# Nuclear Fusion and Plasma Physics

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### The two approaches to fusion energy: Basic concepts and status of research

*(Chen, Chapter 9)*

- Inertial
- Magnetic

### A simple approach to the design of a fusion reactor

*(Freidberg, Chapter 5)*

- The fusion reactor concept
- General layout of a magnetic fusion reactor
- Designs goals and parameters
- Engineering constraints
- Nuclear physics constraints
- Integration of the goals and constraints
  - blanket and shield
  - cost and magnet
  - power density and plasma pressure
  - plasma beta and confinement time

# 1 Summary

Last time we saw that, to produce fusion energy at a faster rate than is lost from the plasma, we need the product between the plasma density $n$ and the energy confinement time $\tau_E$ such that $n\tau_E \geq 10^{20}\,\mathrm{m^{-3}s}$, and the plasma temperature $T \geq 10$ keV, or written differently:

$$\boxed{n\tau_E T \geq 10^{21}\,\mathrm{m^{-3}\,s\,keV} \qquad \text{Lawson criterion for break-even}}$$

This was the first estimate telling us in which parameters we should be. It is of course incomplete, as one needs to consider that:

1. the power that is useful to sustain the plasma is only that carried by the $\alpha$'s (the neutrons are 'lost' from the plasma);
2. engineering systems cannot be 100% efficient;

We then defined a **Physics Fusion Gain Factor (Q):**

$$Q = \frac{P_{\text{tot out}}}{P_{\text{in}}} = \frac{P_{\text{fusion}}}{P_{\text{in}}} \begin{cases} \infty & \text{Ignition} \\ 1 & \text{Break}-\text{even} \end{cases} \tag{1.1}$$

and an **Engineering Gain Factor ($Q_E$):**

$$Q_E = \frac{\text{net electric power out}}{\text{net electric power in}} = \eta\, Q - (1-\eta) \tag{1.2}$$

Where $\eta = \eta_e \times \eta_t$ is the product of the efficiency with which we use electricity to heat the plasma, $\eta_e$, and the total efficiency for converting fusion power into electricity, $\eta_t$.

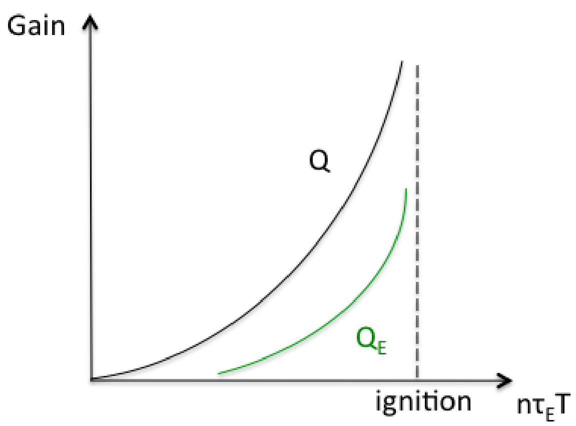


Figure 1: Engineering and physics fusion gain factors.

The first fusion reactors will operate between break-even and ignition, with Q > 10.

We have seen that we need a **plasma** because large energies are required to start nuclear fusion reactions ($\gtrsim$ keV), energies at which ordinary matter cannot exist.

## 2 Fusion energy: basic concepts and status of research

Two approaches to achieving controlled nuclear fusion, in addition to gravity (as in stars), are:

1) Inertial confinement: $n \sim 10^{31}\,\text{m}^{-3}$; $\tau_E \simeq 10^{-11}\,\text{s}$

2) Magnetic confinement: $n \sim 10^{20}\,\text{m}^{-3}$; $\tau_E \sim 1\,\text{s}$

1. **Inertial confinement** (Figure 2): Little pellets containing D-T compressed ($10^{31}\,\text{m}^{-3} \equiv$ 1000 times denser than ice!) and heated ($\sim$ 10 keV) before particles can escape ('inertia' effect).

