

Mathematical Structure and Ontology

Cristian Lopez

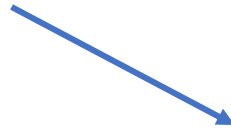
Université de Lausanne – FNS

Philosophical Perspectives on the Exact
Sciences and their History

October 9, 2024

— First Class's last slide...

Philosophy

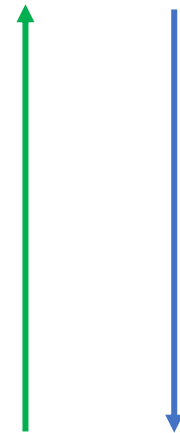


Science



Collaborate in producing **better knowledge** of the world, without any substantival overlapping

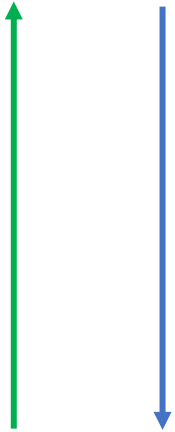
What is a description of?



A description
(of its regularities)

— A development...

What is a description of?

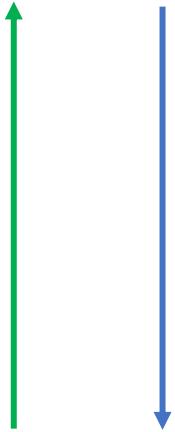


A description

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What is a description of?



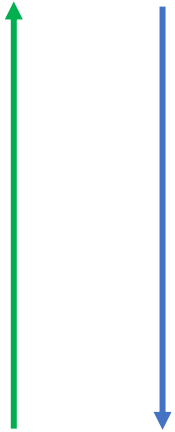
Ontology (what there is and how
is what there is)

A description
(of its regularities)

Theoretical Structure

— A development...

What is a description of?



A description
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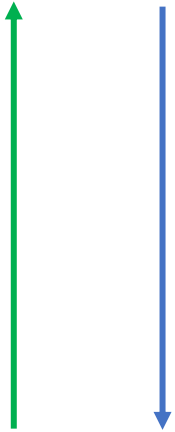
Ontology (what there is and how
is what there is)

Theoretical Structure

- Entities
- Properties (and which kind)
- Relations
- ...

— A development...

What is a description of?



A description
(of its regularities)

Ontology (what there is and how
is what there is)

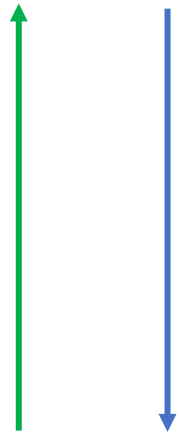
- Entities
- Properties (and which kind)
- Relations
- ...

Theoretical Structure

- Geometrical space (and its structure)
- Symmetries
- Boundary conditions
- Kinematic / Dynamical parameters
- Differential equations...

— A development...

What is a description of?



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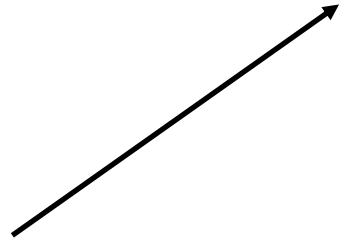
Theoretical Structure

- Geometrical space (and its structure)
- Symmetries
- Boundary conditions
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- Differential equations...



Two attitudes...

Formal Structure



Mere instrument for
prediction

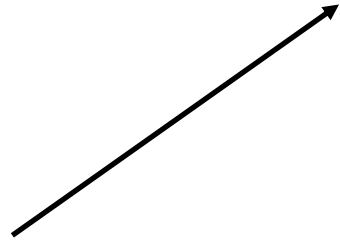


We don't expect a theory to be
informative about what the world is like



Two attitudes...

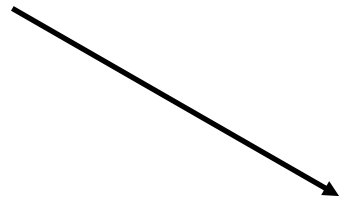
Formal Structure



Mere instrument for
prediction



We don't expect a theory to be informative about what the world is like



More than a mere
instrument for **prediction**



We may expect a theory to be informative about what the world is like –some of its structure would “latch onto reality”

More than a mere instrument...

“More than a mere instrument for **prediction**”

Non-relativistic quantum mechanics (Hilbert space formulation)

More than a mere instrument...

“More than a mere instrument for **prediction**”

Non-relativistic quantum mechanics (Hilbert space formulation)

To represent quantum systems, we need:

- A vectorial state space (Hilbert space and its structure), whose normalized vectors represent possible states of the system,
- A *pure state* is given by a vector (ray) in a Hilbert Space
- A set of preferred operators on Hilbert Space (Hermitian operators) representing observables

More than a mere instrument...

