

Problem Set 5**Exercise 1 - Two derivations of the continuity equation**

Derive the continuity equation for a plasma fluid without sources or sinks, using both the Eulerian and Lagrangian approaches. Ensure to include detailed steps in your derivation.

Hint:

- In the Eulerian approach, consider a specific fixed point in space, with a defined volume, through which the plasma fluid moves. The volume element observed remains constant throughout this process.
- In the Lagrangian approach, track the evolution of a specific set of particles that initially occupy a defined volume element. Here, what remains constant is the number of particles in the volume, not the volume itself. In this context, consider the total (or “convective”) derivative.
- Start both derivations with the principle of mass conservation.

Exercise 2 - Magnetic field diffusion in a resistive plasma

- a) Derive the diffusion equation for the magnetic field in a plasma starting from the resistive MHD equations. Be sure to clearly show how the terms in the equations relate to the diffusion process.
- b) Estimate the diffusion time for the magnetic field in the ITER plasma, using the following parameters: characteristic length $L = 3$ m, electron temperature $T_e = 10$ keV, and density $n_e = 10^{20} \text{ m}^{-3}$. Discuss the physical significance of each parameter in your calculation.

Hint:

- The resistive MHD equations are:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\rho \frac{d\mathbf{u}}{dt} = \mathbf{J} \times \mathbf{B} - \nabla p$$

$$\mathbf{E} + \mathbf{u} \times \mathbf{B} = \eta \mathbf{J}$$

$$\frac{d}{dt}(p\rho^{-\gamma}) = 0$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{J} = 0$$

- A diffusion equation typically includes terms of the form $\frac{\partial}{\partial t}()$ and $\nabla^2()$. Clarify how these terms emerge from the MHD equations.
- Useful vector identities include:

$$\nabla \times (A \times B) = A(\nabla \cdot B) - B(\nabla \cdot A) + (B \cdot \nabla)A - (A \cdot \nabla)B$$

$$\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$$