Markov Chains and Algorithmic Applications

Fall 2023

Mini project: Signal recovery using MCMC

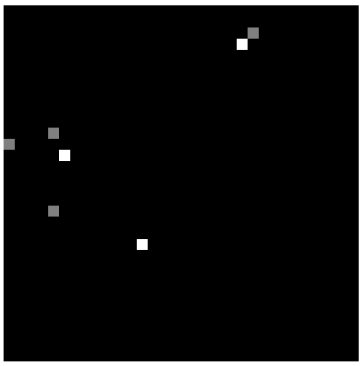
1 Competition Question

Let's put the algorithms you developed to use! You are given noisy measurements of a 128×128 image of small portion of the night sky. There are **at most** 150 stars in the image which are classified into **dim (value 1) and bright (value 2)** stars. Value 0 indicates the absence of any star. Since the number of stars is small, we made only a lower number of measurements (1200).

More formally, we have a signal $\theta^* \in \{\theta \in \{0,1,2\}^d : \|\theta\|_0 \leq s\}$, where $d = 128 * 128$, $s = 150$. Here, θ^* is the row-by-row vectorized version of the true image. Let $X \in \mathbb{R}^{m \times d}$ be a random sensing matrix with its entries sampled i.i.d. from $\mathcal{N}(0,1)$. Let $\xi \in \mathbb{R}^m$ be a noise vector sampled from $\mathcal{N}(0, I_m)$ and independent of X. Let $\Theta \subseteq \mathbb{R}^d$ be a finite set. We have measurements $y \in \mathbb{R}^m$ generated as $y = X\theta^* + \xi$, where $m = 1200$.

You are given (X, y) and tasked to recover the night sky image using the MCMC techniques you have developed. The team that achieves the **minimum squared error** with respect to the original image wins the competition.

You are given the top left portion of the original image for a visual sanity check.



2 Logistics

- You will find sensing matrix X and measurements y with file names 'SensingMatrix_X.csv' and 'Measurements_y.csv' on Moodle. You are required to upload the recovered 128×128 image on Moodle in CSV format. Note that entries in the CSV file you upload should be 0,1 or 2.
- Given below is an example of row-by-row vectorization of a matrix:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{bmatrix}^T.$$