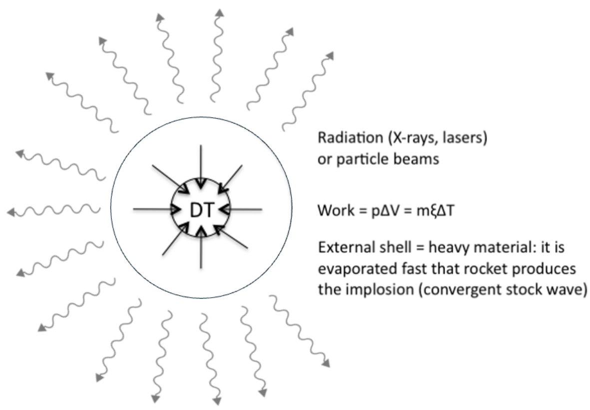


Figure 2: Pellet containing D-T for inertial fusion.

In the process of achieving inertial confinement fusion, several **physics problems** must be addressed to ensure the successful compression and heating of the fuel pellet:

1. **Avoid heating the core** before the shock wave arrives and compresses it. This requires careful pellet design with an internal 'screen' to protect the core and ensure that the heating occurs at the right moment.
2. **Hydrodynamic instability**, which involves several aspects:
   - **Pellet symmetry**
   - **Alignment of beams**: The laser or ion beams must be precisely aligned to target the pellet symmetrically, which is essential for even compression.
   - **Indirect drive**: By using an external structure, such as a hohlraum, to convert laser energy into X-rays that uniformly compress the pellet, asymmetries in the direct drive approach can be mitigated.
3. **Beams cannot penetrate into the pellet**, which presents several challenges:
   - **No electrons, no light ions**: These particles are not suitable for penetrating the pellet due to their low mass and high deflection rates, which prevent them from delivering energy effectively to the core.

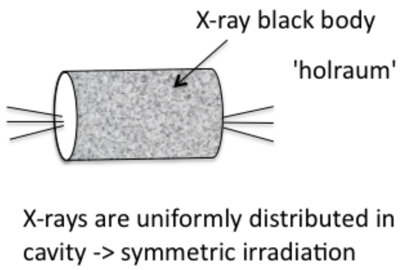


Figure 3: External shell to get uniformly distributed radiation.

- **Photons (lasers)**: Lasers face the challenge of small efficiency when delivering energy to the pellet's core. The energy from the lasers can be absorbed or scattered by the outer layers of the pellet, reducing the amount that reaches the core.
- **Heavy ions**: Heavy ion beams face the issue of small currents, which limits the amount of energy that can be delivered to the pellet.

2. **Magnetic confinement**: is achieved by using the fact that charged particles tend to gyrate around magnetic field lines. Plasma must not touch any material surface directly.

A closed field line configuration is needed, as particles tend to move freely **along** magnetic field lines. (We will analyse particle motion in given B-fields in the next lecture.)

**Note 2.2.1**: the diamagnetic character of the plasma, determined by the particle motion around field lines, sets an ultimate limit for the plasma confinement.

$B_{\text{plasma}} < B_{\text{external}}$: This limit can be expressed (as we will see in the next lecture) in terms of the parameter $\beta$.

$$\boxed{\beta \equiv \frac{\text{plasma pressure}}{\text{magnetic field pressure}} = \frac{nT}{B^2/2\mu_0}} \tag{2.1}$$

Here, plasma pressure ($p = nT$) is what produces fusion power, where $n$ is the plasma density and $T$ is the temperature. Magnetic field pressure ($\frac{B^2}{2\mu_0}$) represents the 'investment' needed to confine the plasma, where $B$ is the magnetic field strength and $\mu_0$ is the permeability of free space.

If $\beta \to 1$, the plasma reduces the magnetic field too much and the confinement becomes impossible. Therefore, $\beta$ is a measure of how efficient a confinement system is.

**Note 2.2.2**: In present devices, $\beta$ is limited by plasma instabilities to values much less than 1 ($\beta \ll 1$).

