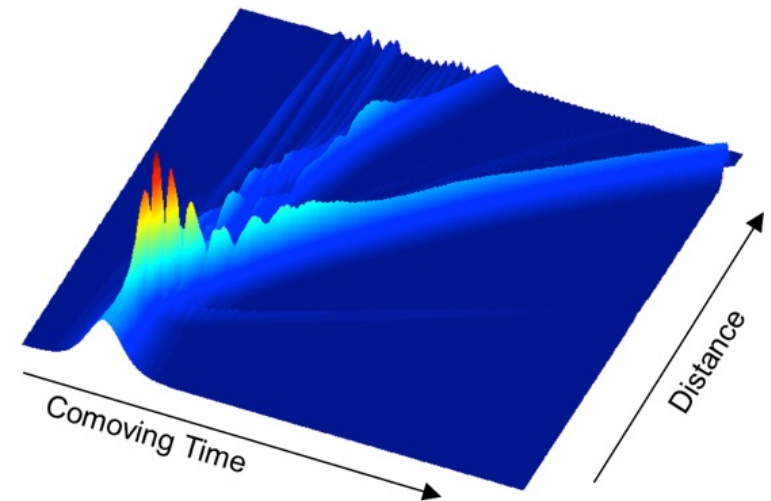
$$\frac{\partial A}{\partial z} = \underbrace{-\frac{\alpha(\omega)}{2}A}_{\text{loss}} + \underbrace{\sum_{k \geq 2} \frac{i^{k+1}}{k!} \beta_k \frac{\partial^k A}{\partial T^k}}_{\text{dispersion}} + \underbrace{i\gamma \left(1 + i\tau_0 \frac{\partial}{\partial T}\right)}_{\text{Self-steepening}} \left(\underbrace{A \int_{-\infty}^{+\infty} R(T') |A(z, T - T')|^2 dT'}_{\text{Nonlinear response (Kerr + Raman)}} \right)$$

Physics: NLSE + perturbations



Three main processes

- Soliton ejection & fission
- Raman – shift to long λ
- Radiation – shift to short λ



K. L. Corwin et al., *Phys. Rev. Lett* **90** (2003) J. M. Dudley et al., *Rev. Mod. Phys.* **78** (2006)

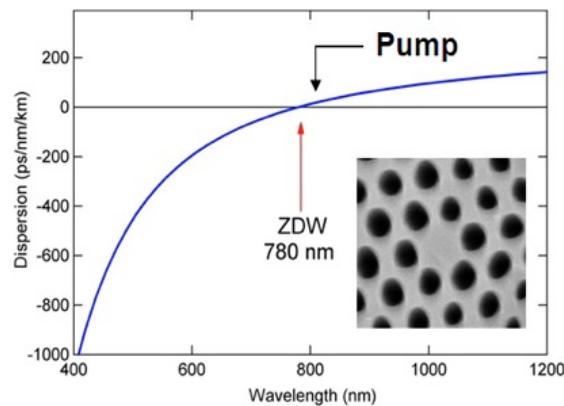
J. M. Dudley & J. Roy Taylor *Nat. Photon* **3** (2009)

Supercontinuum Physics

Modelling with a generalized nonlinear Schrodinger equation (GNLSE)

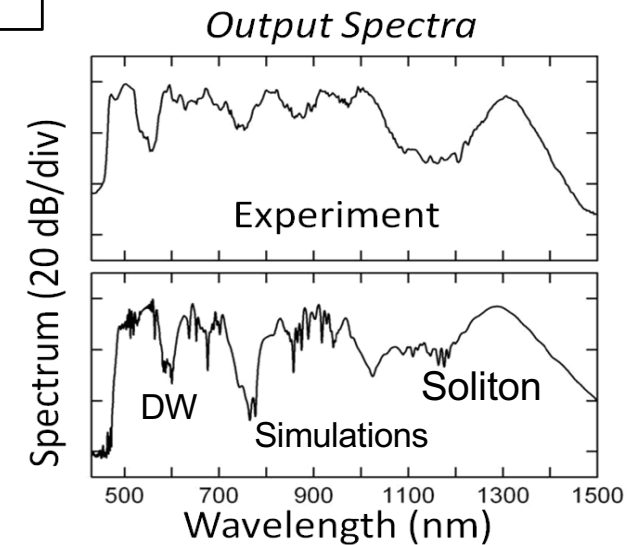
$$\frac{\partial A}{\partial z} = \underbrace{-\frac{\alpha(\omega)}{2}A}_{\text{loss}} + \underbrace{\sum_{k \geq 2} \frac{i^{k+1}}{k!} \beta_k \frac{\partial^k A}{\partial T^k}}_{\text{dispersion}} + \underbrace{i\gamma \left(1 + i\tau_0 \frac{\partial}{\partial T}\right)}_{\text{Self-steepening}} \left(\underbrace{A \int_{-\infty}^{+\infty} R(T') |A(z, T - T')|^2 dT'}_{\text{Nonlinear response (Kerr + Raman)}} \right)$$

Physics: NLSE + perturbations



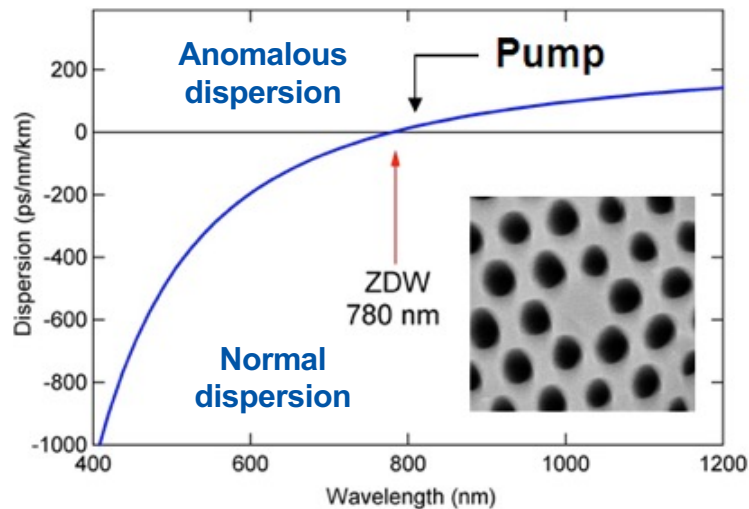
Three main processes

- Soliton ejection & fission
- Raman – shift to long λ
- Radiation – shift to short λ

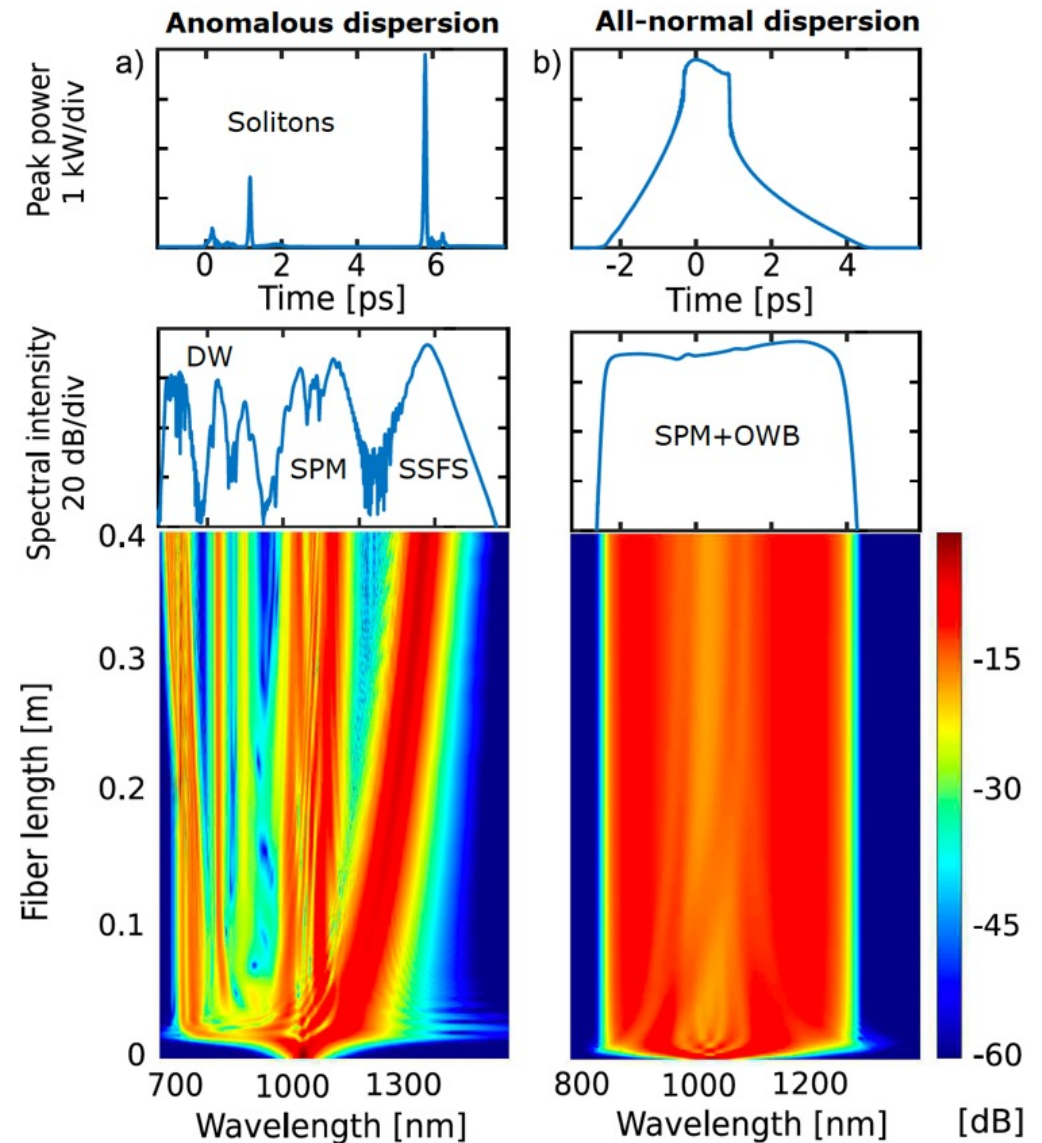


Supercontinuum Physics : femtosecond regime

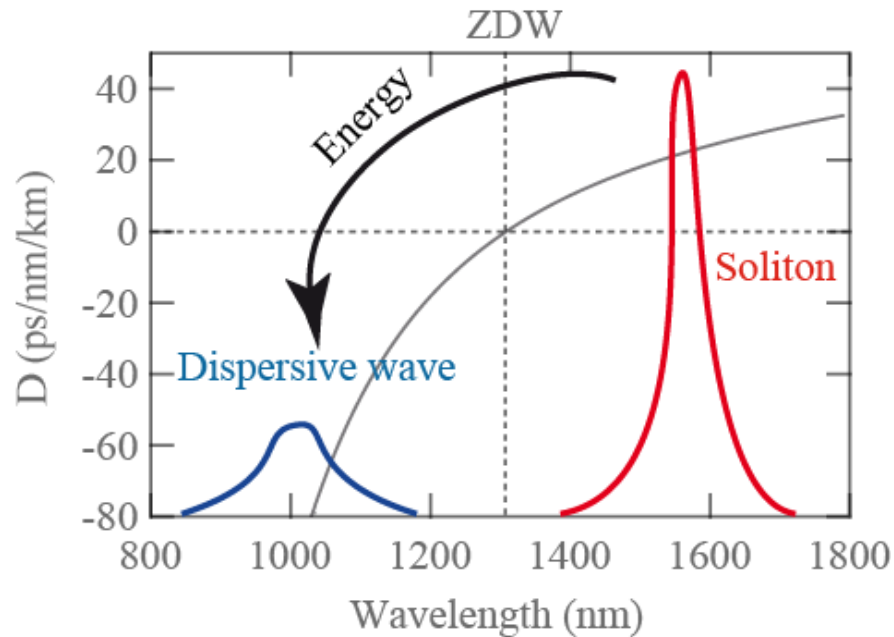
- **Anomalous dispersion ($D > 0, \beta_2 < 0$):**
 - Soliton Formation & Fission,
 - Raman shift (SSFS) to long λ
 - Dispersive-wave (DW) to short λ
- **Normal dispersion ($D < 0, \beta_2 > 0$):**
 - Self-phase modulation (SPM)
 - Optical-wave breaking (OWB)



