Nuclear Fusion and Plasma Physics

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Collisional processes in plasmas

- Basic concept of collisions
  - Coulomb collisions as elastic collisions - main properties
- Multiple collisions in plasmas
  - Rutherford differential cross-section and the small angle approximation
  - Integration over the impact parameter and Coulomb logarithm
- Effective collision frequencies and cross-sections
  - For the exchange of energy and momentum
  - Average over a distribution function
- Relaxation processes and relevant time scales
- Application of collision theory
  - Plasma resistivity
  - Run-away process

1 Basic concept of collisions

Which collisions occur in plasmas for fusion?

- collisions between charged particles and neutrals
- collisions between charged particles and charged particles (Coulomb collisions)

Note 4.1.1:

- To call the interaction between charged particles a "collision" is in fact an approximation. We know that charged particles interact with each other in large numbers (within the Debye sphere). But we *assume* that such interactions can be approximated by a sequence of *binary interactions*.
- We also assume that Coulomb collisions are elastic, meaning that we neglect bremsstrahlung radiation (as $\frac{W_{rad}}{\frac{1}{2}mv^2} \sim \left(\frac{v}{c}\right)^3 \ll 1$).

Fusion plasmas are in general strongly ionized, in the sense that Coulomb collisions dominate over all other kinds of collisions. This situation is described by $\lambda^{\text{mfp}}_{\text{Coulomb}} < \lambda^{\text{mfp}}_{\text{other collisions}}$. Equivalently, Coulomb collisions are more frequent than other types of collisions.

1.1 Theory of Coulomb collisions

We begin by studying the collision between two particles of charges q_1 and q_2 and masses m_1 and m_2 that interact through the Coulomb force. Noting their positions $\mathbf{r}_1$ and $\mathbf{r}_2$ respectively, we can define the center of mass as

$$\mathbf{R} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2}$$

as well as the relative position $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and the reduced mass, $\mu = \frac{m_1 m_2}{m_1 + m_2}$. By writing the equations of motion with respect to these new variables, it appears that the collision can be treated by studying the motion of a test particle q_1 of mass μ in the Coulomb potential of a fixed charge q_2. This is then the classical problem of a particle in a central potential. The situation is presented in Fig. 1.

We assume that the collisions are elastic, and note that the Coulomb force is radial. Thus, $\mathbf{p}$ and E are conserved, the motion is in a single plane and the angular momentum is also conserved.

The conservation laws give:

$$\tan\frac{\theta}{2} = \frac{b_{90}}{b} \qquad \text{with } b_{90} = b_{90}(v) = \frac{q_1 q_2}{4\pi\epsilon_0 \mu v^2}, \tag{1.1}$$

where v is the relative velocity, and b_{90} is the impact parameter that results in a diffusion at an angle of $\theta = 90°$ ($\pi/2$ radians). This expression is valid in the center of mass frame. Note that θ is the angle of deflection only if the target particle is fixed, otherwise one needs to return to the reference frame of the lab in order to calculate this.

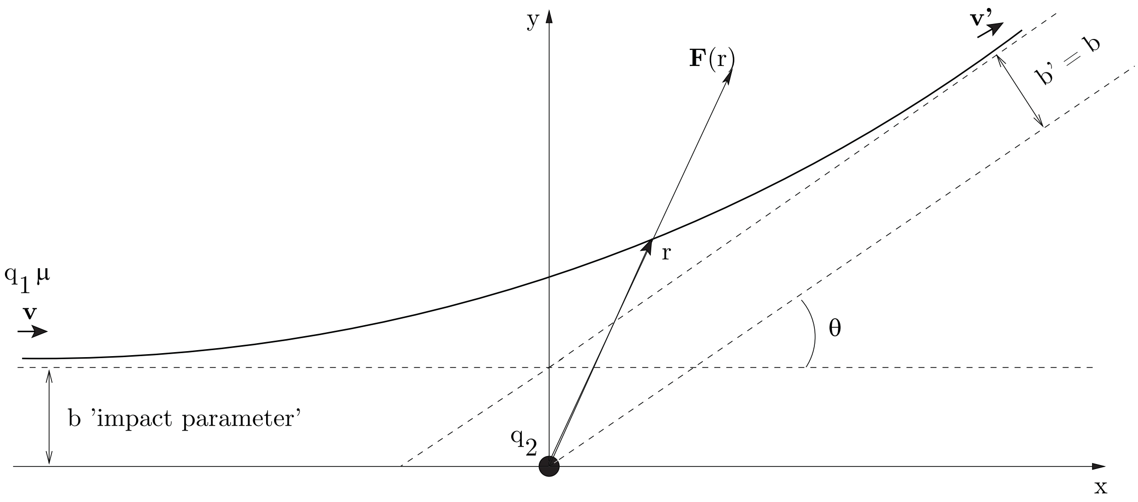

Figure 1: Geometry for one collision and definition of the impact parameter b.