# 3 A simple approach to the design of a (magnetic) fusion reactor (Freidberg, Ch. 5)

For **ignition** we have seen that we need $\begin{cases} n\tau_E T & \gtrsim & 8 \text{ atm s, or } 5\times10^{21}\ \text{m}^{-3}\ \text{keV s} \\ T & \simeq & 15\,\text{keV} \end{cases}$

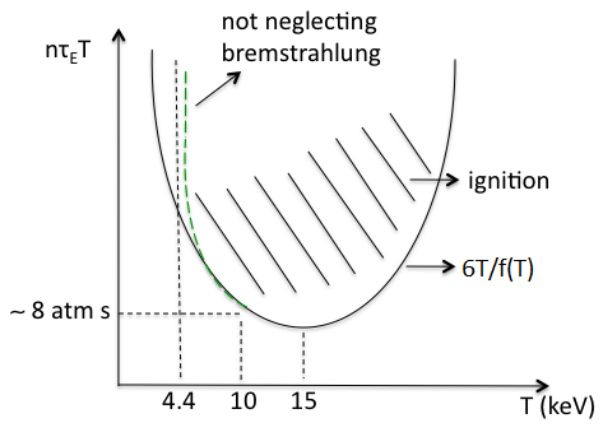


Figure 4: n$\tau_E$T parameter and ignition curve.

Although we don't necessarily need to achieve ignition, we need to approach these values (for a 'decent' fusion gain Q that is of interest for a reactor).

The **question** is what is the best combination of p, $\tau_E, T$? We know roughly that it is reasonable to have $n \sim 10^{20}\,\text{m}^{-3}$, $\tau_E \sim 1\,\text{s}$, but can we be more precise?

What **size** and what B-field should we use? (These are crucial parameters for economic feasibility)

In general, we can try this optimisation even before we know what plasma is.

## 3.1 Concept of magnetic fusion reactor: simplified geometry (toroidal shape with circular cross-section)

What **parameters** describe a reactor?

1. Geometry: $a$, $R_o$, $b$, $c$
2. Plasma:
   - $\tau_E$: confinement time;
   - $n$: plasma density;
   - $T$: plasma temperature;

- $p = nT$: plasma pressure (strictly speaking, $p_j = n_j T_j$ for each species $j = 1, 2, \ldots$);
- $\beta = \frac{nT}{B^2/2\mu_0}$: a measure of the efficiency of the confinement system, also related to the diamagnetic properties of the plasma;

3. Fusion power density $p_f$: power produced per unit volume of plasma.

4. Magnetic field $B$: external magnetic field used to confine the plasma.

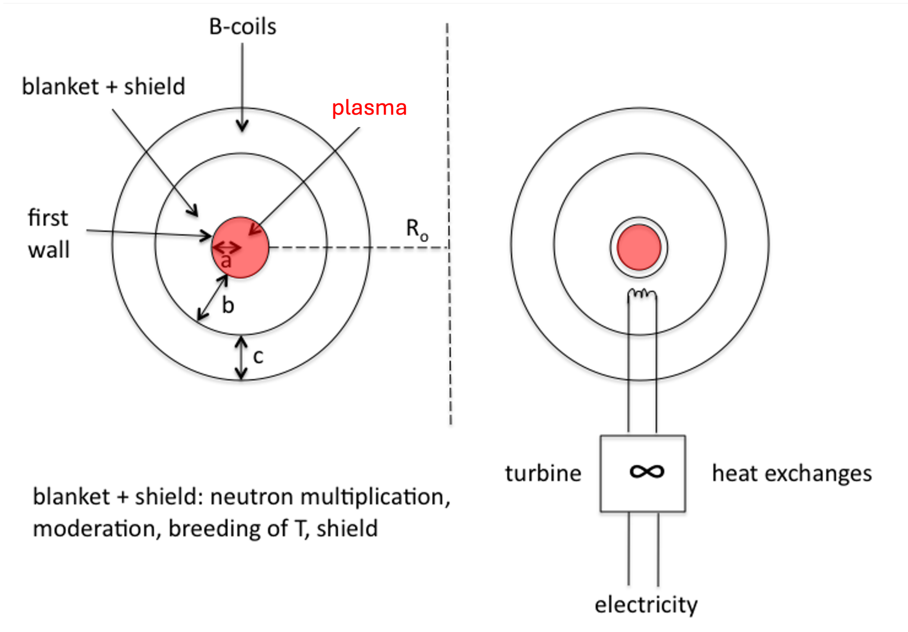


Figure 5: Simplified structure of a toroidal magnetic fusion reactor.