J. M. Dudley et al., Rev. Mod. Phys. 78 (2006)



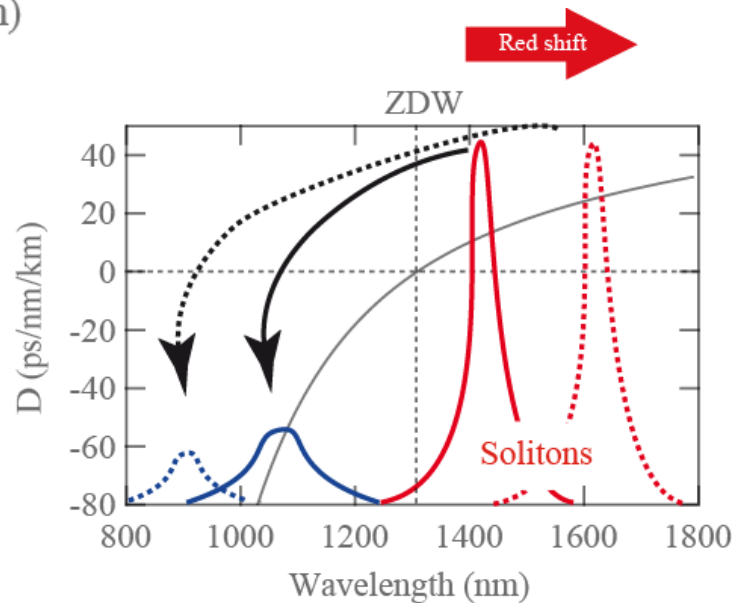
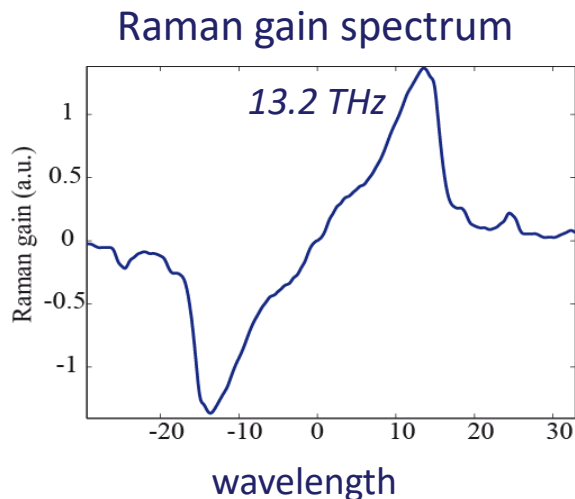
Dispersive Wave (resonant radiation)



Phase-matching condition between solitons and dispersive waves yields:

$$\beta(\omega_{DW}) - \beta(\omega_S) - \beta(\omega_{DW} - \omega_S) - \gamma P_S / 2 = 0$$

$$\Delta\omega_{Disp.Wave} \approx -\frac{3\beta_2(\omega_{sol.})}{\beta_3(\omega_{sol.})} + \frac{\gamma P_{Maxsol}\beta_3(\omega_{sol.})}{3\beta_2^2(\omega_{sol.})}$$



SC red side: soliton self-frequency shift (SSFS)

SC blue side: Dispersive waves trapped by solitons

Dispersive Wave (resonant radiation)

Dispersive wave distance emitted from solitons due to β_3 or β_4

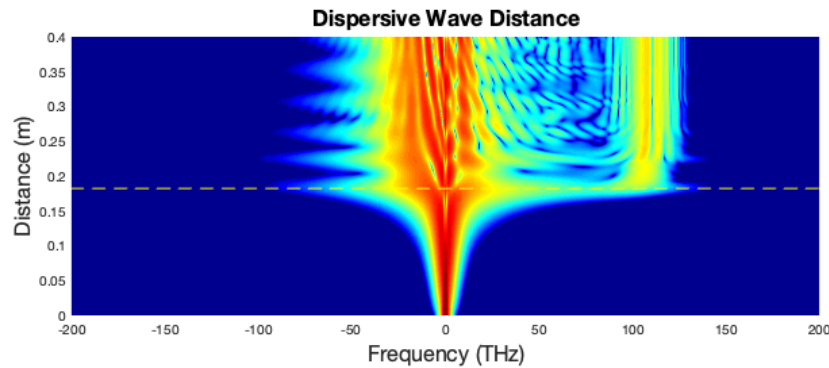
$$\beta_2 = -8e-27 \text{ s}^2/\text{m}; \beta_3=4e-41 \text{ s}^3/\text{m}; \beta_4=0$$

$$P_0=1 \text{ kW}$$

$$T_0=200 \text{ fs}$$

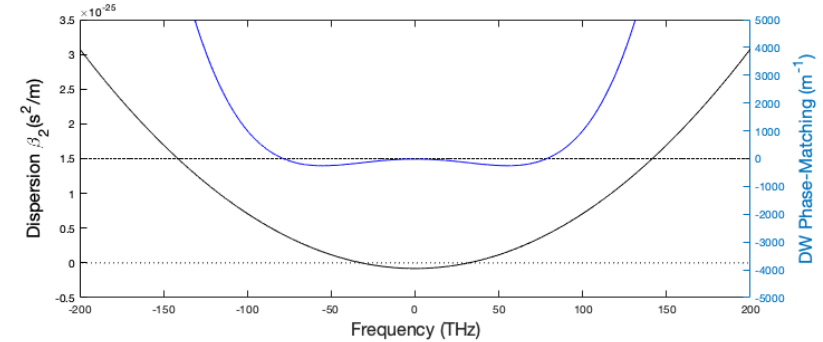
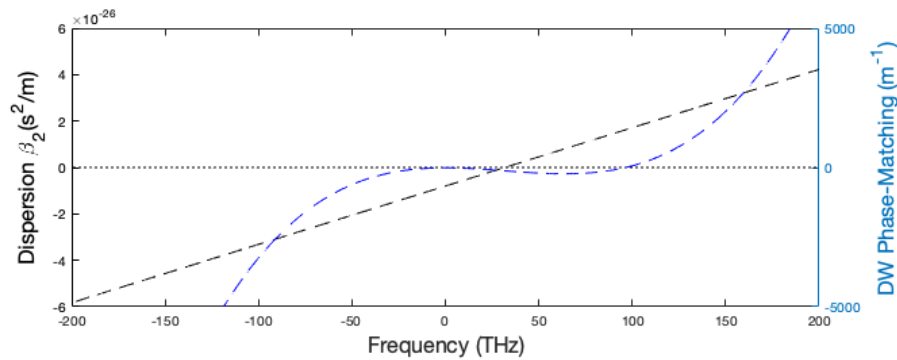
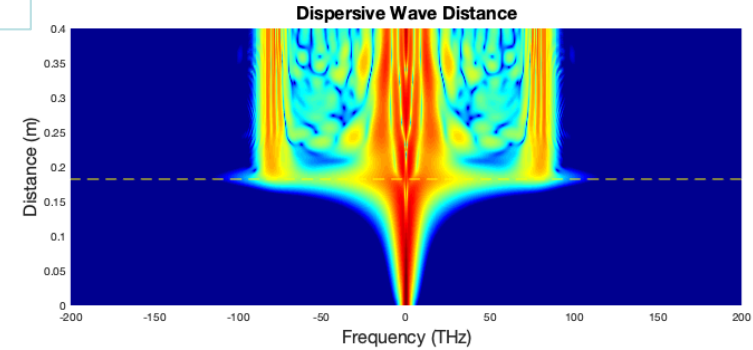
$$\gamma=27 \text{ W}^{-1}\text{km}^{-1}$$

$$\beta_2 = -8e-27 \text{ s}^2/\text{m}; \beta_3= 0; \beta_4=4e-41 \text{ s}^4/\text{m}$$



$$L_{WB} = \sqrt{\frac{3/2}{N^2 + 1}}$$

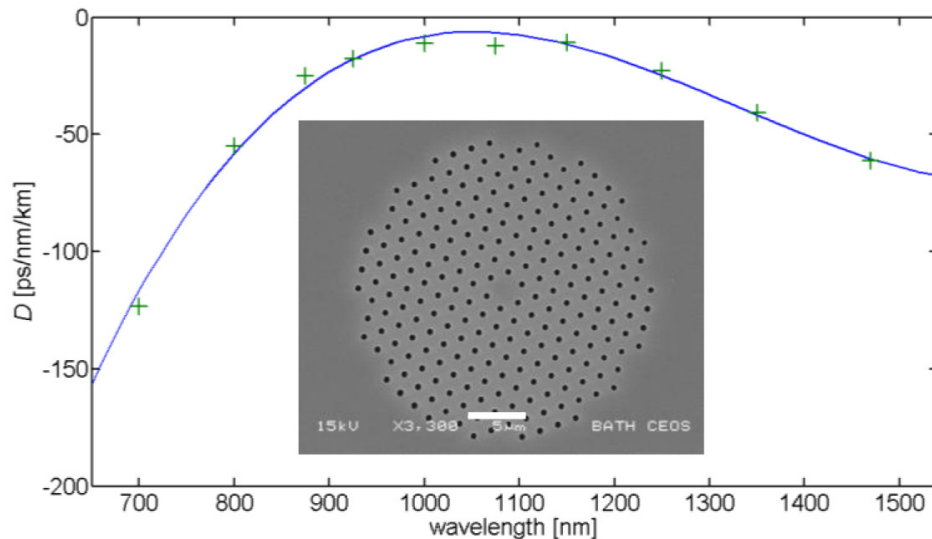
$$N = \sqrt{\frac{\gamma P_0 T_0^2}{|\beta_2|}}$$