1.2 Effect of multiple collisions

We need to look at the different deflection angles/impact parameters.

Rutherford differential cross-section

Rather than knowing the impact parameter b exactly, we suppose that the test particle is incident in a surface $\mathrm{d}S = 2\pi b\,\mathrm{d}b$. The solid angle $\mathrm{d}\Omega$ is given by $\frac{\mathrm{d(area)}}{r^2}$. The geometry gives

$$\begin{aligned} \mathrm{d}\sigma &= 2\pi b|\mathrm{d}b| \\ \mathrm{d}\Omega &= \frac{2\pi r\sin\theta r|\mathrm{d}\theta|}{r^2} = 2\pi\sin\theta|\mathrm{d}\theta| \end{aligned}$$

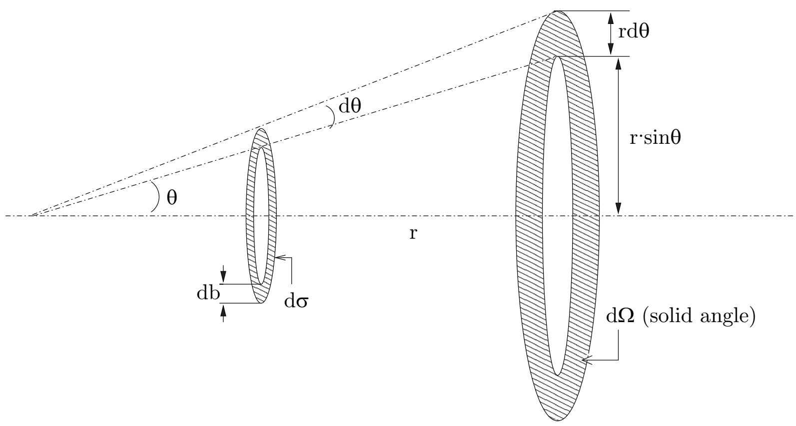

Figure 2: Definition of the differential cross-section.

Definition: The **differential cross-section** $\frac{d\sigma}{d\Omega}$ is defined such that

$$n_{\text{targets}} \left(\frac{d\sigma}{d\Omega}\right) d\Omega \quad \Longleftrightarrow \quad \text{the number of particles per unit path length scattered into solid angle } d\Omega.$$

In other words, the differential cross-section is defined by the probability $\left(\frac{d\sigma}{d\Omega}\right) d\Omega$ that the test particle diffuses into a solid angle $d\Omega$.

By using Eq. 1.1, Rutherford's cross-section can be expressed as:

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \frac{2\pi b|db|}{2\pi \sin\theta |d\theta|} = -\frac{b}{\sin\theta}\frac{db}{d\theta} = \dots = \frac{b_{90}^2}{4}\frac{1}{\sin^4(\theta/2)} \\ &= \frac{q_1^2 q_2^2}{(4\pi\varepsilon_0)^2 \mu^2 v^4}\frac{1}{4\sin^4(\theta/2)}. \end{aligned} \tag{1.2}$$

Example application: To get the cross-section for collisions with angles of deflection larger than 90°, we could calculate:

$$\sigma(\theta \geq \frac{\pi}{2}) = \int_{\theta=\pi/2}^{\theta=\pi} \frac{d\sigma}{d\Omega} d\Omega(\theta) = \frac{b_{90}^2}{4} \int_{\pi/2}^{\pi} \frac{2\pi \sin\theta d\theta}{\sin^4\frac{\theta}{2}} = \pi b_{90}^2,$$

as expected from the definition/meaning of b_{90}.