## 3.2 Design goals

- Minimize the cost of a reactor, to minimize the cost of electricity, as in fusion the cost of fuel should be negligible.
- Minimize requirements on $\tau_E$ and $\beta$ to make it less tough for plasma physics.

## 3.3 Engineering constraints

1. **Power output** ($P_E$): The reactor should aim for a power output of around 1 GW to be economically viable.

2. **Wall loading** ($L_W$): The heat load on the reactor walls should be limited to avoid damage. This is typically kept below a few MW/m$^2$.

   - Plasma losses: These include thermal conduction and radiation losses. Effective design choices in geometry and materials can help mitigate these losses.

   - Neutrons: Neutron flux is an isotropic and more severe constraint as it can cause significant damage to the reactor walls.

   As a limit value, we can assume $L_W^{\max} = 4$ MW/m$^2$ (Freidberg).

3. **Magnets**: The superconducting magnets must operate below certain limits of current density (J), temperature (T), and magnetic field intensity (B) to remain superconducting.

   > **Example:** niobium-tin ($Nb_3Sn$) used in ITER can sustain magnetic fields of 10-15 T. For practical purposes, we take $B_{\max} = 13$ T.

4. **Magnetic field stress**: The stress on the support structure, due to the force from the product of current density and magnetic field ($J \times B$), must be limited. Typically, this stress should not exceed $\sigma_{\max} \approx 300$ MPa.

## 3.4 Nuclear physics constraints

- **Reaction rate** is characterized by $\langle \sigma v \rangle_{DT} \approx 10^{-22}\,\mathrm{m^3/s}$ at 15 keV, dictates the conditions necessary for sustained fusion reactions.
- **Blanket and shield:** These components play multiple roles, including:
  - Neutron multiplication: to sustain the reaction.
  - Slowing down of neutrons: fast neutrons need to be slowed to thermal speeds to breed tritium from lithium.
  - Tritium breeding: necessary for maintaining the fuel supply.
  - Shielding: protects the reactor structure from neutron damage.

  Moderating materials together with lithium are used in the blanket to achieve these goals. The size of the blanket is dictated by the mean free path of neutrons for these processes.

## 3.5 How to put these constraints together, and what are their consequences?

The integration process involves:

- Neutron cross-sections: Determine the required blanket thickness to achieve effective neutron multiplication and shielding.
- Output power and maximum wall loading: Influence the major radius of the reactor, balancing power production and structural integrity.
- Cost and magnet constraints: Affect the thickness of the magnets and coils, as well as the minor radius of the plasma, balancing economic and engineering feasibility.
- Power balance: Achieving the desired energy confinement time ($\tau_E$) and plasma pressure is essential for maintaining a steady fusion reaction.

- Plasma pressure and magnetic field: Influence the value of $\beta$, a measure of the efficiency of the confinement system. High plasma pressure and appropriate magnetic field strength are necessary for optimal reactor performance.

Let's assess these implications in a simplified way one by one.

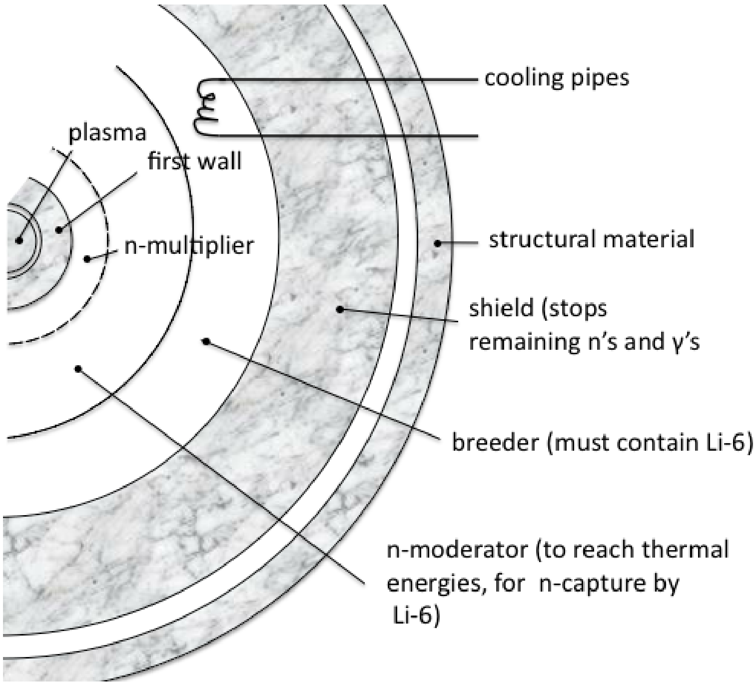


Figure 6: Schematic drawing of a blanket in a fusion reactor.