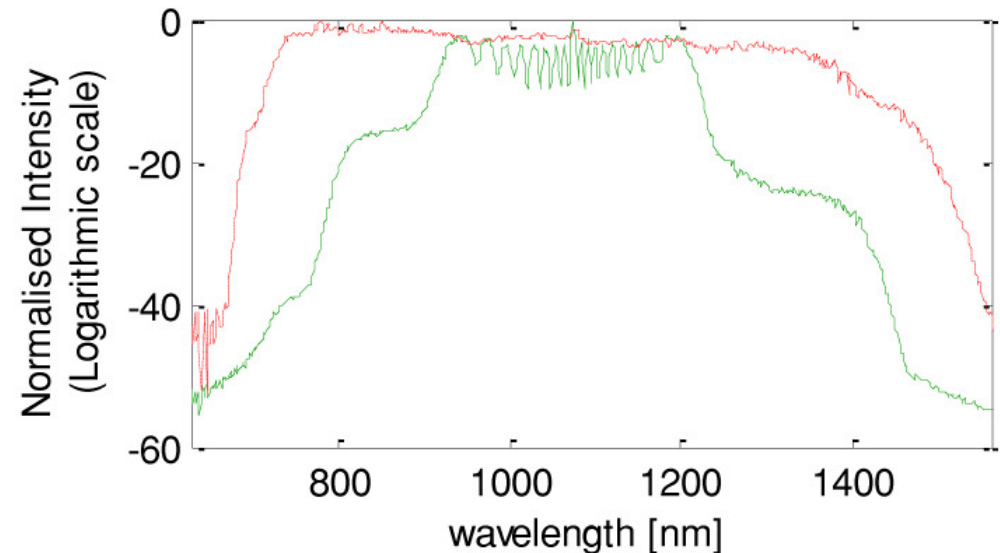
Coherent Supercontinuum generation

- Pumping in the low normal dispersion regime
- Self-phase modulation (SPM) + Optical-Wave Breaking (OWB)
- OWB = Dispersive Shock Wave (DSW)
- Smooth and flat SC spectrum – Pulse-preserved
- Gives low noise and high coherence level

PCF with all-normal group velocity dispersion



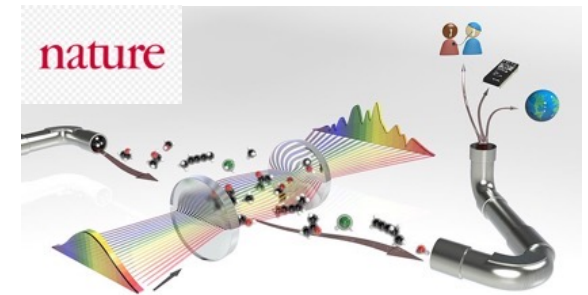
Wideband supercontinuum generation



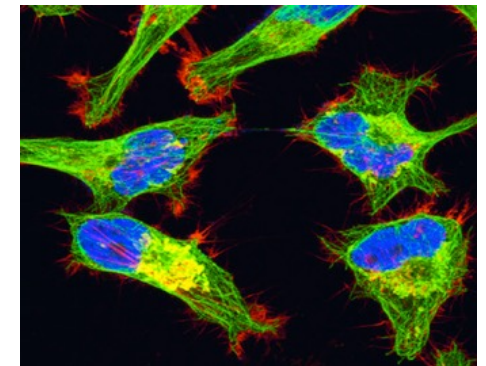
L. E. Hooper, P. J. Mosley, A. C. Muir, W. J. Wadsworth, J. C. Knight, *Opt. Express* **19**, 4902 (2011).

Supercontinuum Applications

- Biomedical imaging and microscopy,
- OCT, Confocal, Fluorescence, CARS imaging
- Cell-Tissue-Material analysis, Air pollution
- Molecular spectroscopy, Material processing
- Remote non-destructive detection
- Optical Frequency Comb (OFC) Metrology
- LIDAR, flow cytometry, and many others...



Molecular spectroscopy



Fluorescence imaging



Confocal microscopy



OCT image of a finger tip

TheScientist
TOP
OF INNOVATIONS
2008

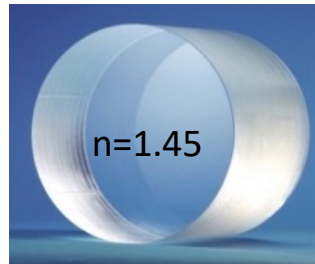
SUPUVIR: SC Fiber materials

Silica glasses

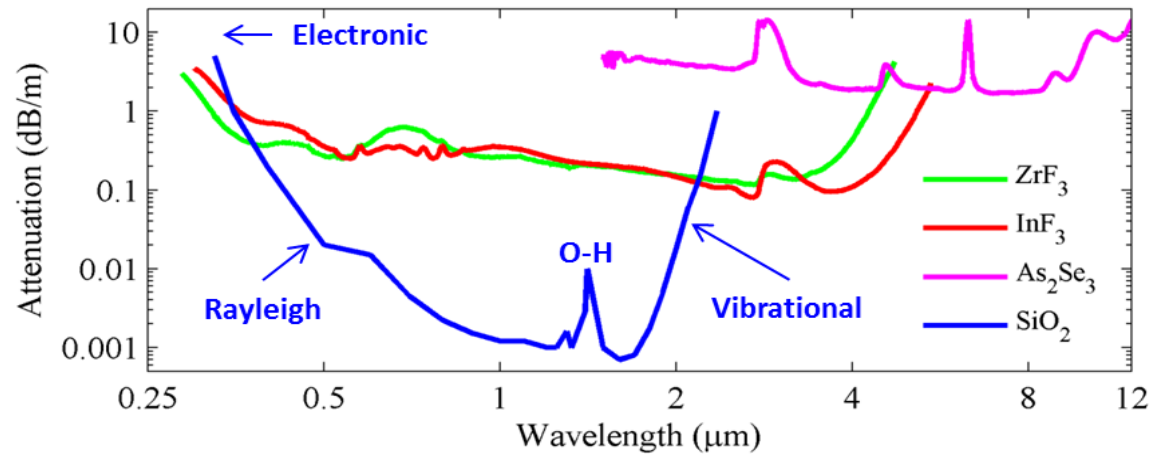
Fluoride glasses


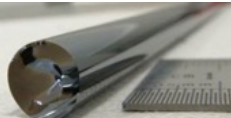
Chalcogenide glasses

UV-grade glass
High-OH dopant
Gas-filled fiber



Heavy-metal-oxide (HMO) glasses
Tellurite glasses (TZN)
Telluride glasses (Te)



	Glass	n	$n_2 (10^{-20} \text{m}^2 \text{W}^{-1})$	T_g (°C)	ZDW (μm)
	Silica	1.45	2.7	~1200	1.26
	ZBLAN	1.48-1.53	2.1-2.55	230-300	1.62-1.71
	As ₄₀ Se ₆₀	2.82-2.9	1400-3000	178-185	7.2-7.4

SUPUVIR: SC Fiber materials

Silica glasses

Fluoride glasses

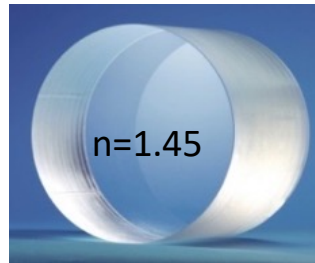
Chalcogenide glasses

Heavy-metal-oxide (HMO) glasses

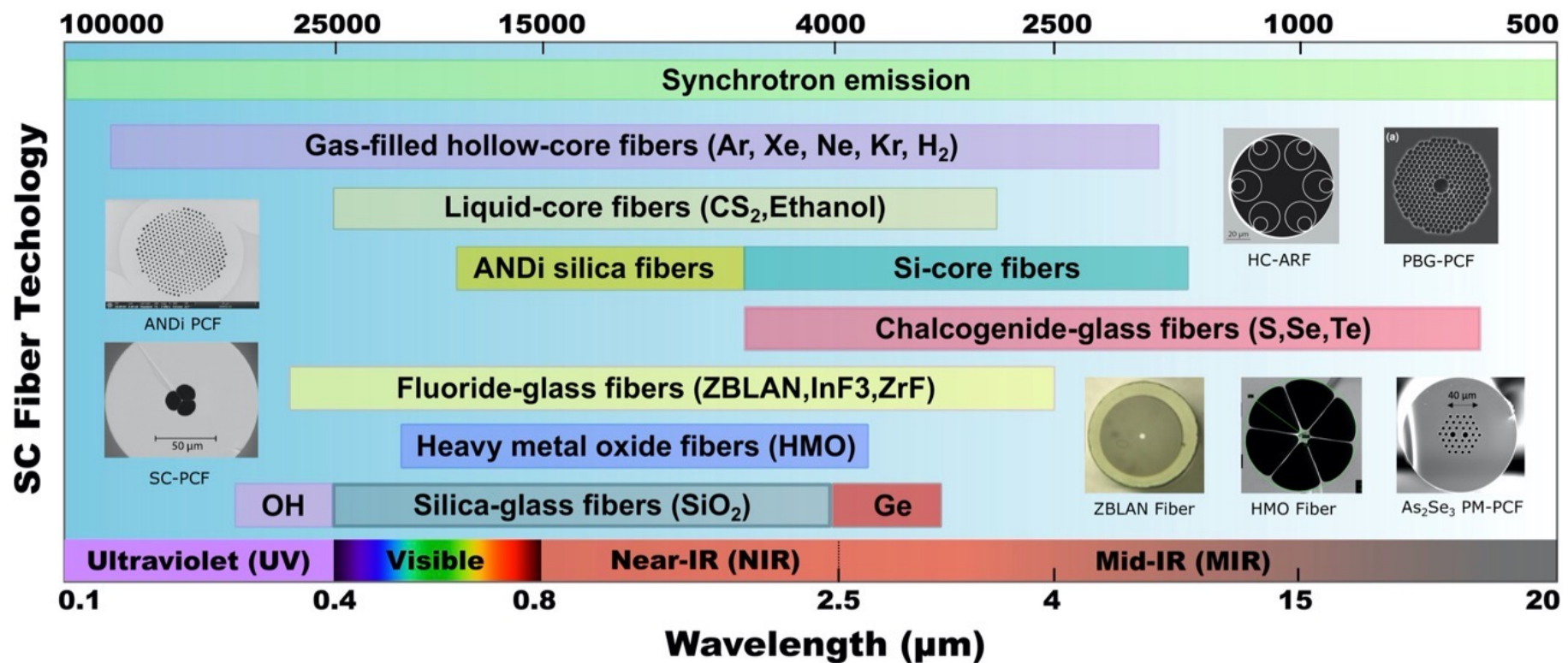
Tellurite glasses (TZN)

Telluride glasses (Te)