Note 4.1.2:

- $\frac{d\sigma}{d\Omega} \propto v^{-4}$: as the collision rate goes like $v\frac{d\sigma}{d\Omega} \propto v^{-3}$, it scales like $T^{-3/2}$ (the *hotter* the plasma, the *less* collisional).
- $\frac{d\sigma}{d\Omega} \propto \frac{1}{\sin^4\frac{\theta}{2}}$ for small angles $\frac{d\sigma}{d\Omega} \propto \theta^{-4}$. The small angle collisions dominate. This means that large diffusions are generally the result of several diffusions at small angles.

From now on, we will consider *only* small angle collisions.

Integration over all possible impact parameters

Energy transfer rate

Noting $E_K = \frac{1}{2}m_1 v_1^2$, the energy exchanged over the collision is:

$$\Delta E_K = E_K \frac{m_1 m_2}{(m_1 + m_2)^2}\theta^2$$

We have assumed that $v_2 = 0$. But for small θ, $\tan\frac{\theta}{2} \sim \frac{\theta}{2} \cong \frac{b_{90}}{b}$. So the energy lost in *one* collision is given by

$$\Delta E_K = E_K \frac{m_1 m_2}{(m_1+m_2)^2}\left(\frac{2b_{90}}{b}\right)^2$$

Per unit path length, for impact parameter in interval $\mathrm{d}b$

$$\left.\frac{\mathrm{d}E_K}{\mathrm{d}l}\right|_{\mathrm{d}b} = \Delta E_K n \mathrm{d}\sigma$$

where $n\mathrm{d}\sigma = \#$ of collisions per unit length. So for all b's,

$$\begin{aligned}\left.\frac{\mathrm{d}E_K}{\mathrm{d}l}\right|_{\text{all } b\text{'s}} &= \int_{b_{min}}^{b_{max}} \left.\frac{\mathrm{d}E_K}{\mathrm{d}l}\right|_{\mathrm{d}b} = \int_{b_{min}}^{b_{max}} \Delta E_K n\mathrm{d}\sigma = \int_{b_{min}}^{b_{max}} E_K \frac{m_1 m_2}{(m_1+m_2)^2}\frac{4b_{90}^2}{b^2} 2\pi n b \mathrm{d}b \\ &= 8\pi \frac{m_1 m_2}{(m_1+m_2)^2} E_K n b_{90}^2 \int_{b_{min}}^{b_{max}} \frac{\mathrm{d}b}{b}.\end{aligned}$$

Discussion: What are b_{min} and b_{max}?

- b_{min} : We are considering only small angles. For $b < b_{90}$ the assumption of small angles would be violated (Eq. 1.1). Thus, $b_{min} \simeq b_{90}$. Note that at very high T_e, b_{90} becomes so small that quantum mechanical corrections must be included. In such cases one can take $b_{min} \simeq \lambda_{\text{DeBroglie}} = h/mv$.
- b_{max} : Remember the Debye screening effect: outside the Debye sphere, the potential is screened, so the "collision" does not "occur". As a result, $b_{max} \simeq \lambda_D$.

Thus, after integration:

$$\boxed{\frac{\mathrm{d}E_K}{\mathrm{d}l} = E_K n 8\pi b_{90}^2 \frac{m_1 m_2}{(m_1+m_2)^2}\ln\Lambda} \tag{1.3}$$

where $\ln\Lambda$ is the so called *Coulomb logarithm* and

$$\boxed{\Lambda = \frac{\lambda_D}{b_{90}}.} \tag{1.4}$$

Note that because of the very weak logarithmic dependence, the *exact* choice of b_{min}, b_{max} is arbitrary.

2 Effective collision frequency for relaxation processes

For the exchange of the quantities of interest, the effective collision frequency is given by

$$\text{Effective collision frequency} = \frac{1}{\text{characteristic time}}$$

The characteristic time τ is the time between collisions; thus noting ν the frequency of collisions, $\nu = \frac{1}{\tau}$. The effective collision frequency for the exchange of energy is,

$$\nu_{E_K} = \frac{1}{E_K}\frac{\mathrm{d}E_K}{\mathrm{d}t} \underset{v=\frac{\mathrm{d}l}{\mathrm{d}t}}{=} \frac{1}{E_K} v \frac{\mathrm{d}E_K}{\mathrm{d}l} = 8\pi n \frac{q_1^2 q_2^2}{(4\pi\varepsilon_0)^2}\frac{\ln\Lambda}{m_1 m_2 v^3}$$

As $v = \frac{\lambda}{\tau}$, $\nu = n\sigma v$. So the effective cross section is given by $\sigma_{E_K} = \frac{\nu_{E_K}}{nv}$.

