Nuclear Fusion and Plasma Physics
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Problem Set 3

Exercise 1 - Plasma production

Semiconductor manufacturers commonly use plasmas for the surface treatment of materials. In a vacuum chamber with dimensions $0.5\,\text{m} \times 0.5\,\text{m} \times 0.5\,\text{m}$, an inert gas such as Argon is partially ionized by radio waves. Consider the case where the gas used is Argon, with a pressure $p = 10^{-4}\,\text{torr}$, electron density $n_e = 10^{16}\,\text{m}^{-3}$, electron temperature $T_e = 3\,\text{eV}$, and ion temperature $T_i = 0.1\,\text{eV}$ (first ionization energy). The temperature of the neutral gas is assumed to be 25 °C.

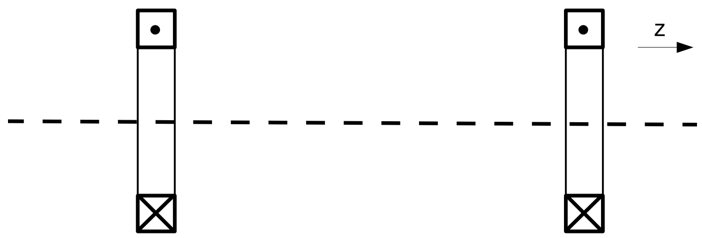
a) Calculate the relative ionization degree of the gas used, α. defined as $\alpha = \frac{n_e}{n_e + n_{gas}}$)

b) Estimate the electron-neutral collision frequency (ν_{en}), assuming a cross section $\sigma = 1000\,\pi\,a_0^2$, where $a_0 = 5.29 \times 10^{-11}\,\text{m}$ is the Bohr radius.

c) Can we consider this gas to be a plasma? Justify your answer.

Exercise 2 - Mirror effect

Consider the following configuration of two cylindrical current-carrying coils:



a) Draw a detailed sketch of the magnetic field lines between and around the coils.

b) Draw a sketch of the magnetic field intensity B_z (along the axis) as a function of the position z.

c) Describe the trajectory of a particle that is initially traveling along the axis with velocity $\mathbf{v} = v_z \mathbf{e_z}$ (i.e., having *no* velocity component orthogonal to the magnetic field).

d) Consider a particle on the z-axis in between the two magnets, having both a velocity component $v_\perp$ perpendicular to the magnetic field, as well as a parallel component $v_{||}$ along the field lines.

Use the adiabatic invariant

$$\frac{mv_\perp^2}{B} = \text{constant}$$

and the conservation of kinetic energy to show that such a particle can be "reflected" by the magnetic field, meaning it reverses its direction along the axis.

e) For a reflected particle, the parallel velocity at the reflection point is $v_{||} = 0$. Use this condition to derive a mathematical relationship on the initial velocity of the particle on the midplane in order for it to be reflected. This defines the so-called loss cone, i.e., the portion of the velocity space that corresponds to particles that are lost from the magnetic mirror confinement.

Exercise 3 - Confinement by a toroidal field

Consider the magnetic field generated by a long, straight current-carrying wire.

We know that in a non-uniform magnetic field, charged particles experience a drift, called the $\vec{\nabla}B$ drift, given by

$$\mathbf{v}_{\nabla B} = \mp \frac{v_\perp^2}{2\omega_c} \frac{\mathbf{B} \times \vec{\nabla}B}{B^2}$$

where the first sign corresponds to negatively charged particles. Additionally, a particle in a curved magnetic field with a radius of curvature $\mathbf{R_c}$ will experience a *curvature* drift (due to the centrifugal force) given by

$$\mathbf{v}_{R_c} = \mp \frac{v_{||}^2}{\omega_c} \frac{\mathbf{R_c} \times \mathbf{B}}{BR_c^2}$$

where ω_c is the particle's cyclotron frequency, which is defined as $\omega_c = \frac{qB}{m}$.

a) Find the expression for the magnetic field $\mathbf{B}$ around an infinitely-long, straight current-carrying wire (hint: use Ampère's law in its differential or integral form). Use this result to derive an expression for the magnetic field gradient $\vec{\nabla}B$ (note that B refers to the *magnitude* of $\mathbf{B}$).

b) Explain why it is not possible to confine a plasma using only a simple toroidal magnetic field. Discuss the effects of particle drifts, such as $\vec{\nabla}B$ drift and curvature drift, on plasma confinement.