Nuclear Fusion and Plasma Physics **EPFL**
Prof. A. Fasoli SPC
Teachers: Umesh Kumar, Luke Simons *09 December 2024*

Problem Set 12

Exercise 1 - Control of Plasma Burn

In principle, burning plasmas can undergo thermal runaway, which involves an uncontrolled transition to ignited conditions, with an ever increasing temperature. Analyze this possibility in ITER using a simple 0-D model, based on power balance and neglecting bremsstrahlung radiation. Consider a situation in which the density is fixed.

a) Assess whether a burning ITER plasma is susceptible to thermal instabilities.

b) Propose strategies to stabilize the plasma temperature and avert instability.

Hints:

1. Begin with the energy balance equation near the point of ignition, neglecting the external power input (why can this be done?). Assuming constant plasma density, derive an expression for the rate of change in plasma temperature over time (the time derivative of the temperature).
2. Deduce the temperature's equilibrium condition from the derived expression.
3. Next, introduce a perturbation to this equilibrium temperature. Estimate whether such a perturbation will grow or decay with time.
4. Use the approximation for $\langle \sigma v \rangle_{DT}$:

$$\langle \sigma v \rangle_{DT} \approx 1.1 \times 10^{-24} \, T^2_{[\mathrm{keV}]} \quad (\mathrm{m^3/s})$$

5. Apply the ELMy H-mode scaling law for τ_E:

$$\tau_E \propto \tau_B \, \rho_*^{-0.7} \, \beta^{-0.9} \, \nu_*^{-0.01},$$

with

$$\tau_B \approx \frac{a^2 B_T}{T_e} \quad \rho_* \approx \frac{\sqrt{T_e}}{a \, B_T} \quad \nu_* \approx \frac{n_e \, a}{T_e^2} \quad \text{and} \quad \beta \approx \frac{n_e \, T_e}{B_T^2/(2\mu_0)}$$

Exercise 2 - On the Use of Tritium in ITER and Fusion Reactors

Utilizing the definitions and discussions from the lectures, estimate the following for ITER and a hypothetical 1 GW electrical fusion reactor:

a) Calculate the tritium burn-up fraction, f_B, in both ITER and the reactor.

b) Determine the daily tritium mass burn rate $\frac{dM}{dt}$ (in grams per day) for each scenario.

c) Estimate the necessary tritium inventory M_0 (in kilograms required to be on-site at all times) for both settings.

Reminder:

$$f_B = \frac{1}{1 + \frac{1}{\tau_T n_T <\sigma v>_{DT}}} \tag{1}$$

$$\frac{dM}{dt} \approx 56 \times \text{fusion power } (\text{GW}_{\text{thermal}}) \; [kg/year] \tag{2}$$

$$M_0 \approx \frac{t_p \frac{dM}{dt}}{\eta_f f_B} \tag{3}$$

Assumptions:

- τ_T is the particle confinement time (assume $\tau_T = 1$ s for ITER and 2 s for a reactor).
- Make a reasonable assumption for the tritium density n_T.
- t_P is the tritium reprocessing time (1 day).
- Use the fusion cross-section approximation from Exercise 1 with plasma temperatures of 10 keV for ITER and 15 keV for the reactor.
- Tritium fueling efficiency η_f is 20% for ITER and 50% for the reactor.