“More than a mere instrument for **prediction**”

Non-relativistic quantum mechanics (Hilbert space formulation)

- A dynamic defined on the Hilbert space for the quantum state. Generally defined as a set of (unitary) transformations which take a state at one time to the state it evolves into at other times (Hamiltonian)
- We may also require that systems' evolutions being invariant under certain transformations (as boosts, spatial translations, etc.). Galilei Invariance
- We may also impose some kinematic requirements –e.g., that Hamiltonians are bounded-from below.

More than a mere instrument...

“More than a mere instrument for **prediction**”

Non-relativistic quantum mechanics (Hilbert space formulation)

$$\blacksquare i\hbar \frac{\partial}{\partial t} \psi(x, t) = \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x, t) \right] \psi(x, t)$$

$$\blacksquare |\psi(t_0)\rangle_z = \frac{1}{\sqrt{2}} (|\uparrow\rangle_z + |\downarrow\rangle_z)$$

We use the **mathematical structure** of a theory to represent physical systems, their properties, how they evolve in time, their possible states...

More than a mere instrument...

“More than a mere instrument for **prediction**”

Non-relativistic quantum mechanics (Hilbert space formulation)

But if they are more than mere instruments, then this is not enough. Strictly speaking such a structure cannot be easily connected with anything in the world. So, it is necessary that they also reflect some **features of the world** (something in the **ontology**), but **how?**

More than a mere instrument...

“More than a mere instrument for **prediction**”

■ ***“Internalist”*** view

More than a mere instrument...

“More than a mere instrument for **prediction**”

■ **“Internalist”** view

A physical theory delivers its own “ontology”

More than a mere instrument...

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■ **“Internalist”** view

A physical theory delivers its own “ontology”

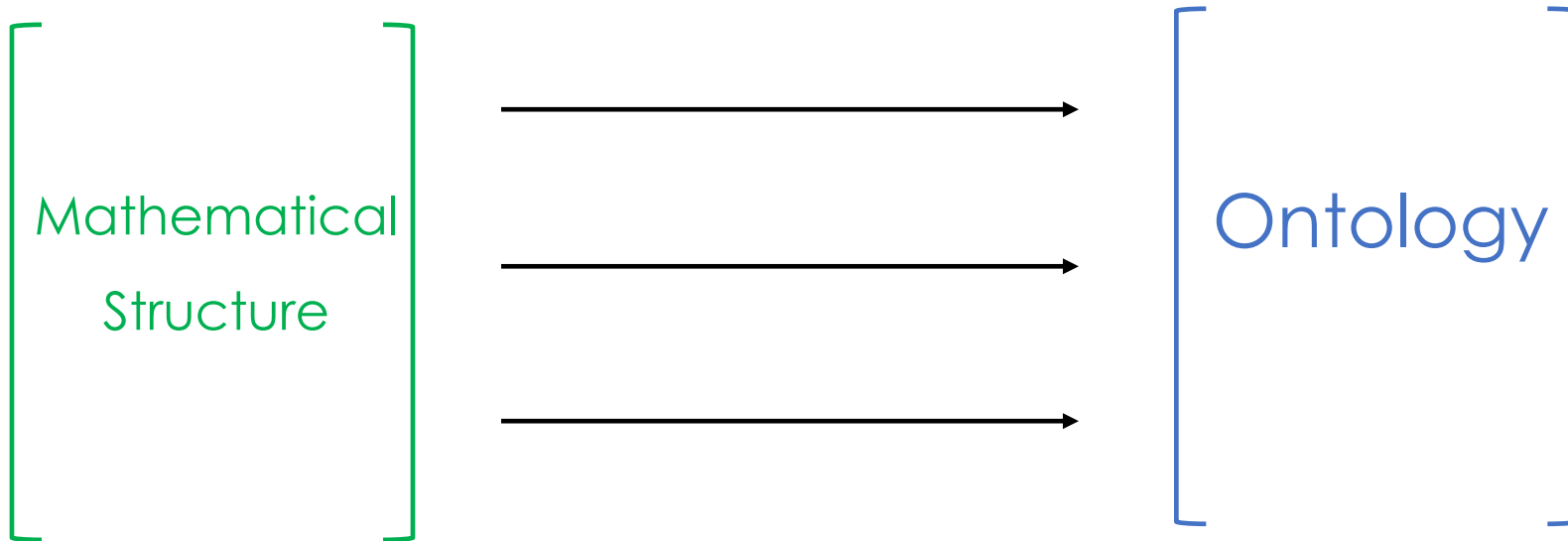
You can read off “what-the-world-is-like” from the mathematical structure of the theory (+some minimal interpretation)

Arg. Non-miracle Argument (IBE)

More than a mere instrument...

“More than a mere instrument for **prediction**”

■ “*Internalist*” view



More than a mere instrument...

