Nuclear Fusion and Plasma Physics | **EPFL**
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Solutions to Problem Set 9

Exercise 1 - Need for additional heating power

a) The resistance of the plasma can be estimated by using

$$R = \eta_{||}\ell/S = 1.65 \times 10^{-9} \frac{\ln\Lambda}{T_e^{3/2}} \frac{2\pi R_0}{\pi a^2} \Longrightarrow R \approx 5.8 \times 10^{-9}\,\Omega \tag{1}$$

b) The ohmic power dissipated is given by

$$P_{ohm} = R\,I_P^2 \approx 145\,\text{kW} \tag{2}$$

c) The power lost by the plasma can be calculated from the definition of the energy confinement time, τ_E, by writing

$$P_{loss} = \frac{W_{kin}}{\tau_E} = \frac{2 \times \frac{3}{2}\,n\,T\,V}{\tau_E} \Longrightarrow P_{loss} \approx 190\,\text{MW} \tag{3}$$

d) The total fusion power produced by the thermonuclear reactions is given by

$$P_{fus} = V\frac{1}{4}n^2\,\langle\sigma\,v\rangle_{15\,\text{keV}}\,\Delta E_f \Longrightarrow P_{fus} \approx 167\,\text{MW} \tag{4}$$

while $P_\alpha = \frac{3.5}{17.1}\,P_{fus} = 34$ MW.

e) The sum of the ohmic power dissipated in the plasma and the power of the alpha particles produced by the fusion reactions is smaller than the power lost by the plasma. This would prevent a steady-state condition and cause the equilibrium temperature to be lower than 15 keV if no additional heating were supplied. In order to maintain the steady-state operation with the given plasma equilibrium parameters (T_e and n_e), we need external heating with power $P_{heating} = P_{loss} - P_{ohm} - P_\alpha = 156$ MW.

f) From the above calculated P_{fus}, P_{ohm} and $P_{heating}$, it is possible to estimate the physics fusion gain factor Q_{fus}:

$$Q_{fus} = \frac{P_{fus}}{P_{in}} = \frac{P_{fus}}{P_{ohm} + P_{heating}} \approx 1.1 \tag{5}$$

Exercise 2 - Efficiency of lower hybrid current drive

a) In order to accelerate a group of electrons localized in a certain region of space from $v_{||}$ to $v_{||} + \delta v_{||}$, an energy density $\delta E = n_e\, m_e\, v_{||}\, \delta v_{||}$ is required. This organized motion of electrons generates a current $\delta j = n_e\, e\, \delta v_{||} = \frac{e\,\delta E}{m_e\, v_{||}}$.

The typical time taken for the particle velocities to be isotropized is $\tau_p = 1/\nu_p$, where ν_p is the collision frequency for momentum transfer. Therefore, the power density required to keep the current $I_{CD} = \delta j \times A$, with A being the cross section of the wave beam, is approximately $p_d = \nu_p\, \delta E$. The ratio I_{CD}/P_{wave} is then calculated as

$$\eta_{CD} = \frac{I_{CD}}{P_{wave}} = \frac{\delta j\, A}{2\pi R_0\, A\, p_d} = \frac{1}{2\pi R_0}\frac{\delta j}{p_d} = \frac{1}{2\pi R_0}\frac{e\,\delta E}{m_e\, v_{||}}\frac{1}{\nu_p\, \delta E} = \frac{1}{2\pi R_0}\frac{e}{m_e\, v_{||}\, \nu_p} \tag{6}$$

Remembering that the typical collision frequency for isotropization is given by $\nu_p = \frac{n_e\, e^4\, ln\Lambda}{4\pi\epsilon_0^2\, m_e^2 v_{||}^3}$, one obtains

$$\eta_{CD} = \frac{I_{CD}}{P_{wave}} = \frac{2\epsilon_0^2\, m_e}{e^3\, ln\Lambda}\frac{v_{||}^2}{R_0\, n_e} \tag{7}$$

b) Assuming that the parallel wave phase velocity which resonates with the particles is $v_{||} \approx 5 v_{th}$ and taking $v_{th} = \sqrt{\frac{T_e}{m_e}}$, one finds

$$\eta_{CD} = \frac{I_{CD}}{P_{wave}} \approx 50\frac{\epsilon_0^2}{e^2\, ln\Lambda\, R_0}\frac{T_e[eV]}{n_e[\mathrm{m}^{-3}]} \approx 9 \times 10^{15}\frac{T_e[\mathrm{eV}]}{R_0\, n_e[\mathrm{m}^{-3}]} \tag{8}$$

c) In order to drive 2 MA in ITER ($R_0 = 6\,\mathrm{m}$, $n_e = 1 \times 10^{20}\,\mathrm{m}^{-3}$ and $T_e = 15\,\mathrm{keV}$), we have that

$$\eta_{CD} \approx 9 \times 10^{15}\frac{15 \times 10^3}{6 \times 10^{20}} \approx 0.225\,\mathrm{MA/MW} \Longrightarrow \mathrm{P_{wave}} = \frac{\mathrm{I_{CD}}}{\eta_{\mathrm{CD}}} \approx 8.89\,\mathrm{MW} \tag{9}$$