Nuclear Fusion and Plasma Physics **EPFL**
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Problem Set 8

Exercise 1 - Alfvèn waves

a) Show that the propagation of a transverse wave along the z axis ($k = k\hat{\mathbf{z}}$) in a string with tension S and mass per unit length M is described by :

$$\frac{\partial^2 y}{\partial z^2} = \frac{M}{S}\frac{\partial^2 y}{\partial t^2}$$

Hint: Consider the tension at two points of the string close to each other in order to derive the tension forces.

b) Considering the ideal MHD model, show that the shear Alfvèn waves propagating along the magnetic field ($\mathbf{k} \parallel \mathbf{B}$, $\mathbf{B} = B_0\hat{\mathbf{z}}$) can be described with the same equation of a transverse wave in a string. Find a correspondence between these results (in the case of Alfvèn waves) and the terms M and S in the equation in (a).

Hint: Start by linearising MHD equations around an equilibrium.

c) The ITER tokamak will operate with a D-T plasma at 13 keV, electron density $n_e = 10^{20}\,\mathrm{m}^{-3}$ (assume that this density is uniform) and a magnetic field $B = 6\,\mathrm{T}$. Evaluate the phase velocity of the Alfvèn waves for that plasma.

d) Fusion reactions D+T $\rightarrow$ He (3.5 MeV)+n(14 MeV) occur when the plasma is heated with ion beams consisting of D at an energy of 1 MeV. Which charged particles can be resonant with the Alfvèn waves (i.e. can have the same phase velocity as the wave)?

Exercise 2 - CMA diagram

The CMA diagram is useful to assess the accessibility of various methods of EC wave heating in tokamaks. The diagram represents an X, Y plane where

$$X = \frac{\omega_p^2}{\omega^2} = \frac{e^2}{\epsilon_0 m_e \omega^2} n_e \quad \text{and} \quad Y = \frac{\Omega_e^2}{\omega^2} = \frac{e^2}{m_e^2\omega^2}B^2$$

As the frequency ω of the wave is fixed by the source, the CMA diagram can be seen as a plot of n_e vs B^2. In this exercise you will draw this diagram and sketch trajectories of EC waves injected perpendicularly in the plasma.

a) Represent the cutoffs and resonances for X mode injection in terms of X and Y and draw them on the CMA diagram.

- Cyclotron resonances: $\omega = n\Omega_e$ where $n = \{1, 2, ...\}$.

- Upper hybrid resonance: $\omega^2 = \omega_p^2 + \Omega_e^2$,
- Cutoff: $(\omega^2 - \omega_R^2)(\omega^2 - \omega_L^2) = 0$ which can be rewritten as $(\omega^2 - \omega_p^2)^2 - (\omega^2 \Omega_e^2) = 0$

b) Since we typically want to heat the plasma center, the injection frequency is chosen as a multiple of the cyclotron frequency Ω_{e0} at the center of the plasma. Sketch the propagation of a wave launched from the low-field side ($B < B_0$) across the plasma to the high field side $B > B_0$. Consider harmonics X1: $\omega = \Omega_{e0}$, X2: $\omega = 2\,\Omega_{e0}$ and X3: $\omega = 3\,\Omega_{e0}$. Remember that the density is highest at the plasma center.

c) On a new diagram, repeat parts a) and b) for O-modes (O1 and O2)

- Cyclotron resonances: Same as X mode.
- Cutoff: $\omega = \omega_p$

d) Based on these CMA diagrams, design two EC heating systems, one for TCV and one for ITER. Take the following constraints into account:

- Toroidal field in ITER: $B = 6\,\mathrm{T}$; in TCV: $B = 1.5\,\mathrm{T}$.
- It is technologically complicated (= expensive) to launch from the high field side in most Tokamaks since the central column and ohmic coils are in the way.
- Existing gyrotron sources of $40 - 140\,\mathrm{GHz}$, $\sim 1\,\mathrm{MW}$ can be bought "off-the-shelf". Higher frequencies need special development and will be more expensive.