Nuclear Fusion and Plasma Physics - Exercises

Prof. A. Fasoli - Swiss Plasma Center / EPFL

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Exercise 1 - Need for additional heating power

For an ITER plasma with minor radius $a = 1\,\mathrm{m}$, major radius $R_0 = 6\,\mathrm{m}$, $B_0 = 5\,\mathrm{T}$, $I_p = 5\,\mathrm{MA}$, and flat profiles of $n = 10^{20}\,\mathrm{m}^{-3}$ and $T_e = T_i = 15\,\mathrm{keV}$:

a) What is the resistance of the plasma ?

Hint: The Spitzer resistivity $\eta_{||} = 1.65 \times 10^{-9} \ln\Lambda / T_e^{3/2} [\Omega\,\mathrm{m}]$ with T_e in keV. Take the Coulomb logarithm $\ln\Lambda = \ln(\lambda_D/r_0)$ to be 17.

b) What is the ohmic power dissipated in this plasma?

c) What is the power lost by the plasma? Assume $\tau_E = 0.45\,\mathrm{s}$

d) What is the fusion power? Assume $\langle\sigma v\rangle|_{T=15\,\mathrm{keV}} = 2 \times 10^{-22}\,\mathrm{m}^3\,\mathrm{s}^{-1}$

e) Are the ohmic and fusion power enough to keep the plasma temperature at 15 keV? If not, how much additional heating power is still needed to equilibrate the power losses in order to keep a steady-state operation?

f) Calculate the physical fusion gain factor $\mathrm{Q_{fus}}$.

Exercise 2 - Efficiency of lower hybrid current drive

Lower hybrid waves are launched with high phase velocity $v_{||}$ parallel to the tokamak magnetic field. High energy electrons (those in the tail of the thermal distribution) can resonate with these waves and be accelerated. The current drive efficiency η_{CD}, i.e. the ratio of the total driven current I_{CD} to the power of the wave P_{wave} needed to drive it, can be evaluated on the basis of very simple qualitative arguments.

a) Show that $\eta_{CD} = I_{CD}/\mathrm{P}_{wave}$ is proportional to $v_{||}^2/(R_0 n_e)$.

b) Estimate η_{CD} for ITER plasmas (Z = 1) assuming $v_{||} \approx 5\,v_{th}$.

c) How much power is needed (order of magnitude) to drive $2\,\mathrm{MA}$ of plasma current in ITER?