**Nuclear Fusion and Plasma Physics** **EPFL**
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# Solutions to Problem Set 7

## Exercise 1 - Some TCV parameters

**A note on Tokamak jargon**: In a Tokamak, the term “Toroidal” refers to the direction that follows the circumference of the torus, equivalent to encircling the central vertical axis. “Poloidal” encompasses all structures lying in the plane that is perpendicular to the toroidal direction, which effectively represents cross-sections of the torus. The illustration provided in this exercise is a poloidal section of TCV. The plasma current and magnetic field in a tokamak predominantly align with the Toroidal direction. The coils generating the toroidal field are paradoxically situated in the poloidal plane and are known as “Toroidal Field coils”. Conversely, the “Poloidal Field coils” and the “Ohmic coils”, which produce fields in the poloidal plane for vertical field control and plasma shaping, are arranged in the toroidal direction.

a) To apply Ampère’s law around a loop encircling the central column along the magnetic axis of the plasma:

$$\oint \mathbf{B} \cdot \mathbf{d}\ell = \oint B_T \, d\ell = \mu_0 I,$$

where $I$ is the total current flowing through all the toroidal coils around the central column.

$$I_0 = 2\pi \times 0.88 \text{ m} \times 1.5 \text{ T}/\mu_0 = 6.6 \text{ MA},$$

This current is equally distributed among 16 coils, each containing 6 windings:

$$I_{winding} = 6.6 \text{ MA}/(16 \times 6) = 68.75 \text{ kA}.$$

b) The magnetic field’s energy density is given by:

$$u = \frac{(1.5 \text{ T})^2}{2\mu_0} = 0.9 \text{ MJ/m}^3.$$

The total volume enclosed by the TF coils measures:

$$V = (2.6 \text{ m} \times 1 \text{ m}) \times (2\pi \times 0.88 \text{ m}) \approx 14.5 \text{ m}^3,$$

yielding a total energy of about $E = 13$ MJ.

c) Assuming a linear current ramp, the required power to achieve this is:

$$P = E/\Delta t = 13 \text{ MW}.$$

d) Each toroidal coil, containing 6 windings and each winding carrying 68 kA, results in a total current per coil of 408 kA. The resulting Lorentz force on both the top and bottom segments of each coil (each segment being 1 m in length) is:

$$\mathbf{F}_{l,i} = L \times \mathbf{I} \times \mathbf{B}_z = \pm L \times I \times B_z \times \mathbf{e}_\phi,$$

$$F_{l,i} = 1 \text{ m} \times 408 \text{ kA} \times 1 \text{ T} = 408 \text{ kN},$$

oriented in the Toroidal direction. It's important to note that since the current is opposite on the top and bottom sections, the forces will be opposite as well.

e) The total twisting moment (torque) along the machine's vertical $z$-axis on the coil structure can be estimated by assuming the Lorentz force acts at the center of the coil segments, at $r = 0.9$ m. The twisting moment at the top (bottom) is:

$$\mathbf{T}_{t,b} = \sum_i \mathbf{r}_i \times \mathbf{F}_{l,i} = \pm 16 \times r \times F_{l,i} \times \mathbf{e}_z,$$

$$T_{t,b} = 16 \times 0.9 \text{ m} \times 408 \text{ kN} = 5875 \text{ kNm},$$

comparable to the moment generated by a 12-ton load (a large truck) held at a distance of 50 m by a crane.

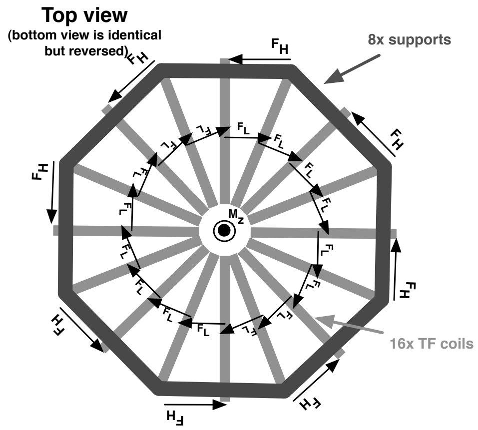


Figure 1: Top view of TF coil support structure

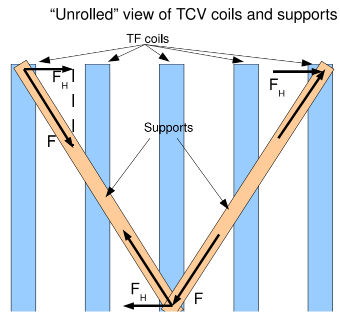


Figure 2: "Unrolled" view of TF coil structure showing 2 out of 8 diagonal supports

f) The eight supports, positioned at the outer edge of the coils at $R = 1.6$ m, must exert a horizontal force to balance the twisting moment:

$$F_H = \frac{5.8 \times 10^6 \text{ Nm}}{8 \times 1.6 \text{ m}} = 459 \text{ kN},$$

given their angular placement, the force along the support's axis is greater. The horizontal distance between supports is:

$$2\pi \cdot (1.6 \text{ m})/8 = 1.25 \text{ m},$$

and the vertical distance is approximately 3 m. This ratio helps calculate the total force along the support's axis relative to its horizontal component:

$$\frac{F}{F_H} = \frac{\sqrt{3^2 + 1.25^2}}{1.25},$$

resulting in $F = 2.6 \times F_H = 1193$ kN.

Finally, the area needed to sustain this force is:

$$A = \frac{1193 \text{ kN}}{300 \text{ MPa}} = 0.004 \text{ m}^2,$$

implying a support radius of 3.5 cm, with a diameter of approximately 7 cm.