Nuclear Fusion and Plasma Physics **EPFL**
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Problem Set 11

Exercise 1 - A steady state Tokamak with copper coils

In this exercise, we aim to design a Tokamak operating at steady-state using coils made from copper (Cu). Consider constructing a Tokamak with a major radius $R = 5$ m and a minor radius $a = 2$ m. The field at the Tokamak's center is 6 T, generated by 20 coils.

a) Calculate the electric current in each coil required to achieve this magnetic field strength.

b) Given a current density in each coil of 5×10^7 A/m^2 (considered high), determine the cross-sectional area of the coil. With copper at room temperature exhibiting a resistivity of $\rho = 1.68 \times 10^{-8}\ \Omega\, m$, calculate the power dissipation due to the resistance.

c) Using the data on the curve of ρ versus T shown in Fig. 1, evaluate the power dissipation when the coil is cooled to 80 K (the boiling point of liquid nitrogen).

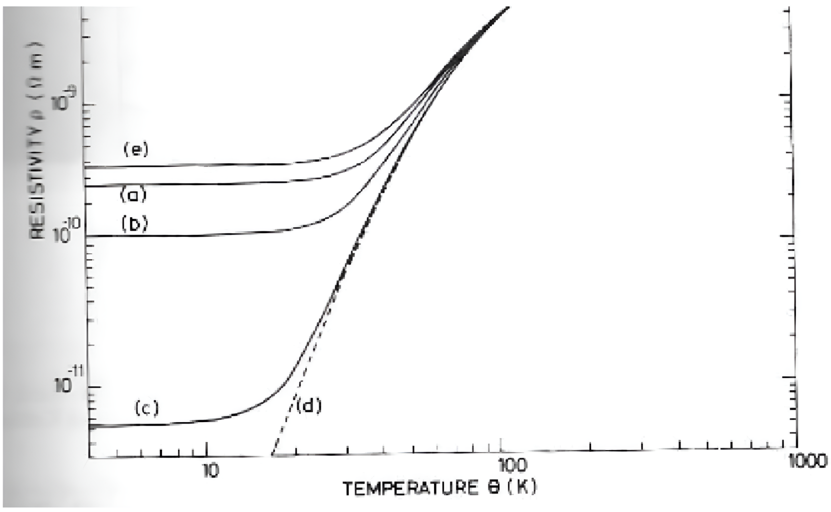

Figure 1: Variation of the resistivity of Copper as a function of temperature. The different curves represent various material treatments.

Exercise 2 - Design of an SC solenoid

This exercise is based on Section 3.1 of the book "Superconducting magnets" by M. Wilson.

Here, we adopt an engineering perspective to design a superconducting magnet. Consider a solenoid of length $2l$ as shown in Fig. 2.

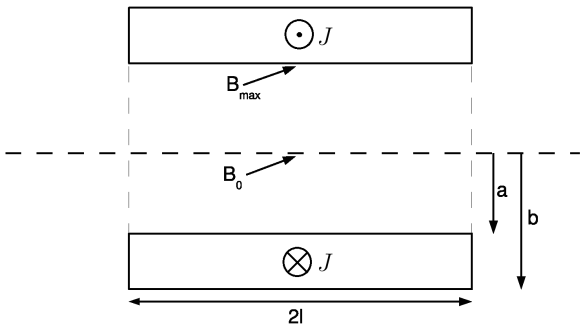

Figure 2: Definition of parameters for the solenoid. Note the location of the maximum field within the conductor.

The magnetic field B at the center is calculated as $B = aJF(\alpha, \beta)$ where J is the average current density, $\alpha = b/a$, $\beta = l/a$ and F is defined by:

$$F(\alpha, \beta) = \mu_0 \beta \ln \left\{ \frac{\alpha + \sqrt{(\alpha^2 + \beta^2)}}{1 + \sqrt{(1 + \beta^2)}} \right\}$$

Figure 3 includes lines of constant $F(\alpha, \beta)$ and a curve showing the parameter values for a minimum volume design.

Assume that we desire a field of $6\,\mathrm{T}$ in a solenoid with bore diameter $2a = 150$ mm.

a) The current limits as a function of the field B of the superconductor are depicted by the upper line in Fig. 5. Calculate the current density considering the upper limit of J as influenced by B.

b) Determine the coil parameters that result in the minimum volume using Fig. 3.

c) From Fig. 4, identify the maximum field in the solenoid, B_w.

d) Given the maximum field calculated in (c), assess whether the magnet can operate at the j_c determined in (b). What field will realistically be achieved?

e) If achieving $B_0 = 6\,\mathrm{T}$ remains a goal, what adjustments are necessary?

