Nuclear Fusion and Plasma Physics - Exercises

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Solutions to problem set 11 - 04 December 2023

Exercise 1 - A steady state Tokamak with copper coils

a) Using Ampere's law on a contour following the magnetic axis of the tokamak yields $2\pi R\,B_0 = \mu_0\,I_{\text{enc}}$ so $I_{\text{enc}} = 150$ MA. Dividing this result by 20 coils gives 7.5 MA/coil.

b) The cross section is $S = I_{coil}/j_{cd} = 0.15\text{m}^2$. Assuming a circular coil its circumference is $\ell = 2\pi a = 12.6$ m. The resistance of the coil is $R = \rho\ell/S = 1.4 \times 10^{-6}\,\Omega$. This gives a power loss of 79 MW.

c) The resistivity is now reduced to $2 \times 10^{-9}\,\Omega\,\text{m}$. This gives a resistance $R = 1.67 \times 10^{-7}\,\Omega$. The power loss now is 9.4 MW.

Exercise 2 - Design of a SC solenoid

This exercise is based on Section 3.1 of the book "Superconducting magnets" by M. Wilson. This solution follows closely the discussion in the book.

a) For a 6 T magnet, Fig. 5 in the problemset shows a critical current density $J \approx 3 \times 10^8\,\text{A}\,\text{m}^{-2}$ (point Q in Fig. 5a).

b) Since $2a = 150\,\text{mm}$, we use the result in (a) to find F:

$$F(\alpha, \beta) = \frac{B}{a\,J} = \frac{6\,\text{T}}{0.075\,\text{m} \cdot 3 \times 10^8\,\text{Am}^{-2}} = 2.7 \times 10^{-7}\,\text{H}\,\text{m}^{-1}$$

Then, using Fig. 3, we find the α and β corresponding to the minimum volume for this value of F. We do this by, first, estimating by eye the curve for $F = 2.7 \times 10^{-7}\,\text{H}\,\text{m}^{-1}$ between $2\times10^{-7}\,\text{H}\,\text{m}^{-1}$ and $3\times10^{-7}\,\text{H}\,\text{m}^{-1}$. Then, we think about the place where this curve would intersect the *min_volume* line. This happens approximately at $(\alpha, \beta) = (1.4, 0.7)$.

c) Figure 4 shows $B_{\text{w}}/B_0 = 1.3$ for the parameter values obtained in (b).

d) If we want the magnet to produce $B_0 = 6\,\mathrm{T}$, the result above tells us that $B_\mathrm{w} = 1.3 \cdot 6\,\mathrm{T} = 7.8\,\mathrm{T}$.

Looking at Fig. 5a, this means that running the design current of $J = 3 \times 10^8\,\mathrm{A\,m^{-2}}$ would actually bring us to point S, which is beyond the limit of what the superconductor can take.

In order to stay within operational limits, we need to go back to point R (still referring to Fig. 5a), that is, we need to reduce the current density to $J = 2.5 \times 10^8\,\mathrm{A\,m^{-2}}$ for a maximum field $B_\mathrm{w} \approx 6.5\,\mathrm{T}$.

Sadly enough, in that case our design would yield $B_0 = B_\mathrm{w}/1.3 \approx 5\,\mathrm{T}$.

e) To be able to achieve $B_0 = 6\,\mathrm{T}$, we need the design to allow for the higher value of B_w, as shown in Fig. 5b.

If we choose $J = 2 \times 10^8\,\mathrm{A\,m^{-2}}$ and $B = 7.5\,\mathrm{T}$, then $F = 5 \times 10^{-7}\,\mathrm{H\,m^{-1}}$. The minimum volume is achieved with $(\alpha, \beta) = (1.7,\, 0.9)$, which yields $B_\mathrm{w}/B_0 = 1.2$.

B_0 can now be as large as $B_0 = 7.5/1.2\,\mathrm{T} = 6.25\,\mathrm{T}$.

This design is able to reach magnetic fields that are higher than the design goal, which may be undesirable (one can anyway always reduce B_0 by using a smaller J) since a larger magnet may be more expensive. One can continue iterating with values not far from these to find the parameters that exactly meet the requirement of B_0 ($F = 4 \times 10^{-7}\,\mathrm{H\,m^{-1}}$).

One can also take a more efficient design by moving away from the minimum volume curve. For example, one can obtain more uniform field lines or achieve less winding volume with a longer magnet.