Exchange of momentum

From the theory of binary collisions, we have

$$\sigma_p = \sigma_{E_K}\frac{m_1+m_2}{2m_1} = \frac{1}{2}\sigma_{E_K}\left(1+\frac{m_2}{m_1}\right) = \begin{cases} \frac{1}{2}\sigma_{E_K} & \text{if } m_2 \ll m_1, \\ \sigma_{E_K} & \text{if } m_2 = m_1, \\ \frac{1}{2}\sigma_{E_K}\frac{m_2}{m_1} \gg \sigma_{E_K} & \text{if } m_2 \gg m_1. \end{cases}$$

The typical case of electrons impinging on ions is characterized for example by $m_2 >> m_1$.

The general form of ν_{E_K} for collisions of particles of species j (projectiles) upon particles of species k (targets) is

$$\boxed{\nu_{E_K}^{j/k} \sim n_k \frac{Z_k^2 Z_j^2 e^4}{2\pi\varepsilon_0^2}\frac{\ln\Lambda_k}{m_j m_k v_{jk}^3}} \tag{2.1}$$

<u>Note 4.2.1</u>:

- $v_{jk} = \left|\vec{v}_j - \vec{v}_k\right|$ is the relative velocity
- $\ln\Lambda_k$ can be considered $\sim$ constant (for example, $\ln\Lambda_e \sim \ln\Lambda_i$). Typical values of $\ln\Lambda$ range between 10 and 20 for plasmas of interest.

From a single velocity to a full distribution

We still need one conceptual step to describe relaxation processes for a whole plasma: to go from a single velocity/energy to a full distribution. For this, we need to average the physical quantity of interest (e.g. the loss/exchange rate of momentum) over a distribution function. But which distribution should we consider? Experiments suggest a Maxwellian for the electrons ($f_e(\mathbf{v}) \sim e^{\frac{mv^2}{2k_bT_e}}$), even in cases for which we do not expect to reach an equilibrium. So, we go from $\nu_{E_K}(v)$ to $\bar{\nu}_{E_K}(v)$ or $\nu_p(v)$ to $\bar{\nu}_p(T)$.
Instead of doing the calculation, we could guess, for example for $\bar{\nu}_p^{e/i}$ that $\bar{\nu}_p^{e/i} = \nu_p^{e/i}(v_{th,e})$, where $v_{th,e} = \sqrt{\frac{T_e}{m_e}}$. In fact, we would not be too wrong. The full calculation gives

$$\bar{\nu}_p^{e/i} = \frac{1}{\langle|\mathbf{p}|\rangle_{t=0}}\left\langle\frac{\mathrm{d}|\mathbf{p}|}{\mathrm{d}t}\right\rangle = \frac{1}{\langle|\mathbf{p}|\rangle_{t=0}}\frac{1}{n_e}\int \mathrm{d}\mathbf{v}\ f_e(\mathbf{v})\underbrace{m_e\mathbf{v}\nu_p^{e/i}}_{\frac{\mathrm{d}|\mathrm{p}|}{\mathrm{d}t}} =$$

$$\cdots = \frac{1}{3}\sqrt{\frac{2}{\pi}}\nu_p^{e/i}(v_{th,e}) \cong 0.26\nu_p^{e/i}(v_{th,e}).$$

Summary of average collision frequencies

The table below presents the collision frequencies of collisions between different species (electrons and ions), for the exchange of momentum and energy.