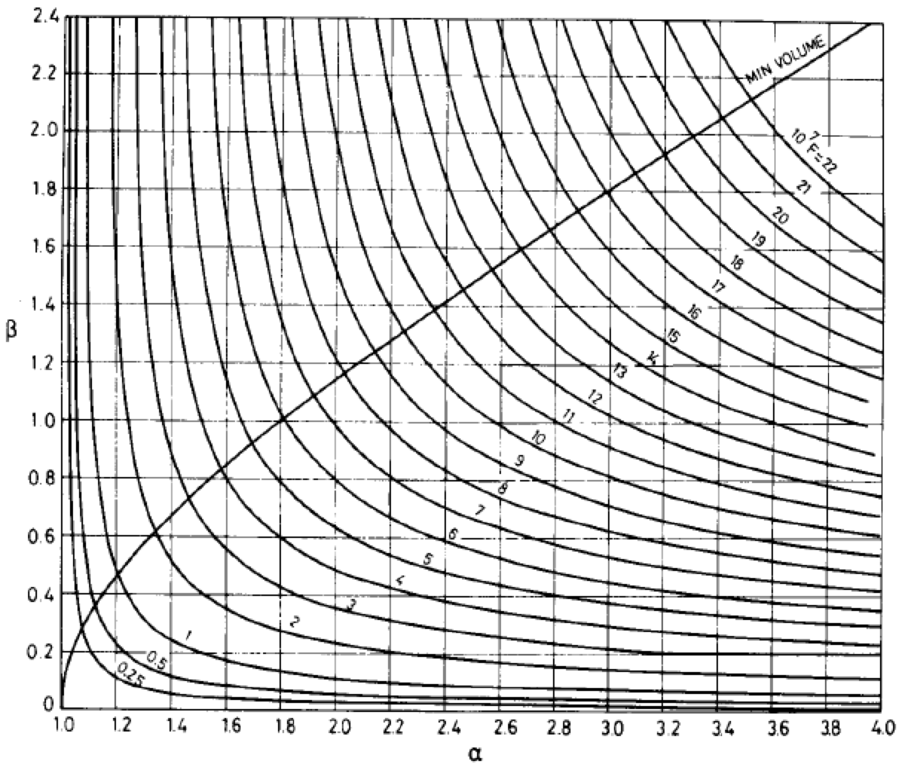

Figure 3: Function F, relating the central field in a simple solenoid to its radius, current density, and shape factors α and β.

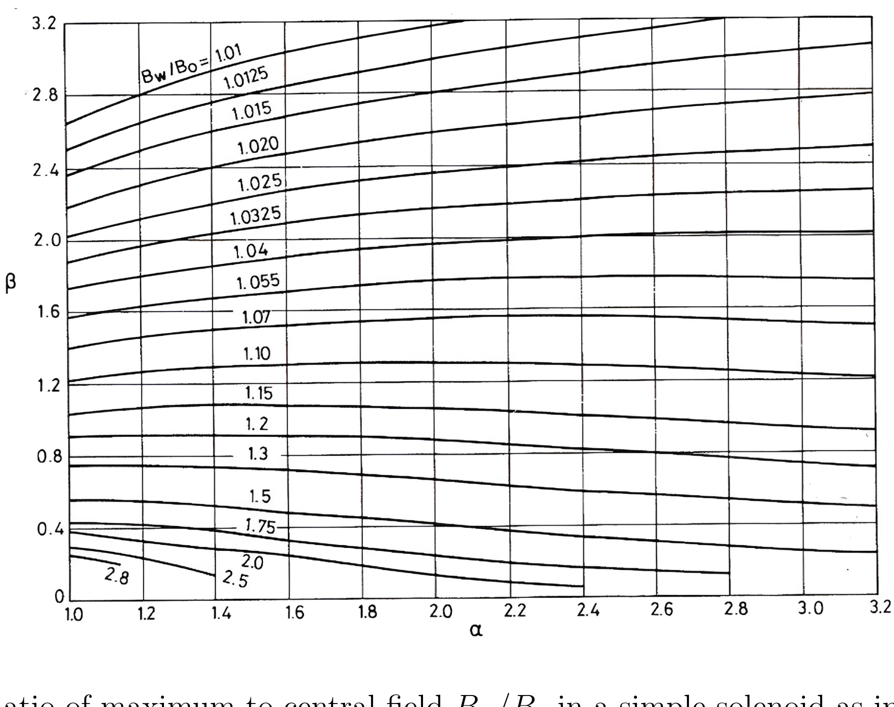

Figure 4: Ratio of maximum to central field B_{w}/B_0 in a simple solenoid as influenced by the shape factors α and β.

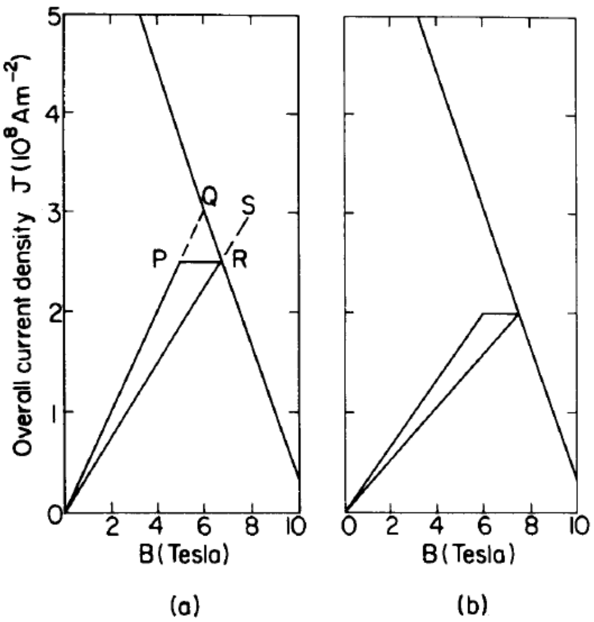

Figure 5: Load lines and current density limit for the solenoid. The different load lines illustrate the effect of the maximum field setting the current density limit.