UV-grade glass
High-OH dopant
Gas-filled fiber

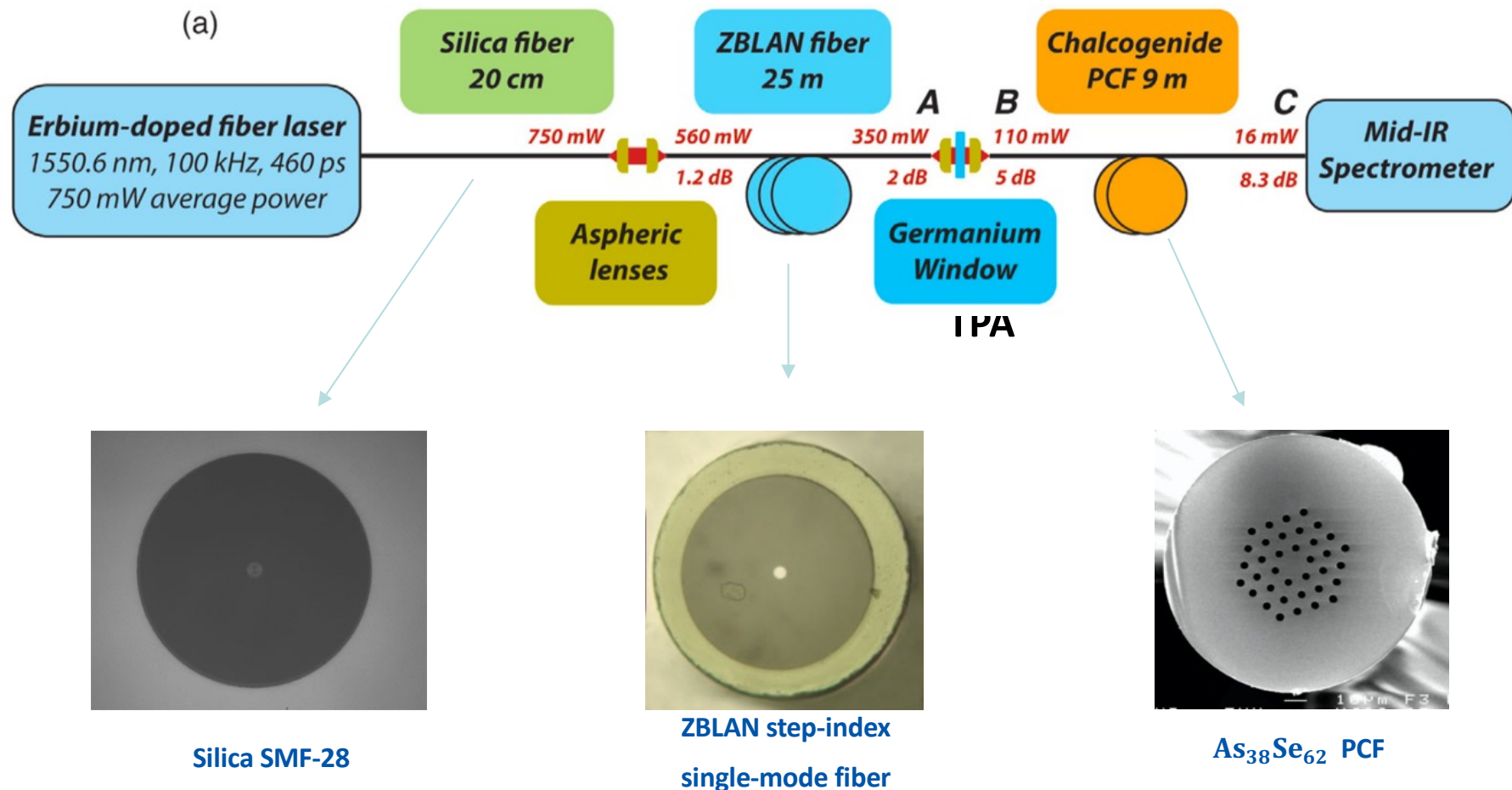


Wavenumber (cm^{-1})



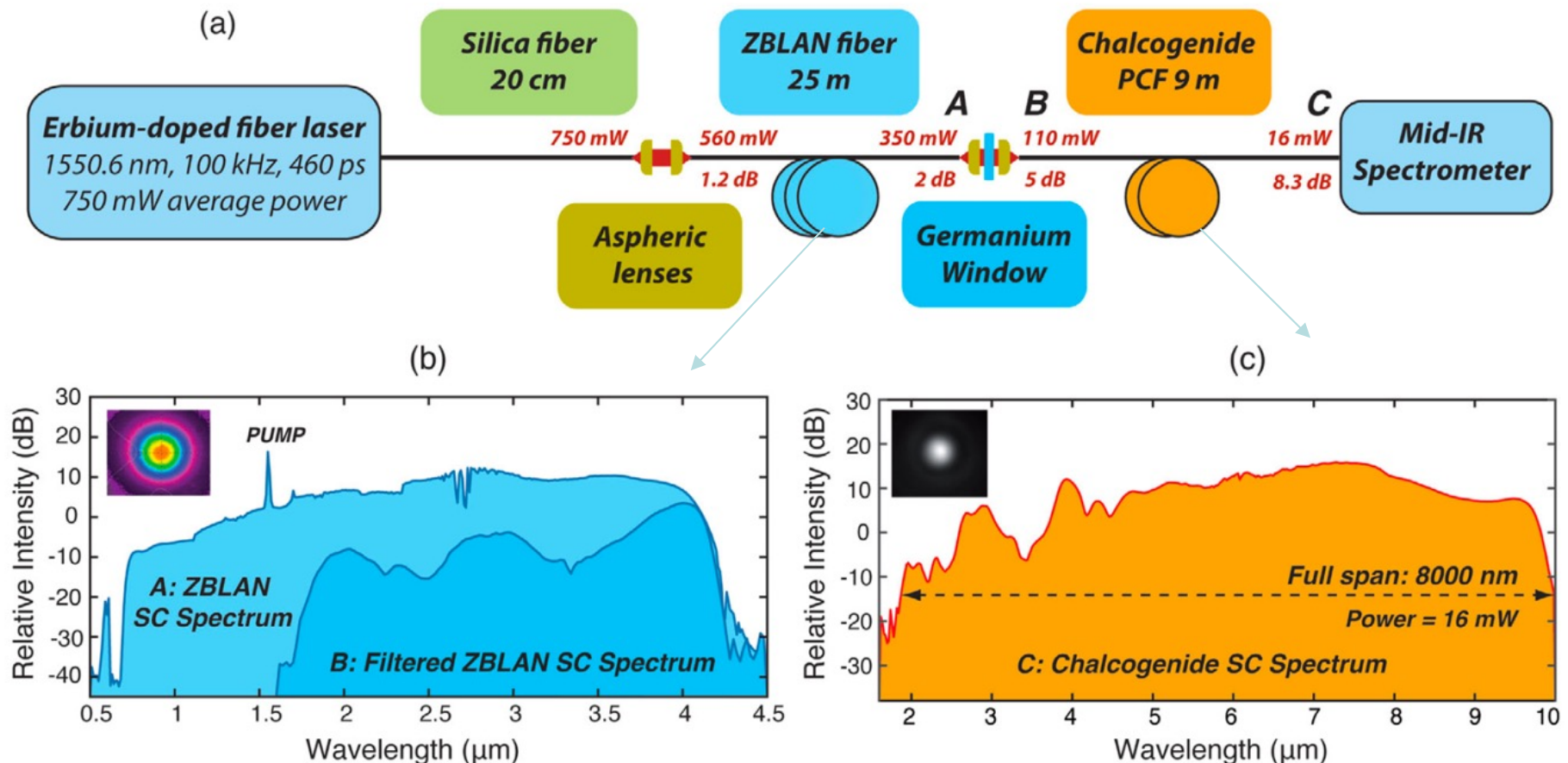
Cascaded Mid-IR SC Source

- **Development of compact cascaded fiber mid-IR SC light sources**
- Combine silica - fluoride - chalcogenide fibers
- Directly pumped by a table-top fiber laser at $1.55 \mu\text{m}$



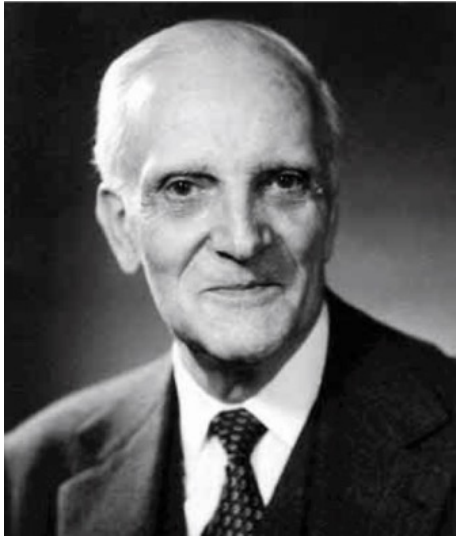
Cascaded Mid-IR SC Source

- Development of compact cascaded fiber mid-IR SC light sources
- Step-wise SC extension towards the MIR by Raman soliton drifting
- 2-10 μm wide SC spectrum with 16 mW output power



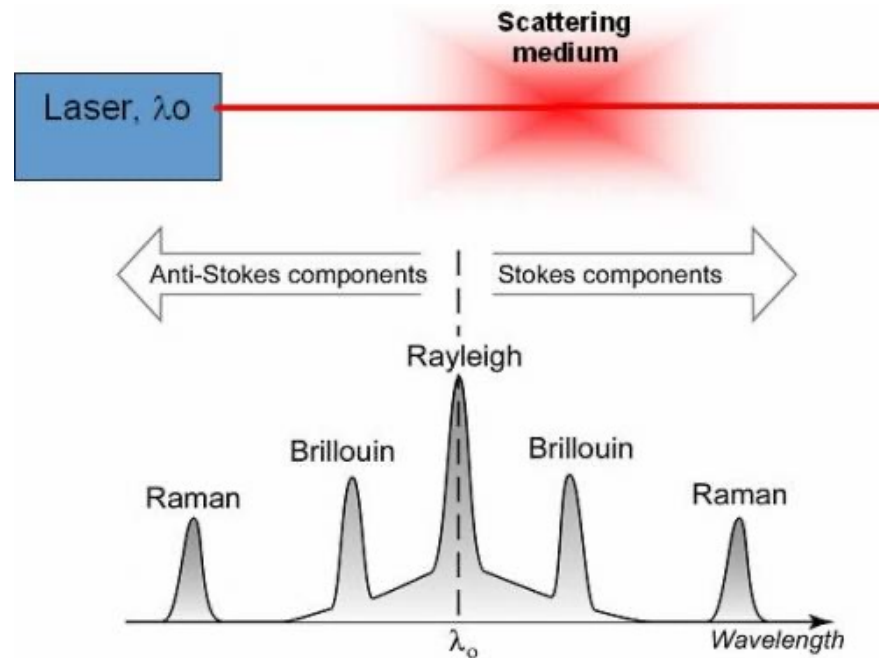
S. Venck et al, Laser and Photonics Rev. 14 (6), 2000011 (2020)

Brillouin scattering in optical fibers



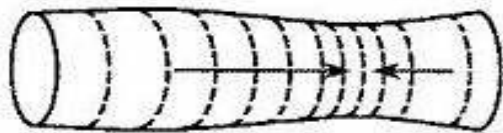
Léon Brillouin (1889-1969)

- Inelastic scattering by hypersound acoustic waves
- Brillouin scattering is a photon-phonon interaction



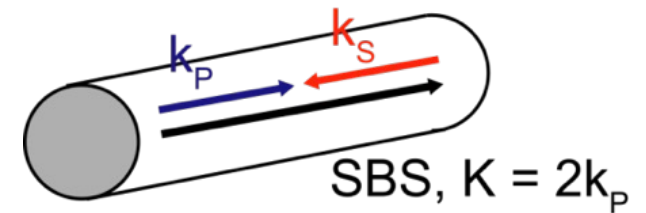
Brillouin scattering in optical fibers

Longitudinal wave



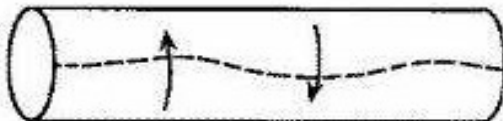
Stimulated Brillouin Scattering (SBS)
Velocity=5960 m/s
Wavelength $\sim 0.5 \mu\text{m}$
Frequency = 11 GHz