> **Note 2.3.1**: Figure 6 is schematic. In reality, the different layers can and will be combined.

> **Note 2.3.2**: In the breeder, $Li^7$ also reacts (endothermically) with neutrons, but with a very small cross-section. In fact, even if $Li^6$ is only $\sim 7\%$, it may not be necessary to enrich Li with it.

### 3.5.1 Thickness?

The thickness needed for each process is equivalent to the mean free path for such process, i.e. 1/(target number density × cross section):

- Neutron multiplication, for example for Be:

$$\lambda_{\text{mult}} = \frac{1}{n_{\text{Be}}\sigma_{\text{mult}}} \sim \frac{1}{1.2 \times 10^{29}\text{m}^{-3} \times 0.6 \times 10^{-28}\text{m}^2} \sim 0.13\,\text{m}$$

- Slowing down of fast neutrons:

$$\lambda_{\text{sd}} = \frac{1}{n_{\text{Li}}\sigma_{\text{sd}}} \simeq \frac{1}{4.5 \times 10^{28}\text{m}^{-3} \times 10^{-28}\text{m}^2} \simeq 0.2\,\text{m}$$

- Tritium breeding with slowed down neutrons, for example at $E_n = 2.5 \times 10^{-2}$ eV and Li in its natural composition (7.5% $Li^6$):

$$\lambda_{\text{br}} = \frac{1}{n_{Li^6}\sigma_{\text{br}}} \sim \frac{1}{0.075 \times 4.5 \times 10^{28}\text{m}^{-3} \times 950 \times 10^{-28}\text{m}^2} \sim 0.3\,\text{cm}$$

- Thickness of shield/breeder to reduce the neutron flux by 100 (i.e. to have 99% of the neutrons that have undergone a breeding reaction, after having slowed down):

$$I = I_0 \exp\left(-\frac{x}{\lambda_{\text{tot}}}\right) \sim I_0 \exp\left(-\frac{x}{\lambda_{\text{sd}}}\right); \qquad \frac{I}{I_0} \sim 0.01 \Rightarrow x_{\text{shield}} \sim \lambda_{\text{sd}} \ln(100) \sim 1\,\text{m}$$

As the different layers are combined, we can consider total thickness of blanket and shield $b \sim \lambda_{\text{mult}} + \lambda_{\text{sd}} + \lambda_{\text{br}} + x_{\text{shield}} \simeq 1.2$ m.

### 3.5.2 Cost and magnet

1. **Cost**

   The cost of electricity is approximately equal to the capital cost. We assume that the cost is proportional to the volume of 'complex' systems. Therefore, the quantity to **minimize** is $\frac{\text{cost}}{\text{power}} \sim \frac{\text{volume}}{\text{power}}$ (volume of coils and blanket only, **not** of plasma), which can be expressed as:

$$\frac{\text{volume}}{\text{P}_\text{E}} = \frac{2\pi R_0 \pi[(a+b+c)^2 - a^2]}{\text{P}_\text{E}}; \text{ with}$$

$$\text{P}_E = \frac{1}{4}\eta_t \left(\Delta E_\alpha + \Delta E_n + \Delta E_{Li}\right) n^2 \langle\sigma v\rangle_{DT} \, 2\pi R_0 \pi a^2 \qquad (\Delta E_{Li} = 4.8 \text{ MeV})$$

   $\eta_t$ is the efficiency with which fusion power is transformed into electricity. To get $R_0$ we use the constraint from the wall, assuming only the isotropic part, the neutrons.