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■ “*Internalist*” view

Mathematical Structure

$\psi(x, t)$



Ontology

‘Quantum State’ (WFR, MWI)

N-dimensional
Hilbert Space



Structure of Physical Space

More than a mere instrument...

“More than a mere instrument for **prediction**”

■ “*Internalist*” view

Mathematical Structure

Ontology

Symmetries

Infer...

Natural Properties



More than a mere instrument...

“More than a mere instrument for **prediction**”

■ **“Externalist”** view

More than a mere instrument...

“More than a mere instrument for **prediction**”

■ **“Externalist”** view

To refer to something, a “commentary” (an ontology) on the mathematical structure is needed

More than a mere instrument...

“More than a mere instrument for **prediction**”

■ **“Externalist”** view

To refer to something, a “commentary” (an ontology) on the mathematical structure is needed

The mathematical structure by itself is silent respect to the ontology. We need to provide it “from outside” –to know what the mathematical representation represents, we need to provide an ontology

More than a mere instrument...

“More than a mere instrument for **prediction**”

■ **“Externalist”** view

Mathematical
Structure

+

Ontology

More than a mere instrument...

“More than a mere instrument for **prediction**”

■ **“Externalist”** view

Mathematical
Structure

+

Ontology
Ontology
Ontology

More than a mere instrument...

“More than a mere instrument for **prediction**”

■ **“Externalist”** view

Mathematical
Structure

+

Ontology
Ontology
Ontology

Different physical theories?

Sum Up...

Mathematical
Structure

“**Internalist**” view

The relationship between MS and ontology is that the latter should be read off from the former

“**Externalist**” view

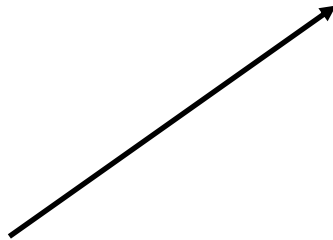
The relationship between MS and ontology is that the latter must be added to the former so as for it to refer to the world

Two attitudes (ontology)...

Ontology

Two attitudes (ontology)...

Ontology



Flat Ontologies

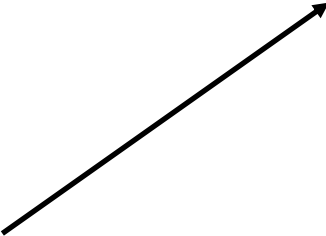


Ontology is about **existence**



Two attitudes (ontology)...

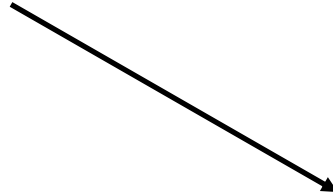
Ontology



Flat Ontologies



Ontology is about **existence**



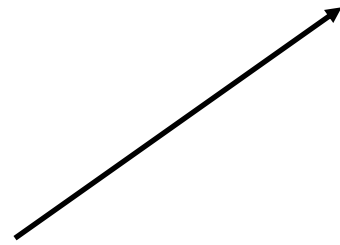
Structured Ontologies



Ontology is about distinguishing *what is **fundamental** what is **derivative***

Sum Up...

Mathematical
Structure

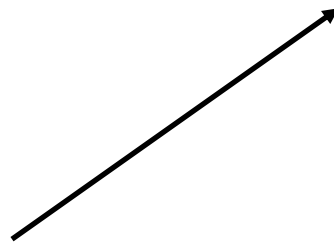


“Internalist” view

The relationship between MS and ontology is that the latter should be read off from the former

Sum Up...

