Astrophysics IV : Stellar and galactic dynamics

Exercises

Problem 1 :

Show that the following relations hold :

$$A(R) \equiv \frac{1}{2}\left(\frac{v_c}{R} - \frac{\mathrm{d}v_c}{\mathrm{d}R}\right) = -\frac{1}{2}R\frac{\mathrm{d}\Omega}{\mathrm{d}R}$$
$$B(R) \equiv -\frac{1}{2}\left(\frac{v_c}{R} + \frac{\mathrm{d}v_c}{\mathrm{d}R}\right) = -\left(\Omega + \frac{1}{2}R\frac{\mathrm{d}\Omega}{\mathrm{d}R}\right)$$
$$\Omega = A - B$$
$$\kappa^2 \equiv \left(R\frac{\mathrm{d}(\Omega^2)}{\mathrm{d}R} + 4\Omega^2\right) = -4B(A-B) = -4B\Omega$$

Problem 2 :

Derive the relation between R and $\dot{R}$ in the $z = 0$ plane (approximation of the third integral), if we assume that the total angular momentum is conserved in an axisymmetric potential.

Problem 3 :

Derive the Hamilton equations of motion of a particle in a potential Φ inside a uniformly rotating reference frame $\vec{\Omega}$.

Problem 4 :

Using the scripts MAPPING.PY and MAPPING-RZ.PY, explore the surface of section discussed during the lectures.