E. Ippen. & R. Stolen *Appl. Phys. Lett.* **21** (1972)



Backward scattering

Torsional wave

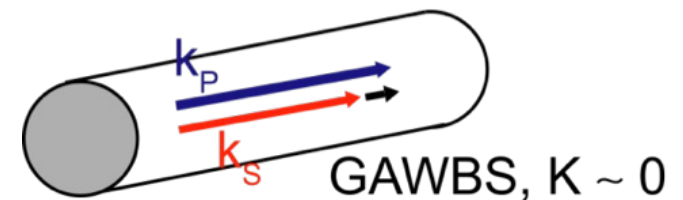


Radial wave



Guided acoustic wave Brillouin Scattering (GAWBS)
Shear velocity=3600 m/s
Wavelength $\sim 0.5 \mu\text{m}$
Frequency [100 MHz-1 GHz]

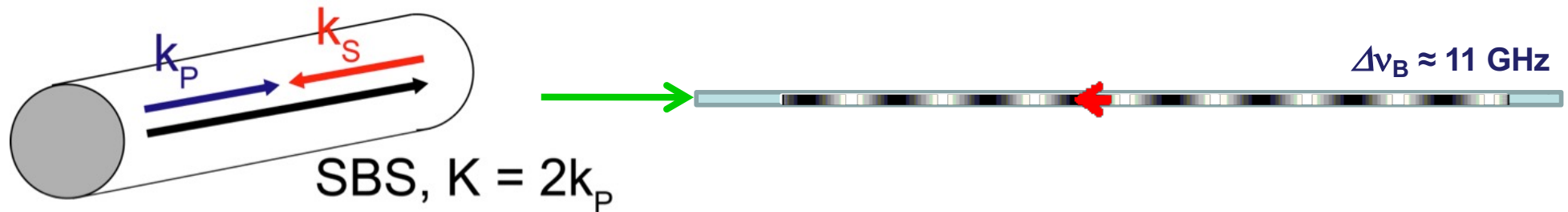
R. M. Shelby et al. *Phys. Rev. B* **31**, 5244 (1985)



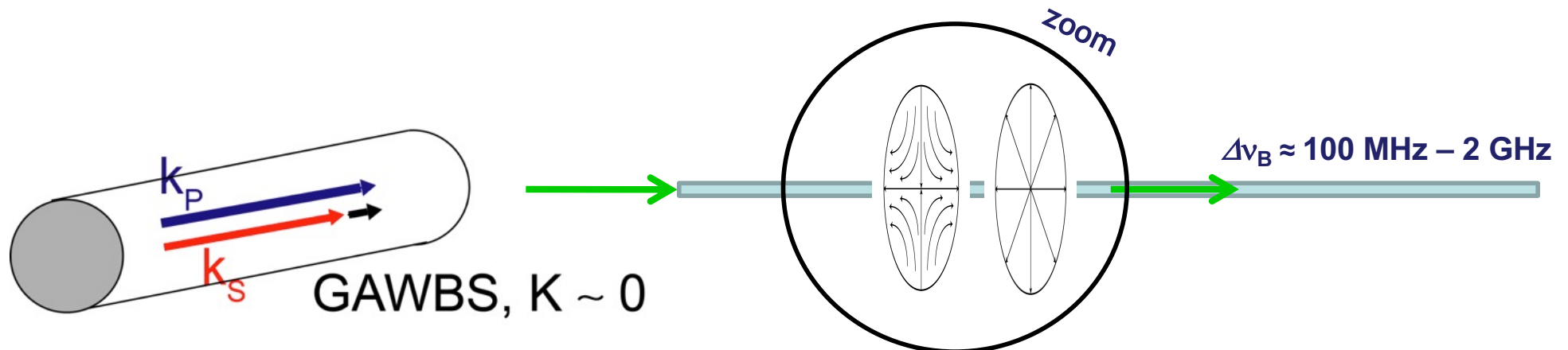
Forward scattering

Brillouin scattering in optical fibers

Backward Stimulated Brillouin scattering (SBS): elastic longitudinal wave (acoustic phonon)



Forward Brillouin scattering: Transverse acoustic wave (Guided Acoustic Wave Brillouin Scattering)

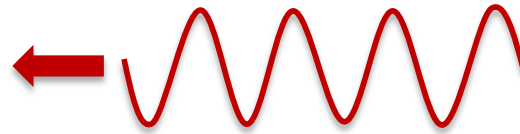


Brillouin scattering in optical fibers

(1) Incident pump beam



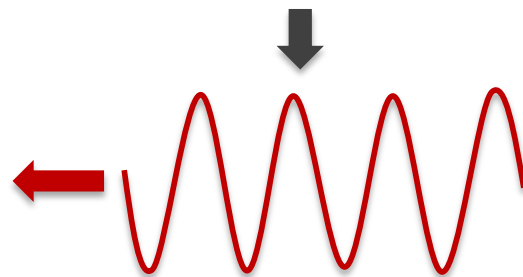
(2) Spontaneous scattering of light by acoustic phonons



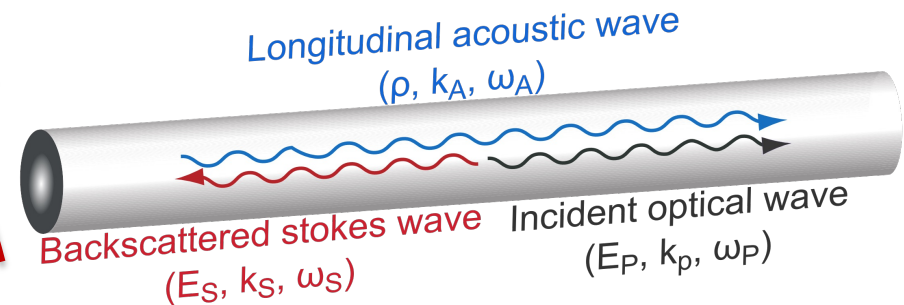
$|(1) + (2)|^2 \rightarrow$ (3) Interference



(3) \rightarrow (4) Acoustic wave created by electrostriction



Exaltation of light scattering: stimulated scattering



Phase matching condition

$$k_B = k_P - k_S$$

Conservation of energy

$$\omega_B = \omega_P - \omega_S$$

Brillouin gain

$$g_B(\omega) = \frac{g_{B0}(\Delta\omega_B/2)^2}{(\omega - \omega_B)^2 + (\Delta\omega_B/2)^2}$$

Brillouin frequency shift (Doppler effect)

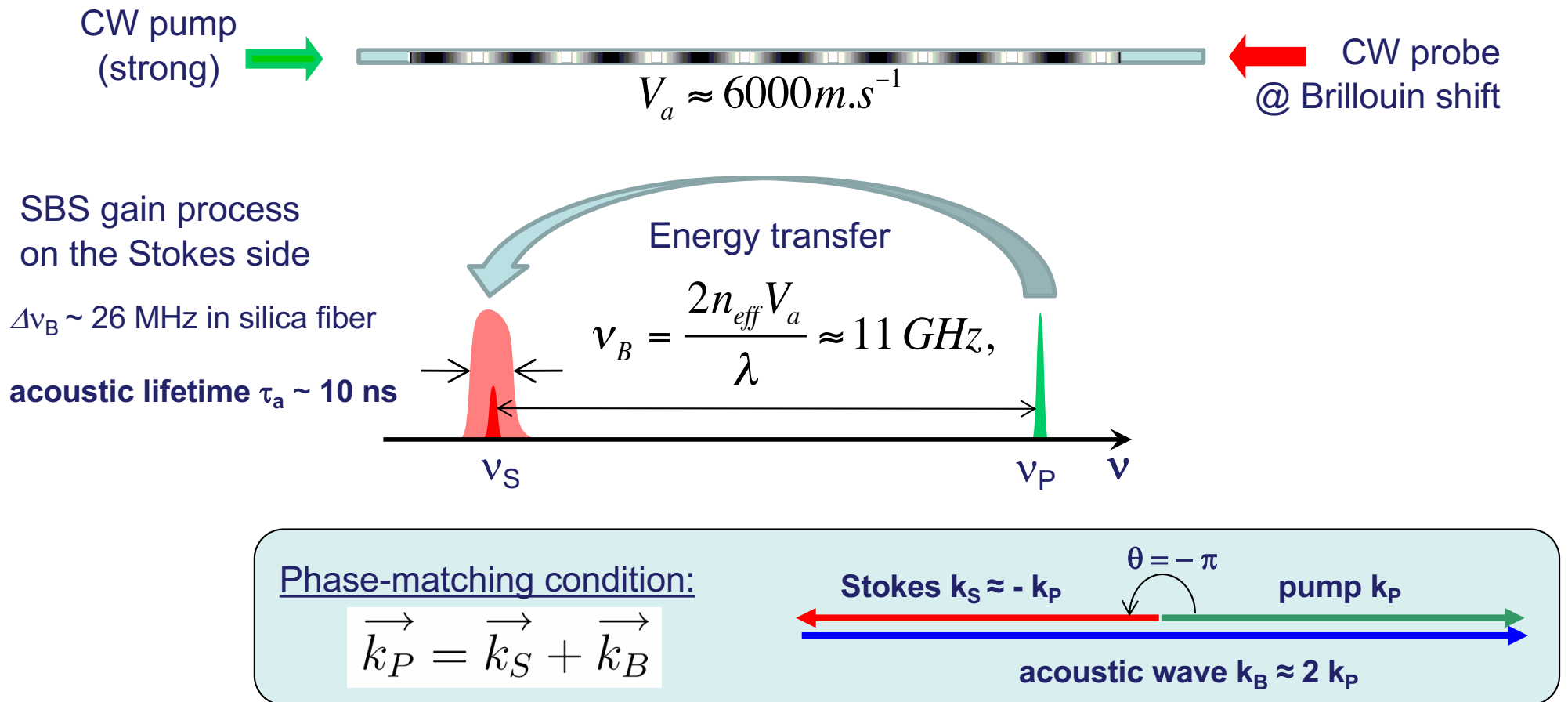
$$\nu_B = \frac{2v_A n_{eff}}{\lambda}$$

Silica $v_A = 5960 \text{ m.s}^{-1}$ $\nu_B \approx 11 \text{ GHz}$ $n_{eff} \approx 1,44$

