Nuclear Fusion and Plasma Physics - Exercises

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Exercise 1 - The frozen flux theorem

The goal of this exercise is to demonstrate the frozen flux theorem: " The total amount of magnetic flux through any closed circuit moving with the plasma fluid velocity is constant"

a) Write the ideal MHD equations. Note that in hot plasmas, the resistivity is so small that we usually take $\eta = 0$.

b) The magnetic flux Φ is the flux of the magnetic field B through a surface that encloses the field lines. Using this definition, find the time derivative of the magnetic flux through a surface S delimited by a contour C.

c) Using Faraday's' and Ohm's laws from the set of equations constituting the ideal MHD model, show that the magnetic flux through any surface moving with the plasma fluid velocity remains constant.

Exercise 2 - Cylindrical equilibrium

A cylindrical plasma column of radius a contains a coaxial magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$ and has a pressure profile

$$p = p_0 \cos^2(\frac{\pi r}{2a})$$

a) Calculate the maximum value of p_0.

b) Using this value of p_0, calculate the diamagnetic current $\mathbf{j}(r)$ and the total field $\mathbf{B}(r)$.

c) Plot $j(r)$, $B(r)$ and $p(r)$ on a graph.