   $\underbrace{L_W^{\text{max}}}_{\text{max wall loading}}$ × surface area of plasma = $\text{P}_n$ (total neutron power)

But $P_n = \frac{P_E}{\eta_t}\frac{\Delta E_n}{\Delta E_\alpha + \Delta E_{Li} + \Delta E_n}$. Therefore, $L_W^{\max} 2\pi a 2\pi R_0 = \frac{P_E}{\eta_t}\frac{\Delta E_n}{\Delta E_\alpha + \Delta E_{Li} + \Delta E_n}$

$$\implies \qquad R_0 = \frac{P_E}{\eta_t}\frac{\Delta E_n}{\Delta E_\alpha + \Delta E_{Li} + \Delta E_n}\frac{1}{4\pi^2 a\, L_W^{\max}} \underbrace{\simeq}_{\eta_t \simeq 0.4} 0.04\frac{P_E}{a\, L_W^{\max}}$$

$$\text{So, } \frac{\text{cost}}{\text{power}} \sim \frac{\text{volume}}{P_E} = \frac{2\pi^2}{P_E}[(a+b+c)^2 - a^2]\, 0.04 \frac{P_E}{a\, L_W^{\max}}$$

$$\simeq 0.8\frac{(a+b+c)^2 - a^2}{a\, L_W^{\max}} \qquad \text{(quantity to minimize)}$$

$b$ was already determined. What about $a$ and $c$? We still have **two** free parameters, but we can fix $c$ from considerations on the magnet.

> **Note 2.3.3**: Naturally, being able to increase $L_W^{\max}$ would reduce cost ($\rightarrow$ materials studies).

2. **Magnet**

The coil thickness should be minimised, compatibly with the **stress** due to $\vec{J} \times \vec{B}$ force. For tensile stress (see Freidberg):

$$c = \frac{2\xi}{1-\xi}(a+b) \quad \text{with} \quad \xi = \frac{B_c^2}{4\mu_0 \sigma_{\max}} \quad (B_c \equiv \text{magnetic field inside the solenoid})$$

We can set $B_c = B_{\max}$, considering that the lower the need for $\beta$, the 'easier' it will be for plasma physics.

$$\Rightarrow \begin{cases} B_c = B_{\max} \simeq 13\,\text{T} \\ \sigma_{\max} \simeq 300\,\text{MPa} = 3 \times 10^8\,\text{N/m}^2 \end{cases} \quad \Rightarrow \quad \xi \simeq 0.1$$

We have then:

$$c = \frac{2 \times 0.1}{1 - 0.1}(a+b) = \frac{2}{9}(a+b)$$

The quantity to minimize becomes:

$$\begin{aligned}
\frac{\text{volume}}{P_E} &= \frac{0.8}{a\, L_W^{\max}}\left\{\left[a + b + \frac{2}{9}(a+b)\right]^2 - a^2\right\} \\
&= \frac{0.8}{a\, L_W^{\max}}\left[(a+b)^2\left(\frac{11}{9}\right)^2 - a^2\right] \\
&= 0.8 \times \left(\frac{11}{9}\right)^2 \frac{1}{a\, L_W^{\max}}\left[(a+b)^2 - \left(\frac{9}{11}\right)^2 a^2\right] \\
&= \frac{1.2}{L_W^{\max}}\left(0.33a + \frac{b^2}{a} + 2b\right) \\
&= f(a) \qquad (3.1)
\end{aligned}$$

The minimal value of $\frac{\text{volume}}{P_E}$ can be found by minimizing $f(a)$:

$$\frac{df(a)}{da} = 0 \quad \Rightarrow \quad 0.33 - \frac{b^2}{a^2} = 0 \quad \Rightarrow$$
$$a = \frac{b}{\sqrt{0.33}} \simeq 1.74b \quad \boxed{a = \frac{b}{\sqrt{0.33}} \simeq 1.74b} \tag{3.2}$$

Since $b \simeq 1.2\,\text{m}$:

$$\boxed{\text{As } b \simeq 1.2\,\text{m} \quad \Rightarrow \quad \begin{cases} a \simeq 1.74\,b \simeq 2\,\text{m} \\ c \equiv \frac{2}{9}(a+b) = 0.7\,\text{m} \end{cases}} \tag{3.3}$$

