Nuclear Fusion and Plasma Physics | **EPFL**
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Problem Set 2

Exercise 1 - Perfect Plasma Power Reactor

Adapted from Freidberg, ex 5.4

In this exercise we examine the economics of an "ideal" fusion reactor. By "ideal" we mean that it has ideal physics performance, so any $\beta < 1$, n and τ_E are achievable. Also, we assume that components of the reactor have no cost, so any set of coils, auxiliary heating systems, etcetera, are available. Essentially, the reactor consists of only a toroidal blanket-and-shield surrounding the first wall. Neutronics show that the required blanket-and-shield thickness is $b = 1.5\,\mathrm{m}$. The first wall material limits the maximum wall loading to[1] $L_{Wmax} = 4\,\mathrm{MW/m^2}$. The electric power output is assumed to be $P_E = 1\,\mathrm{GW}$ at a thermal conversion efficiency of $\eta_t = 0.35$. Assume that each fusion reaction ultimately produces 22.4 MeV of thermal energy (3.5 MeV from the α particles, 14.1 MeV from the neutrons and 4.8 MeV from each fusion neutron when breeding Tritium in the Lithium blanket). Now, define the mass utilization factor as F= reactor mass/electric power. For a fission reactor a typical value of F is 1 ton/MW.

- For a toroidal fusion reactor, determine the values of the major and minor radius (R_0 and a, respectively) that minimize F (i.e., a more economic reactor). Assume that all the produced neutrons pass through the first wall releasing 30% of their energy and are then absorbed by the surrounding blanket-and-shield. The average density of the blanket-and-shield is about $\rho_{blanket} = 3 \times 10^3\,\mathrm{kg/m^3}$.
- Compare the result with the value for a fission reactor.

Exercise 2 - Plasma β and diamagnetism

The plasma β (ratio between plasma and magnetic field pressure, $\beta = \frac{nT}{B_0^2/2\mu_0}$) is a fundamental quantity for the design and optimization of a magnetic fusion reactor. β also measures the plasma diamagnetism, meaning the plasma's ability to reduce the externally applied magnetic field by generating currents that create an opposing magnetic field.

a) Explain how plasma diamagnetism arises by calculating the magnetic field generated by the motion of individual charged particles (electrons and ions) in a Larmor orbit within an infinitely long cylindrical magnetic configuration, and show how this induced field opposes and reduces the externally applied magnetic field.

[1] Note that L_W, the wall power loading in [MW/m^2], is referred to as P_W in [Freidberg]. The notation chosen here is to avoid confusion with P_E which is the electric power in [W]

b) Demonstrate that the diamagnetic field induced by the plasma is proportional to the plasma pressure (nT), i.e. to β.

c) Qualitatively, discuss why β cannot be increased indefinitely.

Reminder: Motion of an electron around a field line: cyclotron frequency $\Omega_j = \frac{q_j B}{m_j}$; Larmor radius $\rho_{L,j} = \frac{v_\perp}{\Omega_j}$, where $v_\perp$ is the velocity perpendicular to B.

Exercise 3 - Violating quasi-neutrality

In the definition of a plasma, it is stated that, although it is an ionized gas made of separate positive and negative charges, it is globally neutral. This means that, under normal conditions, the overall charge density in the plasma is zero, with the number of positive charges (ions) balancing the number of negative charges (electrons).

Considering a typical fusion plasma with electron density $n_e = 10^{20}\,\mathrm{m}^{-3}$, temperature $T = 10\,\mathrm{keV}$, composed of 100% deuterium, and a characteristic size of 1 m,

a) Evaluate the order of magnitude of the force density (force per unit volume) that would be established in the plasma if a small violation of quasi-neutrality is introduced, i.e., if the density of positive charges exceeds the density of negative charges by 1% uniformly throughout the plasma. Clearly state any assumptions you make in your calculation.

b) Compare the result in (a) to other forces that can be present in the plasma volume, such as gravitational forces and pressure gradients. Discuss the relative significance of these forces in the context of plasma confinement.

Hint: Use a simple 1-D model for the plasma, considering only one spatial direction, x, and assume that the densities and temperature are constant and uniform across this direction.