	Momentum	Energy
$e \to i$	$\nu_e \equiv \bar{\nu}_p^{e/i} = \frac{1}{3}\sqrt{\frac{2}{\pi}}\nu_p^{e/i}(v_{th,e}) = \frac{1}{3}\sqrt{\frac{2}{\pi}}\frac{n_i Z^2 e^4 \ln\Lambda}{4\pi\varepsilon_0^2 m_e^{1/2} T_e^{3/2}}$	$\bar{\nu}_{E_K}^{e/i} = 2\frac{m_e}{m_i}\nu_e$
$e \to e$	$\bar{\nu}_p^{e/e} \cong \frac{1}{\sqrt{2}}\nu_e$	$\bar{\nu}_{E_K}^{e/e} = \bar{\nu}_p^{e/e}$
$i \to e$	$\bar{\nu}_p^{i/e} \cong \frac{m_e}{m_i}\nu_e$	$\bar{\nu}_{E_K}^{i/e} \cong 2\bar{\nu}_p^{i/e} \cong \bar{\nu}_{E_K}^{e/i}$
$i \to i$	$\nu_i \equiv \bar{\nu}_p^{i/i} = \frac{1}{\sqrt{2}}\left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{T_e}{T_i}\right)^{3/2}\nu_e$	$\bar{\nu}_{E_K}^{i/i} = \nu_i$

Note 4.2.2:

- We could proportionally relate all frequencies to the e/i case.
- For $T_e = T_i$ and $Z = 1$, we have

$$\frac{\nu_e}{\nu_i} = \sqrt{\frac{2m_i}{m_e}}.$$

Characteristic time scales

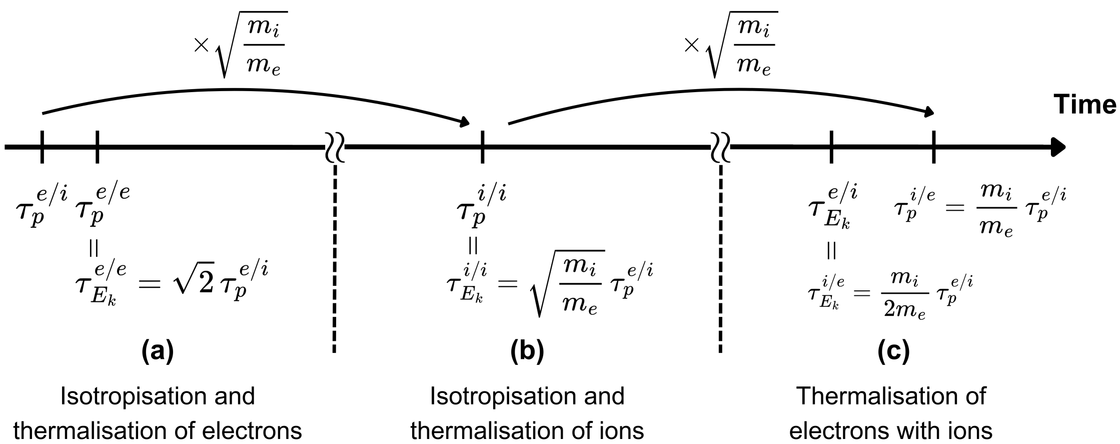

Figure 3: Characteristic time scales for thermalisation and isotropisation in plasmas.

Note 4.2.3:

(a) This time scale corresponds to isotropisation and thermalisation of electrons ($\rightarrow T_e$).

(b) This time scale corresponds to isotropisation and thermalisation of ions ($\rightarrow T_i$).

(c) This time scale corresponds to the thermalisation of electrons with ions ($T_e, T_i \rightarrow T$).

Numerical examples

- Hydrogen-plasma ($Z = 1$):

$$\nu_e \cong 5 \times 10^{-11} \frac{n_{[\mathrm{m}^{-3}]}}{T_{e\,[\mathrm{eV}]}^{3/2}} \mathrm{s}^{-1} \qquad \text{and} \qquad \nu_i \cong 10^{-12} \frac{n_{[\mathrm{m}^{-3}]}}{T_{i\,[\mathrm{eV}]}^{3/2}} \mathrm{s}^{-1}.$$

- For "ITER-like" plasma: $T_e = T_i = 15\mathrm{keV}$, $n = 3 \times 10^{20} \mathrm{m}^{-3}$ so

$$\nu_e \sim 8 \times 10^3 \mathrm{s}^{-1} \rightarrow \tau_e \cong 0.1\mathrm{ms} \qquad \text{and} \qquad \nu_i \cong 160\mathrm{s}^{-1} \rightarrow \tau_i \cong 6\mathrm{ms}.$$

  But, $\tau_{\text{thermal equilibium } e \leftrightarrow i} \sim \frac{m_i}{m_e} \tau_e = 1840 \times 0.1\mathrm{ms} \sim 0.2\mathrm{s}$

Note 4.2.4:

- This large difference in time-scales means that for "not so slow" phenomena we should treat the plasma as made of different species with independent equilibria and, in general, different temperatures as well. For slow phenomena we could also treat the plasma as a single fluid.
- For $T \sim 10 - 20$ keV, $\sigma_{Coul} \gg \sigma_{fusion}$: particles are confined for many collision times before they fuse.
- $\nu_e, \nu_i \ll \Omega_e, \Omega_i$: dynamics is still dominated by Larmor (or drift) motion.

