Nuclear Fusion and Plasma Physics - Exercises

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Exercise 1 - Alfvèn waves

a) Show that the propagation of a transverse wave along the z axis ($k = k\hat{\mathbf{z}}$) in a string with tension S and mass per unit length M is described by :

$$\frac{\partial^2 y}{\partial z^2} = \frac{M}{S}\frac{\partial^2 y}{\partial t^2}$$

b) Considering the ideal MHD model, show that the shear Alfvèn waves propagating along the magnetic field ($\mathbf{k} \parallel \mathbf{B}$, $\mathbf{B} = B_0\hat{\mathbf{z}}$) can be described with the same equation of a transverse wave in a string. Find a correspondence between these results (in the case of Alfvèn waves) and the terms M and S in the equation in (a).

c) The ITER tokamak will operate with a D-T plasma at 13 keV, electron density $n_e = 10^{20}\,\mathrm{m}^{-3}$ (assume that this density is uniform) and a magnetic field $B = 6\,\mathrm{T}$. Evaluate the phase velocity of the Alfvèn waves for that plasma.

d) Fusion reactions D+T $\rightarrow$ He (3.5 MeV)+n(14 MeV) occur when the plasma is heated with ion beams consisting of D at an energy of 1 MeV. Which charged particles can be resonant with the Alfvèn waves (i.e. can have the same phase velocity as the wave)?

Exercise 2 - CMA diagram

The CMA diagram is useful to assess the accessibility of various methods of EC wave heating in tokamaks. The diagram represents an X, Y plane where

$$X = \frac{\omega_p^2}{\omega^2} = \frac{e^2}{\epsilon_0 m_e \omega^2} n_e \quad \text{and} \quad Y = \frac{\Omega_e^2}{\omega^2} = \frac{e^2}{m_e^2\omega^2} B^2$$

As the frequency of the wave is fixed by the source, the CMA diagram can be seen as a plot of n_e vs B^2. In this exercise you will draw this diagram and sketch trajectories of EC waves injected perpendicularly in the plasma.

a) Represent the cutoffs and resonances for X mode injection in terms of X and Y and draw them on the CMA diagram.

- Cyclotron resonances: $\omega = n\Omega_e$ where $n = \{1, 2, ...\}$.
- Upper hybrid resonance: $\omega^2 = \omega_p^2 + \Omega_e^2$,
- Cutoff: $(\omega^2 - \omega_R^2)(\omega^2 - \omega_L^2) = 0$ which can be rewritten as $(\omega^2 - \omega_p^2)^2 - (\omega^2\Omega_e^2) = 0$

b) Since we typically want to heat the plasma center, the injection frequency is chosen as a multiple of the cyclotron frequency Ω_{e0} at the center of the plasma. Sketch the propagation of a wave launched from the low-field side ($B < B_0$) across the plasma to the high field side $B > B_0$. Consider harmonics X1: $\omega = \Omega_{e0}$, X2:($\omega = 2\,\Omega_{e0}$) and X3 ($\omega = 3\,\Omega_{e0}$). Remember that the density is highest at the plasma center.

c) On a new diagram, repeat parts a) and b) for O-mode (O1 and O2)

- Cyclotron resonances: Same as X mode.
- Cutoff: $\omega = \omega_p$

d) Based on these CMA diagrams, design two EC heating systems, one for TCV and one for ITER. Take the following constraints into account:

- Toroidal field in ITER: $B = 6\,\mathrm{T}$; in TCV: $B = 1.5\,\mathrm{T}$.
- It is technologically complicated (= expensive) to launch from the high field side in most Tokamaks since the central column and ohmic coils are in the way.
- Existing gyrotron sources of $40 - 140\,\mathrm{GHz}$, $\sim 1\,\mathrm{MW}$ can be bought "off-the-shelf". Higher frequencies need special development and will be more expensive.