Nuclear Fusion and Plasma Physics - Exercises

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Problem Set 4 - 16 October 2023

Exercise 1 - Small vs. large collision angles

a) For a deuterium plasma with $T_e = T_i = 10\,\mathrm{keV}$ and $n = 10^{20}\,\mathrm{m}^{-3}$, compare the momentum transfer cross-section of electron-ion collisions with small deflection angles to the cross-section of collisions with a deflection angle $\geq 90^o$.

b) Can we assert that in plasmas used for thermonuclear applications most of the collisions have a small deflection angle?

Exercise 2 - Alpha particle thermalisation in a burning plasma

Consider the relaxation process of *alpha* particles (α's) at $3.5\,\mathrm{MeV}$ created by fusion reactions in a deuterium-tritium plasma (50:50 D-T). Evaluate the time-scale for the energy loss of α's in a plasma with $n_e = 10^{20}\,\mathrm{m}^{-3}$. Consider the collisions between three plasma species, assuming $T_e = T_D = T_T = 10\,\mathrm{keV}$.

a) Which species is the most important in the thermalization process of the α's?

b) Which species is heated more by α particles?

Hint: Start with a thermal energy for the α's of $3.5\,\mathrm{MeV}$ and then consider the different regimes corresponding to the different energies of the α's during thermalization. The general form of ν_{E_K} of collisions of particles of species j (projectiles) upon particles of species k (targets), assuming the targets are immobile, is $\nu_{E_K}^{j/k} \sim n_k \frac{Z_k^2 Z_j^2 e^4}{2\pi\epsilon_0^2} \frac{\ln\Lambda_k}{m_j m_k v_j^3}$

Exercise 3 - Runaway electrons

Consider typical parameters for the ITER tokamak $T_e = 15\,\mathrm{keV}$, $I_p = 15\,\mathrm{MA}$, $a = 2\,\mathrm{m}$, $R_0 = 5.3\,\mathrm{m}$, $n_e = 10^{20}\,\mathrm{m}^{-3}$, (D:T) = (50:50).

a) Using the *Spitzer* formula for the resistivity

$$\eta = \frac{5.1 \times 10^{-5}\ Z\ \ln \Lambda}{(T_e[\mathrm{eV}])^{3/2}}$$

and assuming a uniform temperature and resistivity over the entire plasma, calculate the loop voltage necessary to inductively drive the plasma current.

b) Consider the electrons in the *"tail"* of the distribution function ($v >> v_{the}$). The collision frequency for these electrons is:

$$\nu_{se} = \nu_s^{e/e'} + \nu_s^{e/i} = (2+Z)\frac{n_e e^4}{2\pi\epsilon_0^2}\frac{\ln\Lambda}{m_e^2 v^3}$$

Prove that these energetic electrons can be continuously accelerated (*run-away* regime) if their energy is higher then a critical value corresponding to a critical electric field:

$$\frac{1}{2}m_e v^2 > T_e\frac{E_{cr}}{E}$$

Find an expression for E_{cr} and estimate the critical energy for the electric field present in ITER.