Nuclear Fusion and Plasma Physics **EPFL**
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Problem Set 5

Exercise 1 - Two derivations of the continuity equation

Derive the continuity equation for a plasma fluid without sources or sinks, using both the Eulerian and Lagrangian approaches. Ensure to include detailed steps in your derivation.

Hint:

- In the Eulerian approach, consider a specific fixed point in space, with a defined volume, through which the plasma fluid moves. The volume element observed remains constant throughout this process.
- In the Lagrangian approach, track the evolution of a specific set of particles that initially occupy a defined volume element. Here, what remains constant is the number of particles in the volume, not the volume itself. In this context, consider the total (or "convective") derivative.
- Start both derivations with the principle of mass conservation.

Exercise 2 - Magnetic field diffusion in a resistive plasma

a) Derive the diffusion equation for the magnetic field in a plasma starting from the resistive MHD equations. Be sure to clearly show how the terms in the equations relate to the diffusion process.

b) Estimate the diffusion time for the magnetic field in the ITER plasma, using the following parameters: characteristic length $L = 3$ m, electron temperature $T_e = 10$ keV, and density $n_e = 10^{20}\,\mathrm{m}^{-3}$. Discuss the physical significance of each parameter in your calculation.

Hint:

- The resistive MHD equations are:

$$\begin{aligned}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) &= 0 \\
\rho \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} &= \mathbf{J} \times \mathbf{B} - \nabla p \\
\mathbf{E} + \mathbf{u} \times \mathbf{B} &= \eta \mathbf{J} \\
\frac{\mathrm{d}}{\mathrm{d}t}(p\rho^{-\gamma}) &= 0 \\
\nabla \times \mathbf{B} &= \mu_0 \mathbf{J} \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \cdot \mathbf{J} &= 0
\end{aligned}$$

- A diffusion equation typically includes terms of the form $\frac{\partial}{\partial t}()$ and $\nabla^2()$. Clarify how these terms emerge from the MHD equations.

- Useful vector identities include:

$$\begin{aligned}
&\nabla \times (A \times B) = A(\nabla \cdot B) - B(\nabla \cdot A) + (B \cdot \nabla)A - (A \cdot \nabla)B \\
&\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A
\end{aligned}$$