

---

Exercise Set 9  
Quantum Computation

---

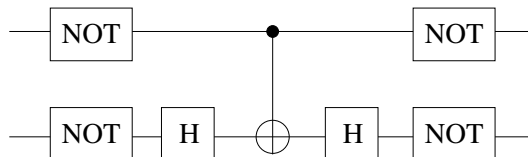
**Exercise 1** *Grover's algorithm for  $N = 4$*

Let  $x \in \{x_0, x_1, x_2, x_3\}$  and  $f(x) = 1$  if and only if  $x = x_0$ . Otherwise  $f(x) = 0$ . We search  $x_0$  thanks to an "oracle" which returns the value of  $f$  when queried with an entry.

- (a) What is the theoretical prediction for the number of queries of the oracle in the quantum setting when we use Grover's algorithm ?
- (b) Show that the following

$$U = \mathbb{I} - 2 \underbrace{|00\dots 0\rangle\langle 00\dots 0|}_{n \text{ times}}$$

is unitary and show also that for  $n = 2$  it can be implemented by the following circuit:



- (c) Take Grover's circuit and for  $N = 4$  compute the quantum state at each step of the algorithm. Draw a geometrical representation in an appropriate two dimensional space (like in class). Confirm that the measurement of the final state indeed gives  $x_0$  and that only one query of the oracle was needed.

