

# Nuclear Fusion and Plasma Physics - Exercises

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## Exercise 1 - Two derivations of the continuity equation

Derive the (mass) continuity equation for a plasma fluid without sources or sinks, in the two approaches, Eulerian and Lagrangian.

*Hint:*

- In the Eulerian approach, one needs to consider a fixed point in space, with a fixed volume, through which the fluid moves. What remains constant is the observed volume element.
- In the Lagrangian approach, one follows the evolution of a set of particles that initially occupy a volume element. What remains constant is the number of particles in the volume, not the volume itself. In this case one needs to consider the total (or “convective”) derivative.
- In both cases, the starting point is the conservation of mass.

## Exercise 2 - Magnetic field diffusion in a resistive plasma

- a) Starting from the resistive MHD equations, find the diffusion equation for the magnetic field in a plasma.
- b) Estimate the diffusion time of the magnetic field in the ITER plasma (characteristic length  $L = 3$  m, electron temperature  $T_e = 10$  keV and density  $n_e = 10^{20} \text{ m}^{-3}$ ).

*Hint:*

- The resistive MHD equations are:

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) &= 0 \\ \rho \frac{d\mathbf{u}}{dt} &= \mathbf{J} \times \mathbf{B} - \nabla p \\ \mathbf{E} + \mathbf{u} \times \mathbf{B} &= \eta \mathbf{J} \\ \frac{d}{dt}(p\rho^{-\gamma}) &= 0 \\ \nabla \times \mathbf{B} &= \mu_0 \mathbf{J} \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \cdot \mathbf{J} &= 0 \end{aligned}$$

- A diffusion equation is an equation which contains terms of the form  $\frac{\partial}{\partial t}()$  and  $\nabla^2()$ .
- You may find the following vector identities useful:

$$\begin{aligned} \nabla \times (A \times B) &= A(\nabla \cdot B) - B(\nabla \cdot A) + (B \cdot \nabla)A - (A \cdot \nabla)B \\ \nabla \times (\nabla \times A) &= \nabla(\nabla \cdot A) - \nabla^2 A \end{aligned}$$