Mathematical
Structure



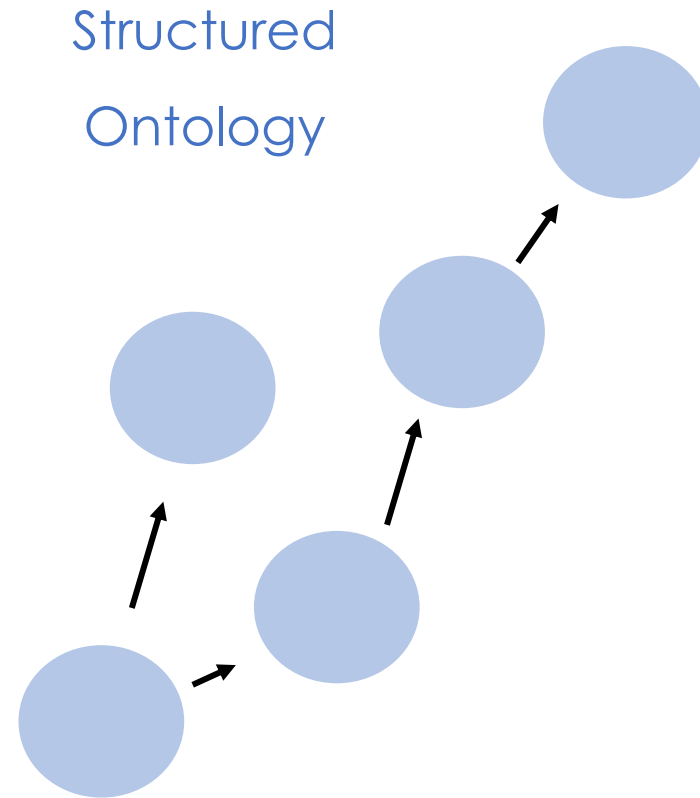
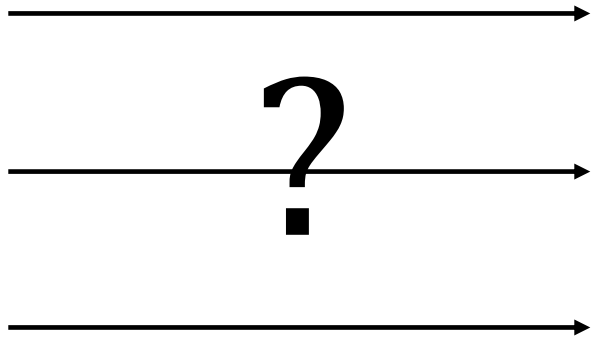
“Internalist” view

The relationship between MS and ontology is that the latter should be read off from the former

- Does it imply a flat ontology where everything what exists exists in the same level? How can a mathematical structure distinguish between what's fundamental or what's derivative in the ontology?

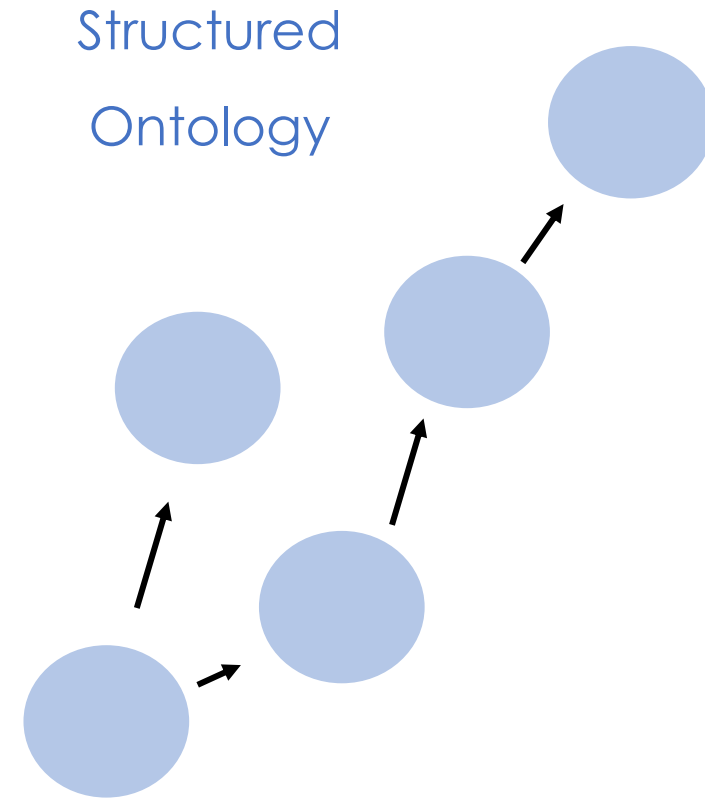
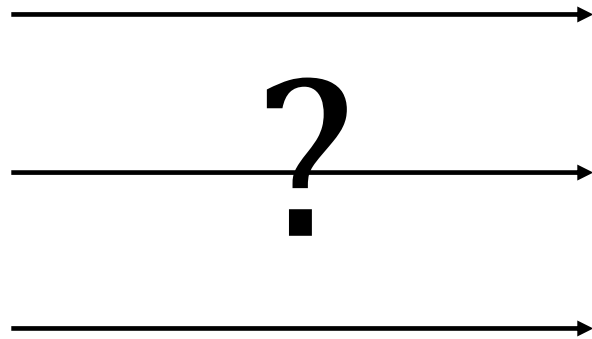
Sum Up...

Mathematical
Structure



Sum Up...

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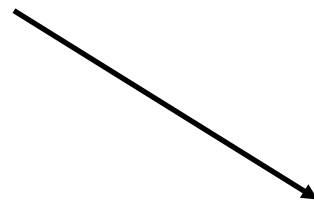


- A simple mapping wouldn't work because there are many different formulations!

Sum Up...

- But where does the ontology then come from? How can we fix what's fundamental or not? We need good philosophical work here!

Mathematical
Structure



“**Externalist**” view

The relationship between MS and ontology is that the latter must be added to the former so as for it to refer to the world