Nuclear Fusion and Plasma Physics - Exercises

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Solutions to Problem set 7 - 6 November 2023

Exercise 1 - Some TCV parameters

A note on Tokamak jargon: "Toroidal" in a Tokamak refers to the direction around the torus, or in other words, following a path around the central vertical axis. "Poloidal" is everything in the plane perpendicular to the toroidal direction, so effectively cross-sections of the torus. The figure in the exercise is a poloidal cut of TCV. The plasma current and magnetic field in a tokamak are mainly in the Toroidal direction. The coils which generate the toroidal field are therefore actually wound in the poloidal plane, but are called "Toroidal Field coils". The "Poloidal Field coils" and the "Ohmic coils" generate fields in the poloidal plane (vertical field, plasma shaping etc) but are wound in the toroidal direction.

a) Apply Ampère's law to a loop around the central column following the magnetic axis of the plasma:

$$\oint \mathbf{B} \cdot \mathbf{d}\ell = \oint B_T \, d\ell = \mu_0 I$$

where I is the total current passing in all the toroidal coils through the central column.

$$I_0 = 2 \times \pi \times 0.88 \text{ m} \times 1.5 \text{ T}/\mu_0 = 6.6 \text{ MA}$$

This current is divided between 16 coils, with 6 windings each:

$$I_{winding} = 6.6 \text{ MA}/(16 \times 6) = 68 \text{ kA}$$

b) The energy density is

$$u = \frac{(1.5 \text{ T})^2}{2\,\mu_0} = 0.9 \text{ MJ/m}^3$$

The total volume inside the TF coils is approximately

$$V = (2.6 \text{ m} \times 1 \text{ m}) \times (2\pi \times 0.88 \text{ m}) \approx 14 \text{ m}^3$$

Which gives a total energy of approximately $E = 13$ MJ.

c) Assuming that the current ramp is linear, we need $P = E/\Delta t = 13$ MW.

d) Each toroidal coil contains 6 windings, each carrying 68kA. This gives a total current per coil equal to 408kA. The resulting lorentz force on the top and bottom segments of each coil (having length 1m) is

$$\mathbf{F}_{l,i} = L\,\mathbf{I} \times \mathbf{B}_z = \pm\, L\; I\; B_z\; \mathbf{e}_\phi$$

$$F_{l,i} = 1 \text{ m} \times 408 \text{ kA} \times 1 \text{ T} = 408 \text{ kN}$$

in Toroidal direction. Note that since the current is opposite on the top and bottom section, the forces will be opposite.

e) The total twisting moment (along the machine vertical z-axis) on the coil structure can be estimated if we assume the lorentz force is acting at the center of the coil segments, at $r = 0.9$m. The twisting moment at the top (bottom) is

$$\mathbf{T}_{t,b} = \sum_i \mathbf{r}_i \times \mathbf{F}_{l,i} = \pm\, 16\; r\; F_{l,i}\; \mathbf{e}_z$$

The twisting moment is

$$T_{t,b} = 16 \times 0.9 \text{ m} \times 408 \text{ kN} = 5875 \text{ kNm}$$

This is the same moment generated by a 12 ton load (a large truck) held by a crane at 50 m distance!

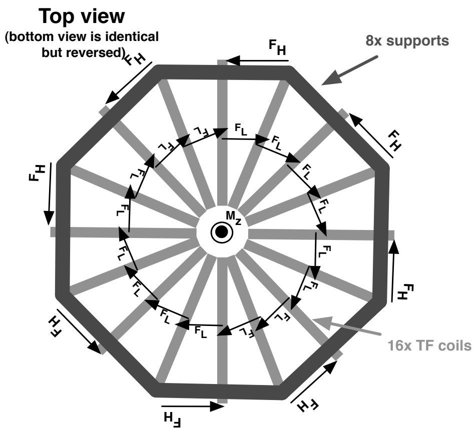

Figure 1: Top view of TF coil support structure

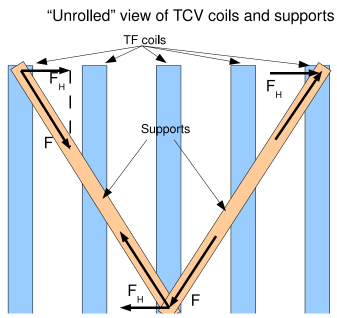

Figure 2: "Unrolled" view of TF coil structure showing 2 out of 8 diagonal supports

The 8 supports are placed at the outer edge of the coils at $R = 1.6$ m. In order to balance the twisting moment, they each have to exert a horizontal force of

$$F_H = \frac{5.8 \times 10^6 \,\text{Nm}}{8 \times 1.6 \,\text{m}} = 459 \text{ kN}$$

to counteract the moment.

Since they are placed at an angle, the force along the axis of the supports will be larger. The horizontal distance between the supports is estimated as

$$2\pi \cdot (1.6 \text{ m})/8 = 1.25 \text{ m}$$

The vertical distance is about 3m. We can use this to calculate the ratio between the total force along the axis of the beam and its horizontal component.

$$\frac{F}{F_H} = \frac{\sqrt{3^2 + 1.25^2}}{1.25}$$

so $F = 2.6 \cdot F_H = 1193$ kN

Finally we can calculate the area of the support required to sustain this force

$$A = \frac{1193\,\text{kN}}{300\,\text{MPa}} = 0.004 \text{ m}^2$$

which gives a radius of 3.5 cm. The diameter of each support should be in the order of 7 cm.