Astrophysics IV, Dr. Yves Revaz | EPFL

4th year physics | Exercises week 3

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Astrophysics IV: Stellar and galactic dynamics

Exercises

Problem 1:

Derive the density corresponding to the Plummer-Schuster potential:

$$\Phi(r) = -\frac{GM}{\sqrt{b^2 + r^2}}$$

Problem 2:

Derive analytically the circular velocity corresponding to the following potentials:

a) Point mass:

$$\Phi(r) = -\frac{GM}{r}$$

b) Homogeneous sphere of radius a:

$$\Phi(r) = \begin{cases} -\frac{GM}{a}\left(\frac{3}{2} - \frac{1}{2}\frac{r^2}{a^2}\right) & \text{if } r < a \\ -\frac{GM}{r} & \text{if } r \geq a \end{cases}$$

c) Plummer-Schuster potential:

$$\Phi(r) = -\frac{GM}{\sqrt{b^2 + r^2}}$$

d) Miyamoto-Nagai potential:

$$\Phi(R, z) = -\frac{GM}{\sqrt{R^2 + (a + \sqrt{b^2 + z^2})^2}}$$

Compute for $z = 0$.

Problem 3:

Using the scale length h_R and scale height h_z :

$$h_R = a + b$$
$$h_z = b$$

in the Miyamoto-Nagai potential, verify that the rotation curve is independent of the scale height h_z.

Problem 4:

Derive the density and circular velocity corresponding to the NFW potential

$$\Phi(r) = v_s^2 \left[1 - \frac{\ln(1 + r/r_s)}{r/r_s}\right]$$

Problem 5:

The isochrone potential is given by

$$\Phi(r) = -\frac{GM}{b + \sqrt{b^2 + r^2}}$$

What is the density profile that generates this potential? What is the corresponding circular velocity?

Problem 6:

Using Gauss' theorem, derive the surface density for the Kuzmin disk potential at z=0

$$\Phi_K(R, z) = -\frac{GM}{\sqrt{R^2 + (a + |z|)^2}}$$

Problem 7:

The surface density of a Mestel disk is defined as:

$$\Sigma(R) = \begin{cases} \frac{v_0^2}{2\pi G R} & (R < R_{\max}) \\ 0 & (R \geq R_{\max}) \end{cases} \tag{1}$$

Show that in the limit $R_{\max} \to \infty$, the circular velocity is the constant v_0.