3 Plasma resistivity and run-away process

Take a fully ionised plasma to which we apply an external electric field $\mathbf{E}$. Electrons and ions will be accelerated in opposite directions, but will also be subject to a *friction force* due to Coulomb collisions. This friction force is responsible for the finite *resistivity* of the plasma. In order to calculate it, we assume:

- Only electrons carry currents[1]
- Only $e \rightarrow i$ collisions occur (ignore $e \rightarrow e$)

[1] $m_e \ll m_i$; for similar energies $\longrightarrow |v_i| \ll |v_e|$

- The distribution of electrons remains Maxwellian with a drift v_{d}

The momentum equation[2] parallel to **E** (and **B**, or with $\mathbf{B} = 0$) can be written in scalar form:

$$m_e \frac{dv_{\mathrm{d}}}{dt} = \underbrace{-eE}_{\text{acceleration}} - \underbrace{\frac{m_e v_{\mathrm{d}}}{\tau_p^{e/i}(v)}}_{\text{deceleration}} \tag{3.1}$$

Note that for electrons the directions of $\mathbf{v}_{\mathrm{d}}$ and **E** are opposite. To solve Eq. 3.1 we need to evaluate $\tau_p^{e/i}$; but for which velocity? Two cases can be distinguished:

1. $v_{\mathrm{d}} \ll v_{\mathrm{th}e}$
2. $v_{\mathrm{d}} \geq v_{\mathrm{th}e}$

Case $\mathbf{v}_{\mathrm{d}} \ll \mathbf{v}_{\mathrm{the}}$

In this case, the velocity that dominates in the definition of the relative velocity in the collision corresponds to the electron thermal motion and does not depend on v_{d}. We have a steady–state solution ($\frac{d}{dt} = 0$), in which the acceleration due to the electric field is balanced by the collisional drag exerted by the ions:

$$\tau_p^{e/i} eE = -m_e v_{\mathrm{d}} \quad \Rightarrow \quad v_{\mathrm{d}}^{\mathrm{terminal}} = -\frac{\tau_p^{e/i} eE}{m_e}. \tag{3.2}$$

As the current $j = -en_e v_{\mathrm{d}}$, the previous equation can be recast as

$$\tau_p^{e/i} eE = \frac{m_e j}{en_e} \quad \text{or} \quad j = \frac{e^2 n_e}{m_e \bar{\nu}_p^{e/i}} E. \tag{3.3}$$

With the definition of the resistivity η, $j = \eta^{-1} E$, we find

$$\eta = \frac{m_e \bar{\nu}_p^{e/i}}{e^2 n_e} = \frac{m_e}{e^2 n_e} \frac{1}{3} \sqrt{\frac{2}{\pi}} \frac{(n_i Z) Z e^4 \ln \Lambda}{4\pi \varepsilon_0^2 m_e^{1/2} T_e^{3/2}} = \frac{\sqrt{2}}{\pi^{3/2}} \frac{m_e^{1/2} Z e^2 \ln \Lambda}{12 \varepsilon_0^2 T_e^{3/2}}. \tag{3.4}$$

We observe that:

- There is no dependence on the plasma density. In fact, increasing the density, increases both the number of carriers and the number of collisions, so the two effects cancel out.
- $\eta \propto T_e^{-3/2}$. For a metal, $\eta \propto T_e^{\alpha}$, with $\alpha > 0$: very different!
- Our simple calculation *over*–estimates η by a factor of 2 because we did not account for the acceleration of electrons by **E**: faster electrons are less subject to collisions and carry more current.

[2] Note that we need to consider *momentum* exchange collisions, as we have to do with directed velocity.

