Astrophysics IV: Stellar and galactic dynamics

Exercises

Problem 1:

Demonstrate that the condition:

$$\dot{L}_R - L_\theta \dot{\theta} = 0$$

is equivalent to

$$L_z = \text{constant}$$

Can you explain why ?

Problem 2:

For the following potentials, derive the analytic expression for the acceleration $\vec{a}$ at an arbitrary point $\vec{x}$ in Cartesian coordinates. Also, derive the orbital circular period for a radius r.

a) Point mass:

$$\Phi(r) = -\frac{GM}{r}$$

b) Plummer-Schuster:

$$\Phi(r) = -\frac{GM}{\sqrt{e^2 + r^2}}$$

c) Miyamoto-Nagai:

$$\Phi(R, z) = -\frac{GM}{\sqrt{R^2 + (a + \sqrt{b^2 + z^2})^2}}$$

d) Harmonic potential:

$$\Phi(x, y, z) = \frac{1}{2}\omega_x^2 x^2 + \frac{1}{2}\omega_y^2 y^2 + \frac{1}{2}\omega_z^2 z^2$$

Problem 3:

Derive the epicycle frequencies for the following potentials:

a) Point mass:

$$\Phi(r) = -\frac{GM}{r}$$

b) Plummer-Schuster:

$$\Phi(r) = -\frac{GM}{\sqrt{e^2 + r^2}}$$

c) Miyamoto-Nagai:

$$\Phi(R, z) = -\frac{GM}{\sqrt{R^2 + (a + \sqrt{b^2 + z^2})^2}}$$