

Problem Set 4

Exercise 1 - Small vs. large collision angles

- a) For a deuterium plasma with $T_e = T_i = 10 \text{ keV}$ and $n = 10^{20} \text{ m}^{-3}$, compare the momentum transfer cross-section of electron-ion collisions with small deflection angles to the cross-section of collisions with a deflection angle $\geq 90^\circ$.
- b) Can we assert that in plasmas used for thermonuclear applications most of the collisions have a small deflection angle?

Exercise 2 - Alpha particle thermalisation in a burning plasma

Consider the relaxation process of *alpha* particles (α 's) at 3.5 MeV created by fusion reactions in a deuterium-tritium plasma (50:50 D-T). Evaluate the time-scale for the energy loss of α 's in a plasma with $n_e = 10^{20} \text{ m}^{-3}$. Consider the collisions between three plasma species, assuming $T_e = T_D = T_T = 10 \text{ keV}$.

- a) Which species is the most important in the thermalization process of the α 's?
- b) Which species is heated more by α particles?

Hint: Start with a thermal energy for the α 's of 3.5 MeV and then consider the different regimes corresponding to the different energies of the α 's during thermalisation. The general form of ν_{EK} of collisions of particles of species j (projectiles) upon particles of species k (targets), assuming the targets are immobile, is $\nu_{EK}^{j/k} \sim n_k \frac{Z_k^2 Z_j^2 e^4}{2\pi\epsilon_0^2} \frac{\ln \Lambda_k}{m_j m_k v_j^3}$

Exercise 3 - Runaway electrons

Consider typical parameters for the ITER tokamak: $T_e = 15 \text{ keV}$, $I_p = 15 \text{ MA}$, minor radius $a = 2 \text{ m}$, $R_0 = 5.3 \text{ m}$, $n_e = 10^{20} \text{ m}^{-3}$, and a deuterium-tritium plasma (50:50 D-T).

- a) Using the *Spitzer* formula for the resistivity

$$\eta = \frac{5.1 \times 10^{-5} Z \ln \Lambda}{(T_e [\text{eV}])^{3/2}}$$

and assuming a uniform temperature and resistivity over the entire plasma, calculate the loop voltage necessary to inductively drive the plasma current.

Hint: Use potential as the voltage difference between the extremities of the cylinder.

- b) Consider the electrons in the “*tail*” of the distribution function ($v \gg v_{the}$). The collision frequency for these electrons is:

$$\nu_{se} = \nu_s^{e/e'} + \nu_s^{e/i} = (2 + Z) \frac{n_e e^4}{4\pi\epsilon_0^2} \frac{\ln \Lambda}{m_e^2 v^3}$$

Prove that these energetic electrons can be continuously accelerated (*run-away* regime) if their energy is higher than a critical value corresponding to a critical electric field:

$$\frac{1}{2} m_e v^2 > T_e \frac{E_{cr}}{E}$$

Find an expression for E_{cr} and estimate the critical kinetic energy for the electric field present in ITER.