- From more complete calculations:

$$\eta \quad [\Omega \text{m}] = \frac{Ze^2\sqrt{m_e}\ln\Lambda}{4\pi\epsilon_0^2 3\sqrt{2\pi}T_e^{3/2}} = 5.1\cdot 10^{-5}\times\frac{Z\ln\Lambda}{(T_e[\text{eV}])^{3/2}} \tag{3.5}$$

This is known as the Spitzer resistivity. The estimated value from this equation agrees reasonably well with experiments.

Examples:

1. Plasma at 100 eV: $\eta \sim 6\cdot 10^{-7}$ Ωm [$\sim \eta$ of stainless steel]
2. Plasma at 1 keV: $\eta \sim 2\cdot 10^{-8}$ Ωm [$\sim \eta$ of copper][3]
3. For $T \gg 1$ keV plasma becomes almost a perfect conductor

The decrease of the resistivity with temperature has two consequences:

1. The magnetic flux becomes 'frozen' within the plasma – a general property of superconductors[4]
2. Heating by current ('ohmic heating') becomes less and less effective at high T_e. The increase in energy per unit volume is

$$\frac{\text{Power}}{\text{Volume}} = \text{force}\times\text{velocity}\times\text{density} = e|E|\times v_\text{d}\times n = \eta j^2 \propto T_e^{-3/2}. \tag{3.6}$$

Note that in the presence of **B** (with **B**||**E**), we would have $\eta_{||}\approx\eta$ and $\eta_\perp > \eta$: particles move preferentially along the magnetic field lines, therefore the resistivity in this direction is smaller than in the direction perpendicular to **B**.

Case $\mathbf{v}_\text{d} \gtrsim \mathbf{v}_\text{the}$

If the field E is sufficiently high such the the relative drift speed becomes not much smaller than the electron thermal speed, $\tau_p^{e/i}$ cannot be considered independent of v_d and we do not necessarily reach a steady–state solution. In this case we cannot take the value of $\nu_p^{e/i}$ averaged over a Maxwellian distribution, but we need to retain the velocity dependent expression of $\nu_p^{e/i}(v_\text{d})$ *and* the time derivative d/dt.

Thus

$$m_e\frac{dv_\text{d}}{dt} = -eE - \nu_p^{e/i}(v_\text{d})m_e v_\text{d}. \tag{3.7}$$

The key question is the sign of the term on the right hand side. For

$$e|E| > \nu_p^{e/i} m_e v_\text{d} \tag{3.8}$$

we have acceleration, otherwise deceleration. If we have *acceleration*, an increase in v_d leads to a decrease in $\nu_p^{e/i}$. Then there is even more acceleration and so on. This is called the

[4]e.g. solar wind carrying B–field with it.

run-away regime: electrons with sufficiently high velocity are accelerated more and more by E as the collisional drag due to the friction force is insufficient to balance the acceleration given by the electric field (Fig. 4).

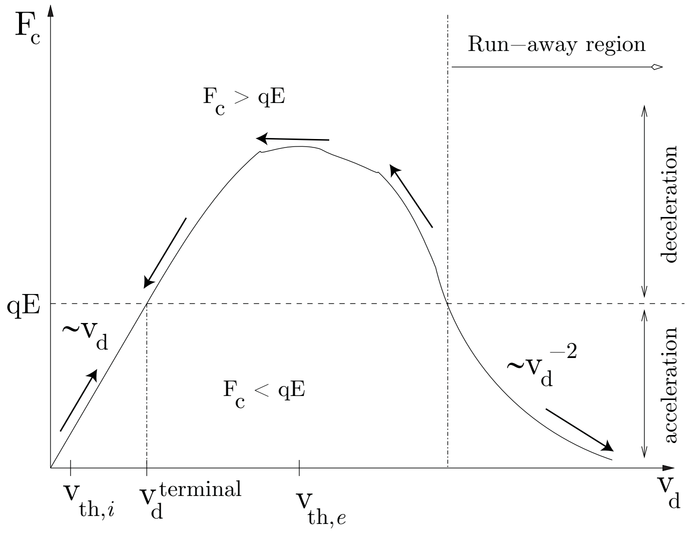

Figure 4: Sketch of the collisional drag F_c acting on electrons as a function of their velocity v_d for $E > E_D$. The black arrows indicate the overall acceleration or deceleration.