The minimal value of $\frac{\text{volume}}{P_E}$ thus becomes:

$$\text{Min}\left(\frac{\text{volume}}{P_E}\right) \simeq \frac{1.2}{L_W^{\max}}\left(0.33 \times 2 + \frac{1.2^2}{2} + 2 \times 1.2\right) \simeq \frac{4.5}{L_W^{\max}} \underbrace{\simeq}_{L_W^{\max} \simeq 4\text{MW/m}^2} 1\,\text{m}^3/\text{MW}$$

Also:

$$R_0 \simeq \frac{0.04}{a}\frac{P_E}{L_W^{\max}} \simeq \frac{0.04 \times 10^9}{2 \times 4 \times 10^6} \simeq 5\,\text{m} \quad \Rightarrow \quad \begin{cases} \text{Plasma volume}: \ = 2\pi R_0 \pi a^2 \simeq 400\,\text{m}^3 \\ \text{Plasma surface area}: \ = 2\pi R_0 2\pi a \simeq 400\,\text{m}^2 \end{cases}$$

### 3.5.3 Total power density in the plasma

$$\frac{\text{P}_\alpha + \text{P}_n}{\text{volume}} \simeq 5\ \text{MW/m}^3 \text{ (small compared to fission reactor)} \left(= \frac{\Delta E_\alpha + \Delta E_n}{\Delta E_\alpha + \Delta E_n + \Delta E_{Li}} \times \frac{\text{P}_E}{\eta_t\,\text{volume}}\right)$$

### 3.5.4 $\beta$

The minimum requirement for ignition is $P\tau_E \simeq 8$ atm s; remember that to reach the minimum of the ignition curve, we need a temperature $T \cong 15$ keV and a plasma density $n \cong 10^{20}$ m$^{-3}$, which gives an energy confinement time $\tau_E \approx 1$ s.

$$\beta = \frac{P}{B_0^2/2\mu_0}$$

We cannot take $B_0 \equiv B_c$, as this $B_0$ value is the value in the plasma, and $B_c$ is inside the magnet. Since $B \propto \frac{1}{R}$ (figure 7), and we know the distances ($R_o$, $a$, ...), we can calculate [1] $B_0$ and $\beta$:

$$\boxed{B_0 \sim 5\ \text{T}} \quad \Rightarrow \quad \boxed{\beta \simeq 8\ \%}$$

This value of $\beta$ is quite high, but we are not far from it in tokamaks.

[1] $B(R) = \frac{\text{const}}{R}$; at $B(R = R_0 - a - b) = \frac{\text{const}}{R_0 - a - b} = B_{\max} \Rightarrow B(R = R_0) = B_{\max}\frac{(R_0 - a - b)}{R_0}$.

This exercise has given to us a very simplistic, yet not too far from reality, estimate of the characteristics of an ideal fusion reactor. So, where is plasma physics?

We must:

1. create a plasma,
2. confine it, with its energy, for a macroscopic time ($\sim$ 1 s),
3. heat it to $\sim$ 15 keV
4. keep it in a stable equilibrium with $\beta \sim 8$ %,

There are many other aspects of plasma physics that enter into the functioning of a reactor, but at least these provide a general frame, and already a strong motivation to study it.

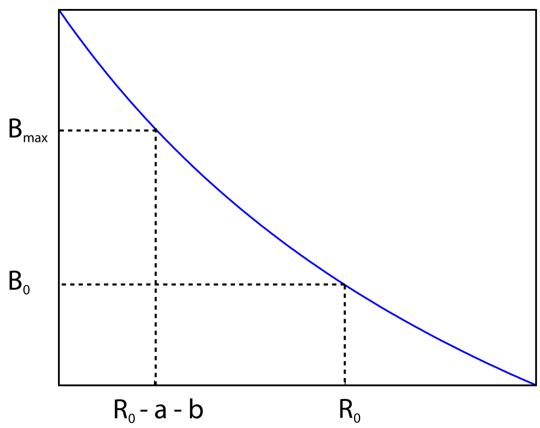


Figure 7: Toroidal magnetic field as a function of R.