Brillouin scattering in optical fibers

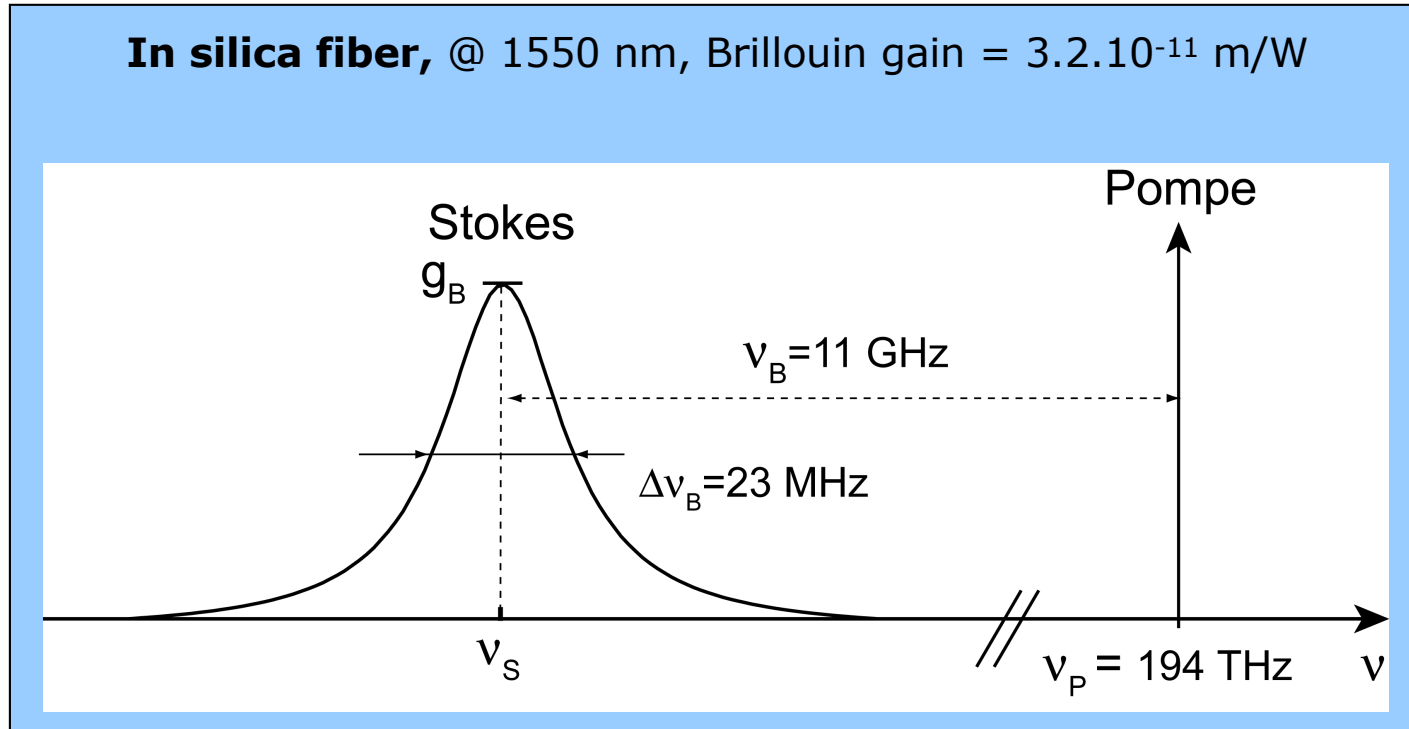
Pump and Probe at slightly different frequencies ν_P and ν_S are interacting through acoustic wave at Brillouin frequency ν_B . ($\nu_P = \nu_S + \nu_B$ energy conservation)

Moving index Bragg grating generated along the fiber core by electrostriction



* Starts from thermal agitation, i.e. spontaneous scattering, when no probe

Brillouin scattering in optical fibers



➤ Brillouin frequency

$$\nu_B = \frac{2 n v_a}{\lambda_P}$$

➤ Brillouin spectrum

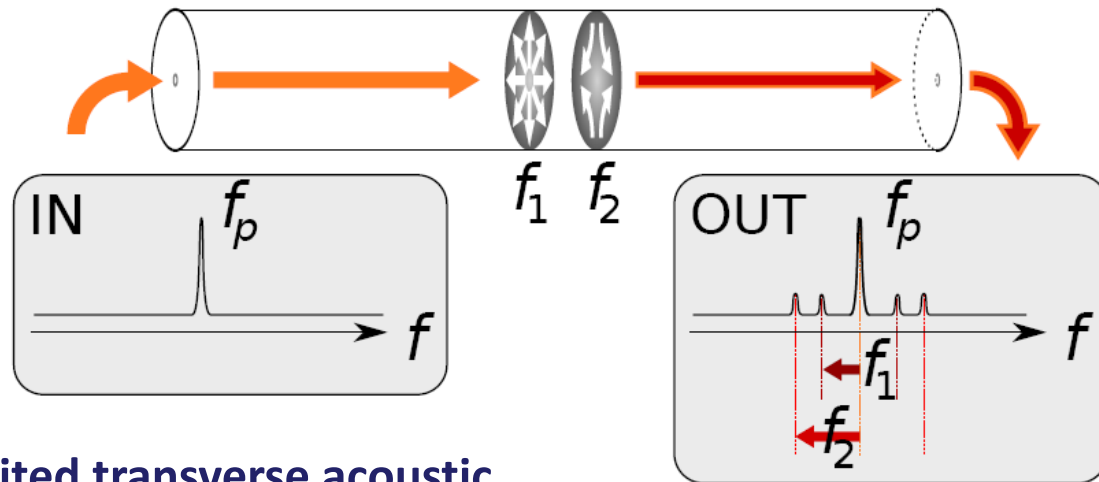
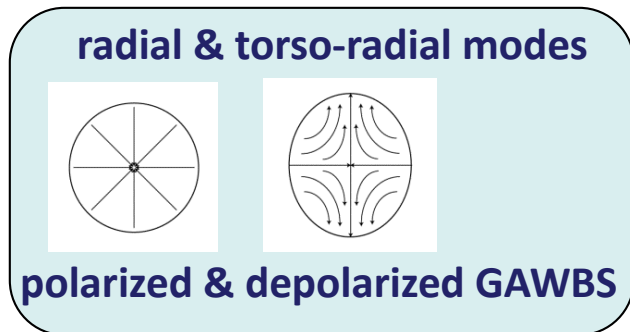
$$g_B(\nu) = \frac{(\Delta\nu_B/2)^2}{(\nu - \nu_B)^2 + (\Delta\nu_B/2)^2} g_B$$

➤ Brillouin gain at the resonance

$$g_B = \frac{2 \pi n^7 p_{12}^2}{c \lambda_P^2 \rho_0 v_A \Delta\nu_B}$$

Forward Brillouin scattering in optical fibers

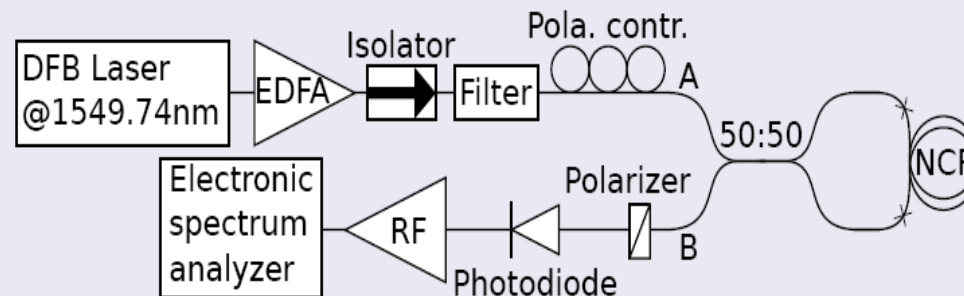
Forward scattering by transversely trapped acoustic resonances



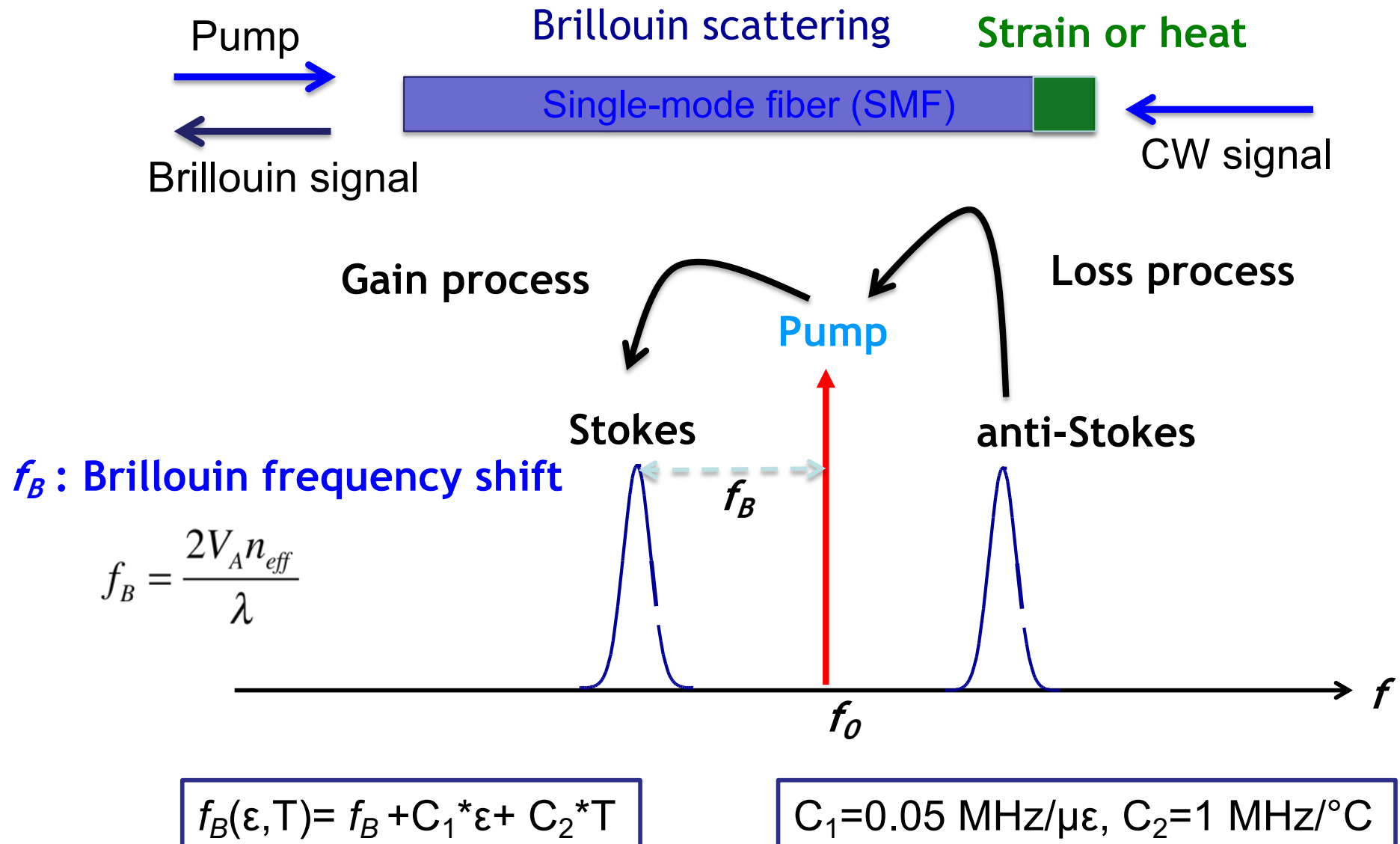
- Originates from thermally excited transverse acoustic phonon of the fiber (the whole cylinder)
- Acoustic modes alter the refractive index periodically
→ phase or polarization modulation of the light wave

$f_{1,2} \sim 100 \text{ MHz} - 2 \text{ GHz}$
(backscattering : $\sim 11 \text{ GHz}$)

Setup



Stimulated Brillouin scattering (SBS)



Brillouin-based applications

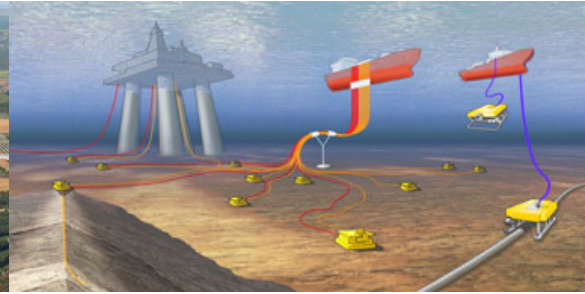
Applications to civil engineering and petroleum industry