By expressing $\nu_p^{e/i}$ in terms of v_d, $\nu_p^{e/i} = \nu_p^{e/i}(v_\mathrm{d})$ we have[5]

$$e|E| > \frac{(n_i Z)Ze^4}{4\pi\epsilon_0^2}\frac{\ln\Lambda}{m_e^2 v_\mathrm{d}^3} m_e v_\mathrm{d}$$

which implies

$$|E| > \frac{n_e Z e^3 \ln\Lambda}{4\pi\varepsilon_0^2 m_e v_\mathrm{d}^2} \qquad \text{or} \qquad \frac{1}{2} m_e v_\mathrm{d}^2 > \frac{n_e Z e^3 \ln\Lambda}{8\pi\varepsilon_0^2 |E|} \tag{3.9}$$

Let's divide by T_e:

$$\frac{1}{2}\frac{m_e v_d^2}{T_e} > \frac{E_D}{|E|} \tag{3.10}$$

where we have introduced the Dreicer electric field $E_D := \frac{n_e Z e^3 \ln\Lambda}{8\pi\varepsilon_0^2 T_e}$.

The meaning of this equation is that for $|E| = E_D$, the run-away regime is reached at

$$E_\mathrm{drift} = \frac{1}{2} m_e v_\mathrm{d}^2 = T_e$$

.

The production of run-away electrons is a serious problem in tokamaks. For typical parameters of fusion plasmas the Dreicer field can be as low as 1 V/m. The probability of generating run-away electrons is then quite high, and these electrons can reach energies of the order of a few MeV. If their number is sufficiently high they give rise to 'electron beams' that are no longer confined inside the plasma. In fact, they are thought to be responsible for damage to the vacuum vessel walls and to other components installed inside the vessel following loss of

[5] We have done this calculation in the exercise session

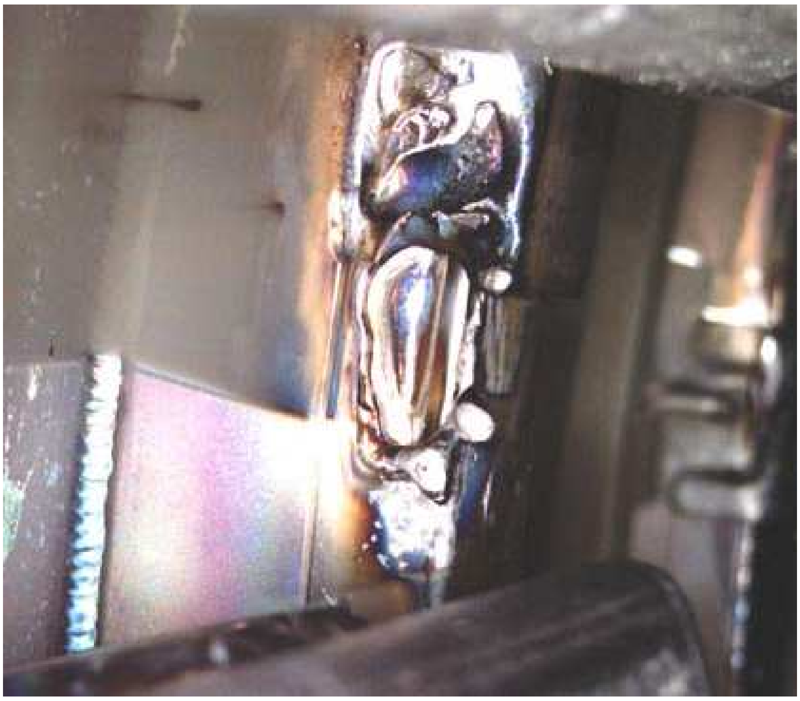

Figure 5: Melting damage to the upper inner wall of JET, thought to be caused by run-away electrons.

confinement of the runaway electron population and their collisions with the wall (Fig. 5).

One of the problems to be solved for ITER is how to avoid, or mitigate, the generation of run-away electrons following a plasma disruption (a sudden loss of current, hence of confinement).

Once an electron exceeds the critical velocity (Eq. 3.10), it is continuously accelerated and can reach energies of several tens of MeV. Because of the toroidal acceleration, electrons emit synchrotron radiation. A relativistic limit to the maximum energy an electron can reach is given by a balance between the amount of power that is absorbed from the accelerating electric field and the amount of power lost by electromagnetic radiation.