

# Nuclear Fusion and Plasma Physics - Exercises

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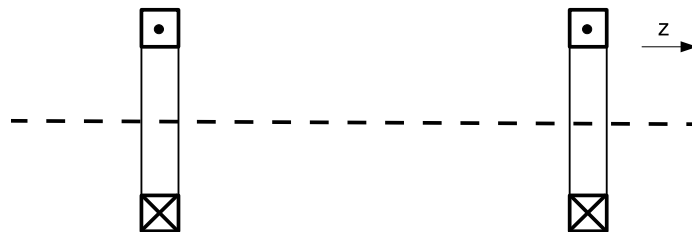
## Exercise 1 - Plasma production

Semiconductor manufacturers use plasmas for surface treatment of materials. In a vacuum chamber of dimensions  $0.5\text{ m} \times 0.5\text{ m} \times 0.5\text{ m}$  an inert gas is partially ionized by radio waves. Consider the case where the gas used is Argon ( $p = 10^{-4}\text{ torr}$ ,  $n_e = 10^{16}\text{ m}^{-3}$ ,  $T_e = 3\text{ eV}$ , and  $T_i = 0.1\text{ eV}$  - first ionisation), and the temperature of the neutral gas is assumed to be  $25\text{ }^\circ\text{C}$ .

- Calculate the relative ionization degree of the gas used,  $\alpha$ . (def.  $\alpha = \frac{n_e}{n_e + n_{gas}}$ )
- Estimate the electron-neutral collision frequency ( $\nu_{en}$ ) assuming a cross section  $\sigma = 1000\pi a_0^2$ , where  $a_0 = 5.29 \times 10^{-11}\text{ m}$  is the Bohr radius.
- Can we consider this gas to be a plasma? Why?

## Exercise 2 - Mirror effect

Consider the following configuration of two cylindrical current carrying coils:



- Draw a sketch of the magnetic field lines.

- b) Draw a sketch of the magnetic field intensity  $B_z$  (along the axis) as a function of  $z$ .
- c) What is the trajectory of a particle that is initially traveling along the axis with velocity  $\mathbf{v} = v_z \mathbf{e}_z$  (i.e. having *no* velocity component orthogonal to the magnetic field)?
- d) Consider a particle on the  $z$ -axis in between the two magnets, having a velocity component  $v_\perp$  perpendicular to the magnetic field, as well as a parallel component  $v_\parallel$ .

Use the adiabatic invariant

$$\frac{mv_\perp^2}{B} = \text{constant}$$

and the conservation of kinetic energy to show that such a particle can be “reflected” by the magnetic field.

- e) For a reflected particle, the parallel velocity at the reflection point is  $v_\parallel = 0$ . Use this to derive a condition on the initial velocity of the particle on the midplane in order for it to be reflected. This defines the so-called loss cone, i.e. the portion of the velocity space that corresponds to particles that are lost from the mirror.

### Exercise 3 - Confinement by a toroidal field

Consider the magnetic field generated by a long current carrying wire.

We know that in a non-uniform magnetic field the particles experience a drift, called the  $\vec{\nabla}B$  drift, given by

$$\mathbf{v}_{\nabla B} = \mp \frac{v_\perp^2}{2\omega_c} \frac{\mathbf{B} \times \vec{\nabla}B}{B^2}$$

where the first sign corresponds to negative charges. Additionally, a particle in a curved magnetic field with radius of curvature  $\mathbf{R}_c$  will have a *curvature* drift (due to the centrifugal force) given by

$$\mathbf{v}_{R_c} = \mp \frac{v_\parallel^2}{\omega_c} \frac{\mathbf{R}_c \times \mathbf{B}}{BR_c^2}$$

where  $\omega_c$  is the particle’s cyclotron frequency  $\omega_c = \frac{qB}{m}$

- a) Find  $\mathbf{B}$  for an infinitely-long current carrying wire (hint: use Ampère’s law in differential or integral form). Use the result to find an expression for  $\vec{\nabla}B$  (notice that  $B$  is the *magnitude* of  $\mathbf{B}$ ).
- b) Explain why you cannot confine a plasma with a simple toroidal field.