Pipeline (~ 100 km)



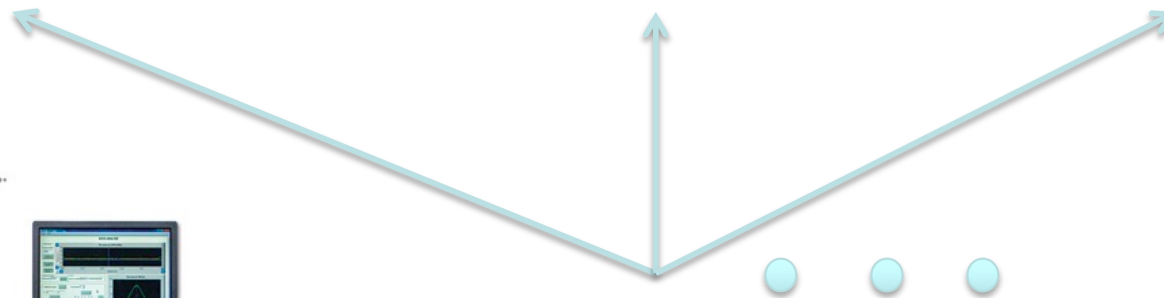
Millau Bridge (2460 m)



Petroleum platform



Aeolian



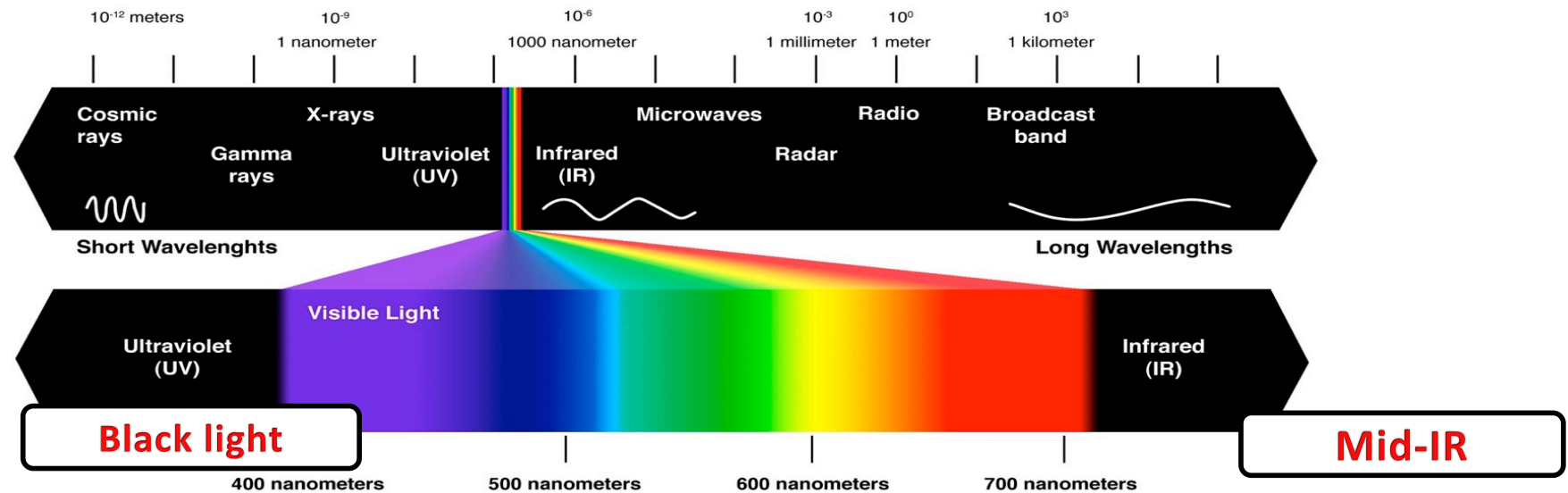
Brillouin optical time domain analysis sensors
(Current Performances : Range =30 km; Resolution=1m)

Conclusions and Outlooks

- ✓ Nonlinear optical effects are remarkable physical phenomena that offer unique and exciting potential applications, such as devices in which light can be controlled by light.
- ✓ Due to their strong light confinement capabilities, optical fibers have been early recognized as an ideal medium to exploit the nonlinear effects for all-optical processing applications in telecommunication and fiber laser industries, supercontinuum sources, etc...
- ✓ This research field has recently been stimulated by the development of a new generation of highly nonlinear tiny optical waveguides, e.g., photonic crystal fibers, micro and nanowires, photonic chip, microresonators.

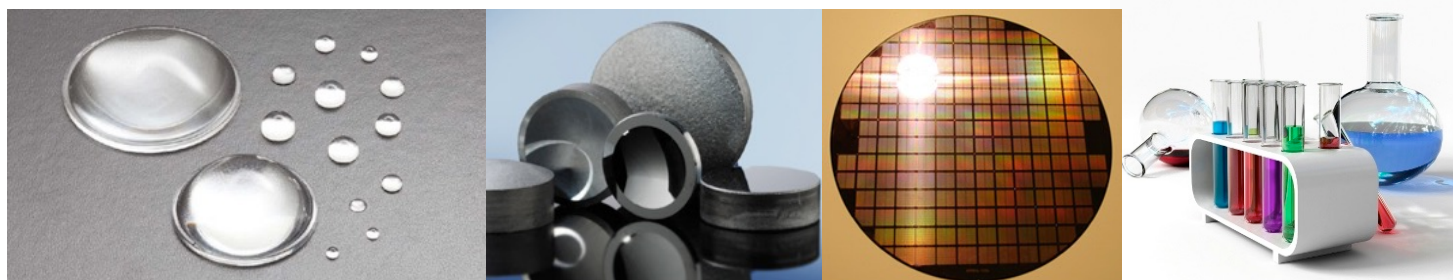
What next ?

Extending applications to UV & Mid-IR wavelength ranges



Exploiting new nonlinear materials

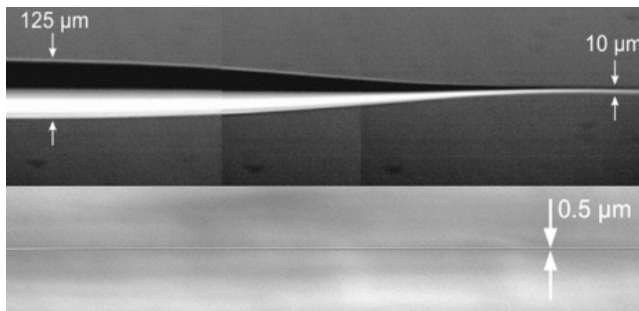
UV Silica, Chalcogenide, Tellurite, Silicium, Liquids, Gas, Crystal fibres (LiBNO_3)



What next ?

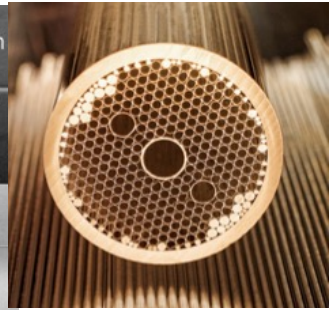
Reducing the dimensions (sub- λ) and the power levels

Nanowires, Tapers

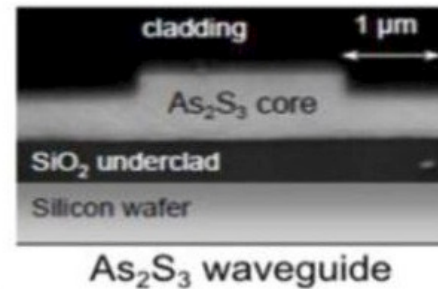


L Tong *et al.* *Nature* 426, 816 (2003)

Nanostructured fibers

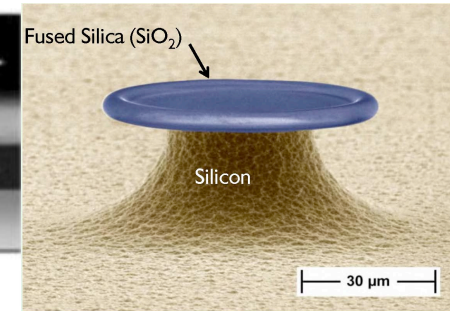


Chip waveguides

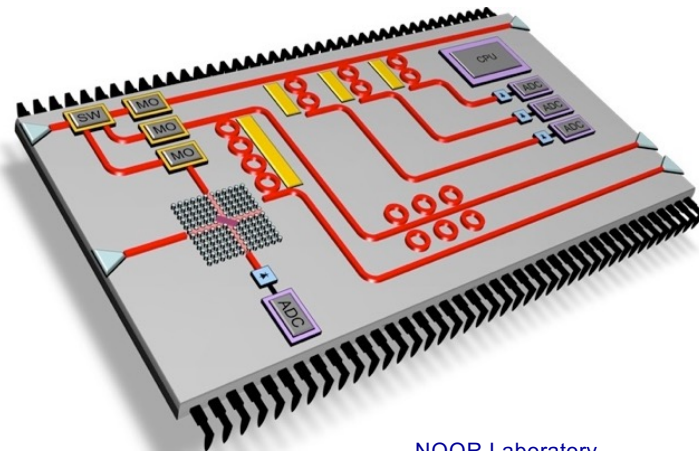


B. Eggleton., *Nat. Phot.* 5, 141 (2011)

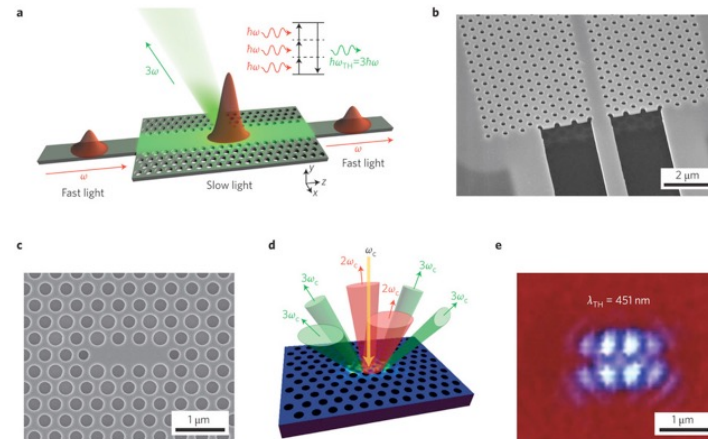
Microresonators



From systems to integration: The photonic circuit



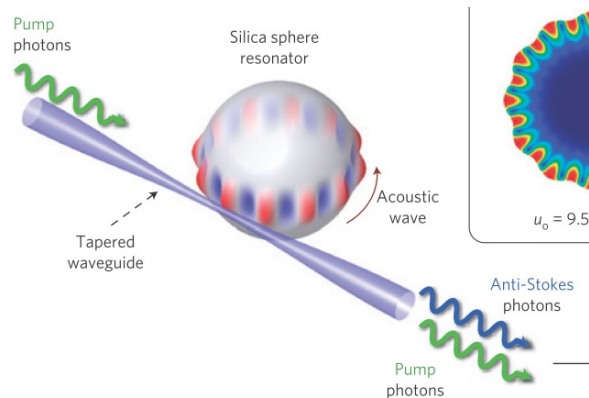
NOOR Laboratory



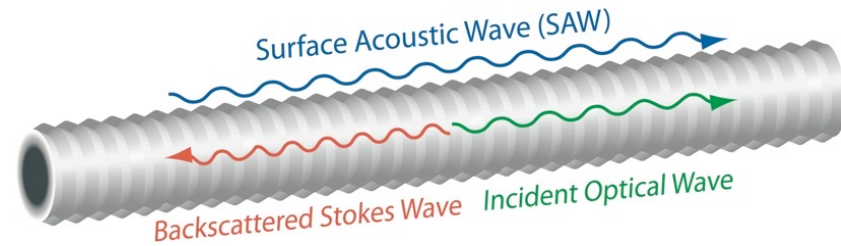
T. Krauss, *Nature Nanotechnology* 9, 19 (2014)

What next ?

Harnessing new nonlinear effects (opto-mechanical & radiation pressure)

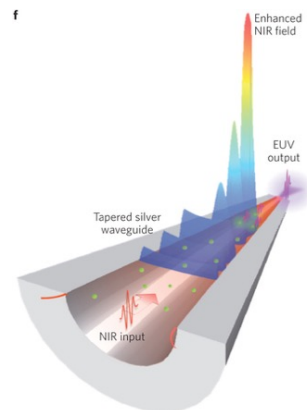


G. Bahl *et al.*, Nat. Comm., 2, 403 (2011)

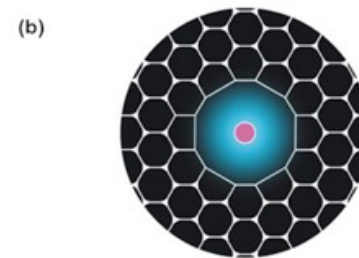
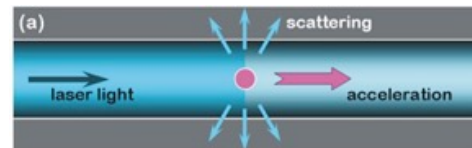


Beugnot *et al.*, Nat. Comm. (2014)

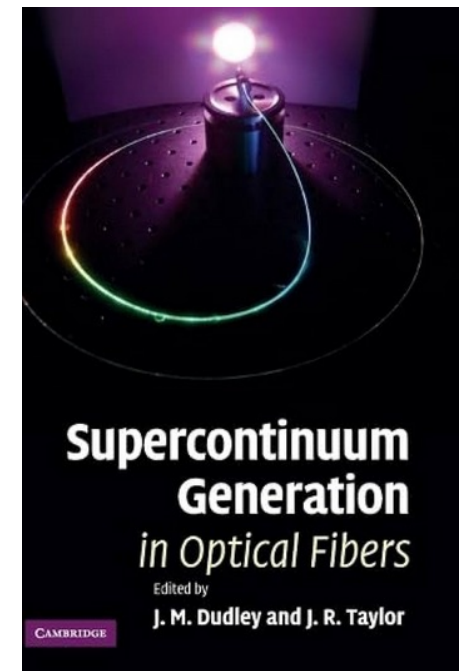
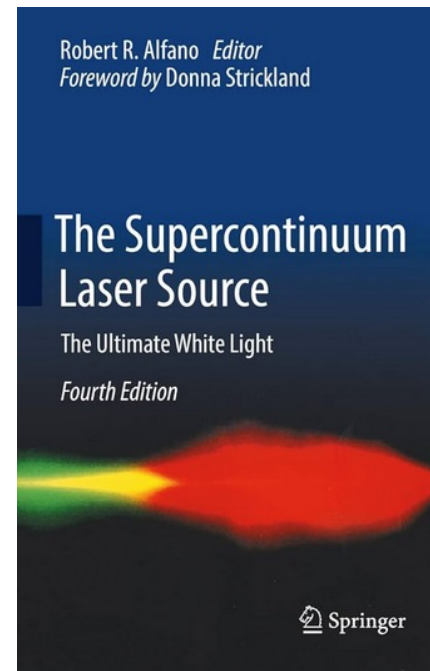
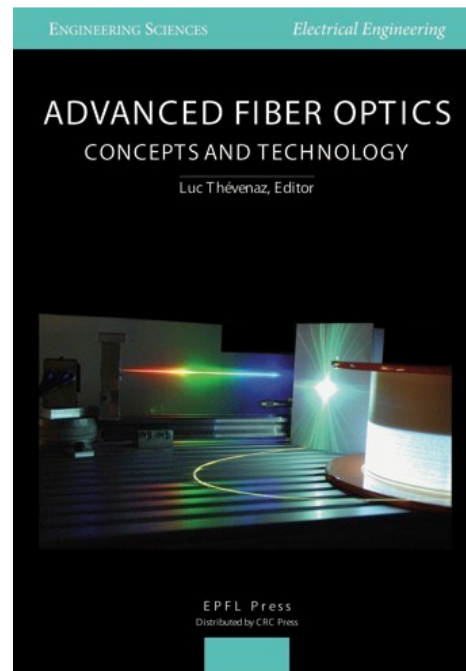
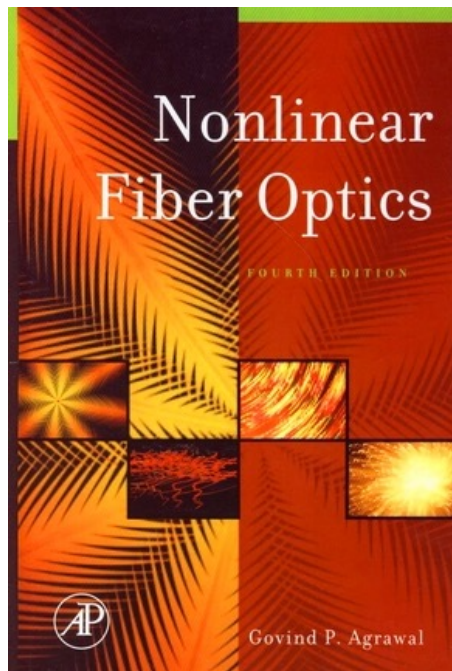
Combining nonlinear optics with nanophotonics, plasmonics and microfluidics



Kauranen *et al.*, Nat. Phot. 6, 737–748 (2012)



Accessible books on nonlinear fiber optics



Questions ?



International Day of Light
Opening Ceremony
UNESCO, Paris, May 16, 2018[©]

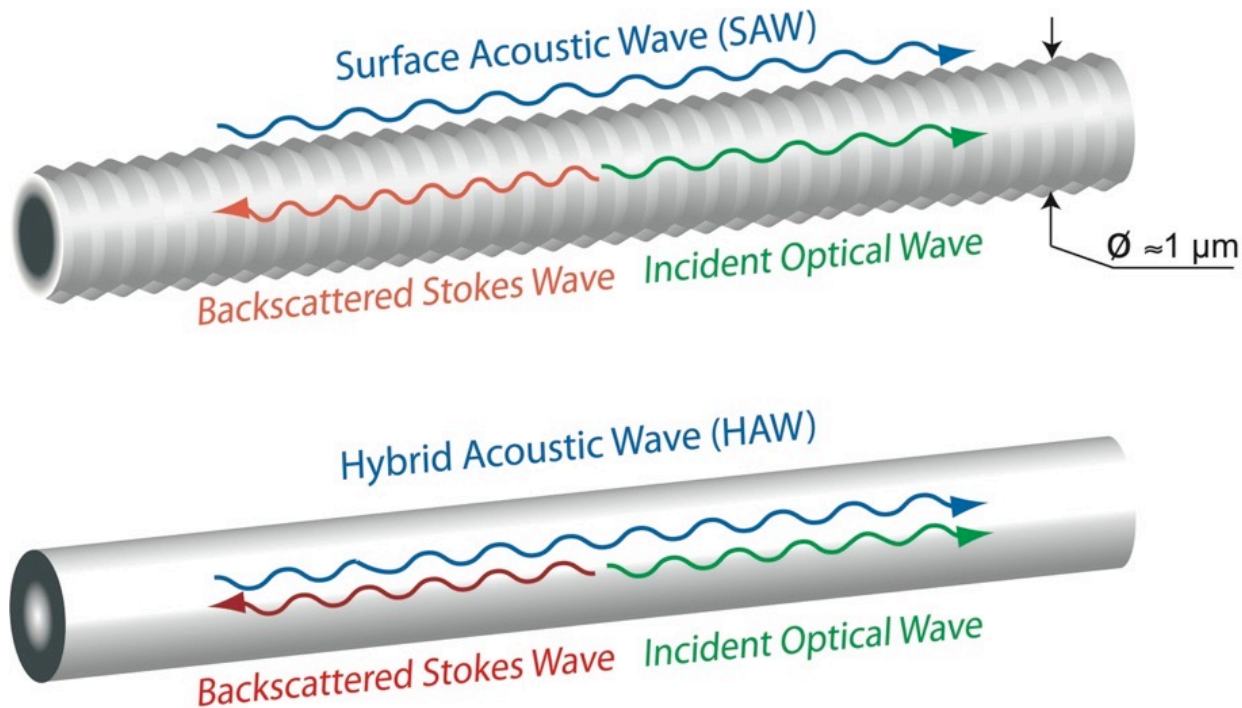
IDE 18 LIGHT

Questions for students :

1. What are the 3 main nonlinear optical effects in silica optical fibers ?
2. What are their physical origin and their typical time scale ?
3. Why second-order nonlinear effects can be neglected in optical fibers ?
4. What is a temporal optical soliton ?
5. What is a dispersive wave ?
6. Is Raman scattering an elastic or an inelastic scattering ?
7. What is the difference between spontaneous and stimulated Raman scattering ?
8. Why the Raman gain band is very large in silica fibers ?

Brillouin scattering in a fiber taper

Unlike optical fibres, optical microwires can actually exhibit both surface acoustic waves (SAW) and hybrid acoustic waves (HAW).



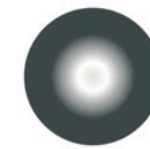
Acoustic energy density

SAW mode



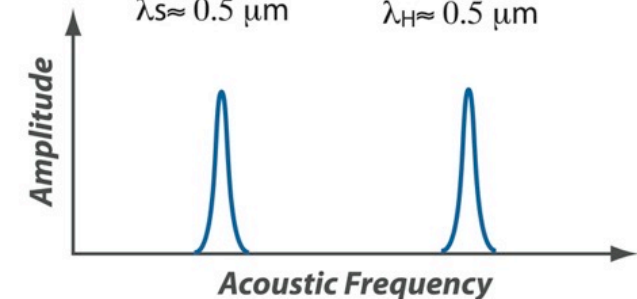
$v_s \approx 3400 \text{ m/s}$
 $\nu_s \approx 6 \text{ GHz}$
 $\lambda_s \approx 0.5 \mu\text{m}$

HAW mode



$v_H \approx 5900 \text{ m/s}$
 $\nu_H \approx 9 \text{ GHz}$
 $\lambda_H \approx 0.5 \mu\text{m}$

Max
Min



Surface waves travel at a velocity of 3400 m/s leading to new Brillouin lines frequency-shifted by 6 GHz in the backscattered spectrum